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Cost effective management of animal and plant disease incursions

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Managing the risks of the incursion and spread of plant and animal diseases is a key component of Australia's agricultural policy. Establishing the best mix of preventative, detection, control and eradication instruments in place presents a complex problem. The level of risk associated with the likelihood and costs of disease incursions is high, and as a consequence so are the costs and benefits of a given management strategy. This risk may have a considerable impact on the choice of an optimal policy response.

In this paper, a stochastic control framework is developed to examine the impact of risk on the choice of an optimal disease management strategy for a hypothetical animal or plant disease incursion. The key sources of risk examined are the likelihood of incursion and the rate of spread. In addition, it is recognised that the likelihood that a disease is detected amongst the population is also uncertain. The management instruments considered include preventative measures, such as border controls, measures to restrict the spread of a disease, such as quarantine, and measures to increase the probability of detection. The model is solved numerically using collocation techniques.

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Introduction

The objective of this paper is to develop an extended framework estimating the net benefits of alternative plant and animal disease management strategies. The hypothesis examined is that risk about different management decisions is an important consideration in determining an optimal response to a disease threat or existing incursion. Risk in disease management can arise from many sources. For example, the likelihood of introduction and detection may be relatively difficult to predict, as may be the rate of spread and the costs of disease. These can all have a bearing on the appropriate choice of control measure and level of application.

To explore this problem an epidemiological model of a hypothetical disease outbreak is embedded within a stochastic optimal control framework. Three controls are specified: first, a border control which limits the likelihood of an incursion; second, a detection and treatment program; and third, a control which inhibits the spread of the disease. Given the initial state of the population, the expected costs of an incursion are then minimised. The model is solved numerically under a number of scenarios using collocation techniques.

Theoretical framework

The framework is based on a managed animal population or area of crop production. Abstracting from the animal or plant reproduction dynamics, let there be two observable states:

- the proportion of the population in which a disease or pest incursion has not been detected (S_{nd}); and
- the proportion of the population in which a disease has been detected (S_d).

There are correspondingly four actual population states:

- the proportion of the population which is disease or pest free (S_{ni});
- the proportion of population which is infected or infested but not detected (S_{ind});
- the proportion of the population which is infected or infested and detected (S_{id}); and
- the proportion of the population which is not infected but has tested positive.

While the last state is of some potential interest it will not be considered further here (ie $S_d = S_{id}$). Three controls are considered:

- c_1 – a border control applied to the general population to reduce the probability of a disease incursion;
- c_2 – a control applied to increase the probability that an infestation will be detected and subsequently treated; and

- c_3 – a non-targeted control applied to the whole of the population which reduces the spread of a disease.

In designing an optimal response strategy, the only observable state on which to base a control decision is the extent to which a disease has been detected. Given that detection is uncertain there is a corresponding degree of risk about the true and likely future state of the population. In the case where little is known about a disease it is not possible to characterise this risk and only a reactive strategy may be pursued. However, given some understanding of the epidemiology of a disease it maybe possible to characterise both the expected progress of a disease and the risk associated with that progress. The use of a stochastic optimal control framework to model management options for a disease incursion may then provide some useful insights into the design of a cost effective strategy. A good general reference that outlines the scope of the stochastic control framework is Mangel (1995).

Model specification

There are a number of mathematical models commonly used to represent the progression of a disease as discussed by Kranz (1974). This analysis is limited to the spread of a disease over time for which the most common model is a logistic growth equation (Madden and Campbell 1986). Here the discrete form of the logistic progression curve is used:

(1) $$S_i(t+1) = S_i(t) + \alpha X(t)[1 - X(t)]$$

where

$$X(t) = \min[S_i(t) + \sigma(t), 1]$$

and S_i is the total proportion of the population infected, α is a parameter and σ_i is a non-negative stochastic disturbance representing an incursion from outside the population. The stochastic disturbance is drawn from a winsorised normal distribution.

The border control is introduced as a modification to the stochastic incursion process:

(2) $$z(t) = \sigma(t)\left[1 - \frac{\beta_{1\max}}{1 + \exp\left(\frac{\beta_{11} - c_1}{\beta_{12}}\right)}\right]$$

where β_{1max} is the maximum detection rate expressed as a proportion, β_{11} is the inflection point of the response function and β_{12} determines the curvature of the response function about the point of inflection. Similarly, the infected proportion of population which can be treated is given by:

$$S_d(t) = S_i(t)\left[1 - \frac{\beta_{2\max}}{1+\exp\left(\frac{\beta_{21} - c_2 + \sigma_d}{\beta_{22}}\right)}\right] \tag{3}$$

where σ_d is a stochastic disturbance, drawn from a normal distribution, representing the risk associated with detection. The control function for the rate of spread is given by:

$$\alpha(t) = \alpha\left[1 - \frac{\beta_{3\max}}{1+\exp\left(\frac{\beta_{31} - c_3}{\beta_{32}}\right)}\right] \tag{4}$$

The modified equation specifying the spread of the disease through time is:

$$S_i(t+1) = S_i(t) + \alpha(t)X(t)[1 - X(t)] \tag{5}$$

where

$$X(t) = \min[S_i(t) - S_d(t) + z(t), 1]$$

Without loss of generality the annual cost of an outbreak affecting the entire population can be normalised to one and a linear cost function specified as:

$$Cost(t) = S_i(t) + \sum_{i=1}^{3} w_i c_i + w_d S_d(t) \tag{6}$$

where w_i is the per unit cost of the *i*th control and w_d is the treatment cost per decimal per cent of the population infected.

The problem is then to minimise the expected cost of a disease outbreak through the choice of the controls:

(7) $$\min_{c} E\sum_{t=0}^{\infty}\left[S_i(t)+\sum_{i=1}^{3}w_i c_i + w_d S_d(t)\right](1+r)^{-t}$$

subject to

$$S_i(t+1) = S_i(t)+\alpha(t)X(t)[1-X(t)]$$

$$S_d(t) = S_i(t)\left[1-\frac{\beta_{2\max}}{1+\exp\left(\frac{\beta_{21}-c_2+\sigma_d}{\beta_{22}}\right)}\right]$$

$$S_i(0) = S_i^0$$

where

$$X(t) = \min[S_i(t)-S_d(t)+z(t),1]$$

$$z(t) = \sigma(t)\left[1-\frac{\beta_{1\max}}{1+\exp\left(\frac{\beta_{11}-c_3}{\beta_{32}}\right)}\right]$$

$$\alpha(t) = \alpha\left[1-\frac{\beta_{3\max}}{1+\exp\left(\frac{\beta_{31}-c_3}{\beta_{32}}\right)}\right]$$

E is the expectation operator and *r* the discount rate.

Solution method

The approach adopted here is to find an approximate numerical solution to Bellman's equation for the stochastic control problem through collocation. The technique is discussed in detail by Von Stryk (1993), Miranda and Fackler (1997), Beare and Bell (1999). Bellman's equation for the problem given by equation (7) is:

(8) $$V[S_i(t)] = \min_{c} E\left\{S_i(t)+\sum_{i=1}^{3}w_i c_i + w_d S_d(t)+(1-r)V[S_i(t+1)]\right\}$$

A Galerkin approximation is used to replace the Bellman functional equation (8), such that:

(9) $$V(S_i) \approx \Phi S_i c$$

where Φ is an n dimensional interpolation matrix and c is an n-vector of basis coefficients which are to be determined. The discrete approximation to Bellman's equation is then:

(10) $$V[S_i(t)] = \min_c E\left\{ S_i(t) + \sum_{i=1}^{3} w_i c_i + w_d S_d(t) + (1-r)\Phi S_i(t+1)c \right\}$$

A search algorithm is employed to find values for the vector of basis coefficients which solves equation (10) at each of the n nodes. Equation (9) is then utilised to interpolate the solution for states falling between nodes. Random sampling through Gaussian quadrature was used to incorporate the stochastic components of the model (Judd 1997; Rust 1997). In a Gaussian quadrature scheme each continuous random variable in the state transition function is replaced with a set of discrete approximates, the value of which the variable takes on with an assumed known probability. The full procedure is specified in the appendix.

Experimental design and results

A baseline solution to the model is developed in which all three controls are cost effective. That is, even if only one of the three controls were available it would still be cost effective to use on its own. In addition, the costs in the baseline were chosen so that border controls are relatively low cost.

The assumed values for the model parameters in the baseline scenario are given in tables 1 and 2. While the parameter values are chosen arbitrarily and are only for illustrative purposes, the disease control function is fairly linear about the point of inflection.

Table 1: Non–stochastic parameter values

Parameter		Value
α		1.0
β_{imax}	i=1,3	1.0
β_{i1}	i=1,3	10
β_{i2}	i=1,3	5
w_i	i=1,3	0.005
w_d		10

Table 2: Stochastic parameters

Disturbance	Mean	Standard Deviation	Winsorised Percentile
σ_i	0	0.05	50
σ_d	0	5	na

na: not applicable.

The baseline scenario is then compared with two counter experiments. In the first counter experiment the costs of detection and treatment are higher than the direct cost of the disease, excluding the cost due to the increased spread of the infection. Specifically, the cost of treatment was increased from 10 to 25 units. The net present cost if the current population remained totally infected is, only 20 units at a discount rate of 5 per cent.

In the second counter experiment the maximum efficacy of all the controls is reduced from 100 per cent to 90 per cent. That is, at most 90 per cent of incursions can be intercepted at the border, 90 per cent of the infected population can be detected and the rate of spread can be reduced by a maximum of 90 per cent.

Discussion

The phase diagrams for the base and two counter experiments are shown in figure 1. The initial proportion of the population infected is plotted against the expected proportion of the population infected in the next period, given the optimal choice of controls.

In the base case the solution is for complete control. In the first counter experiment where the costs of treatment are high, a strategy of complete control is employed at low levels of initial infection. However, at higher levels of infection, the rate of progression of the disease is not fully arrested. In the second counter experiment, with reduced efficacy, a strategy of maximum control is employed but again the progress of the disease is not fully controlled.

The choices of controls under the three scenarios are shown in figures 2a through 2c. In the baseline scenario the level of effort spent on detection and control increases with the level of infection. This is largely due to the fact that effort is related to the proportion detected. The number detected therefore increases as the percentage of infected members increases. The optimal level of border controls remains relatively constant. This reflects the fact that nearly all of the infected population will be treated and the only new source of infection is through an incursion.

Figure 1: Phase diagram

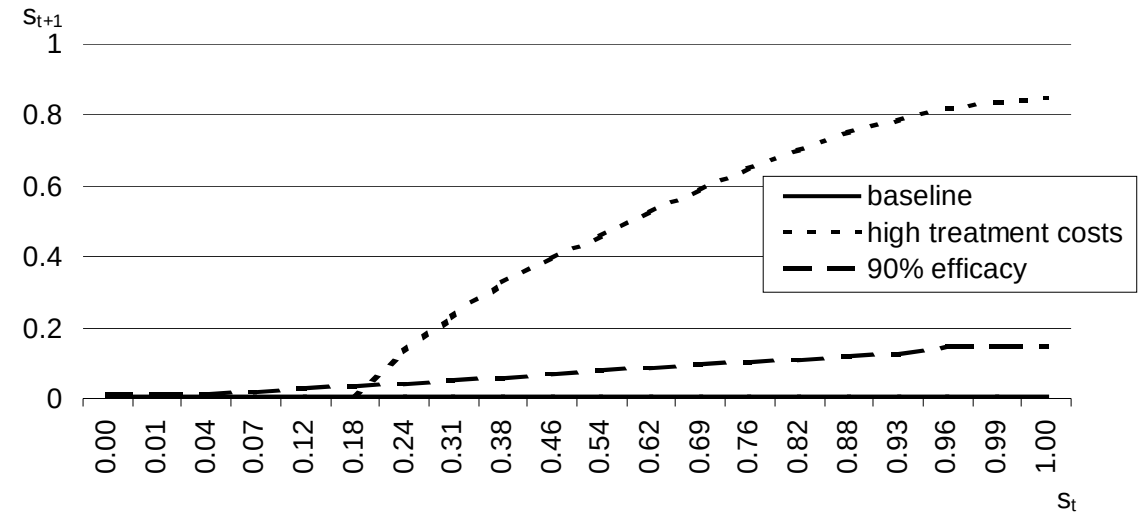
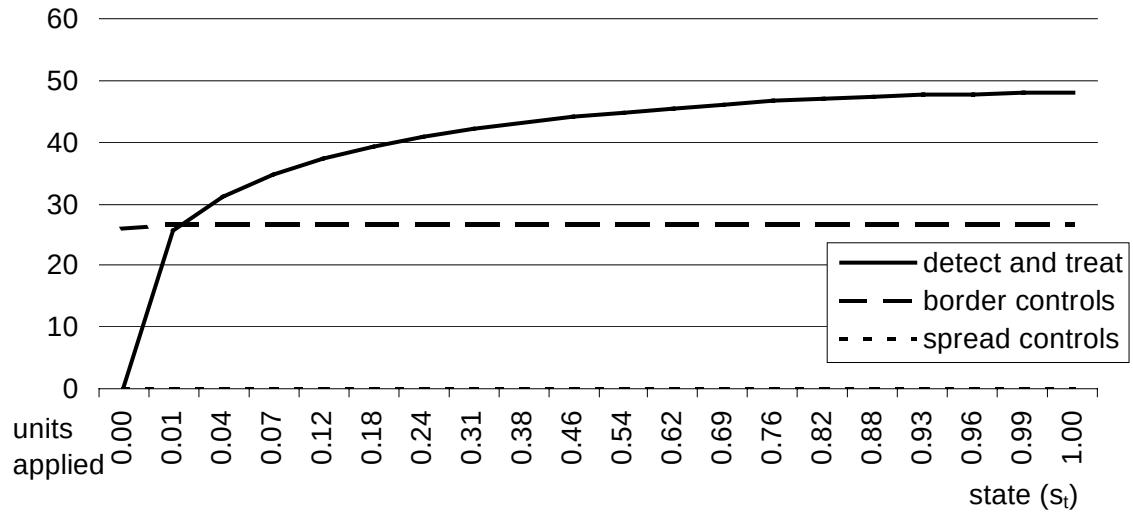


Figure 2a: Controls in baseline scenario

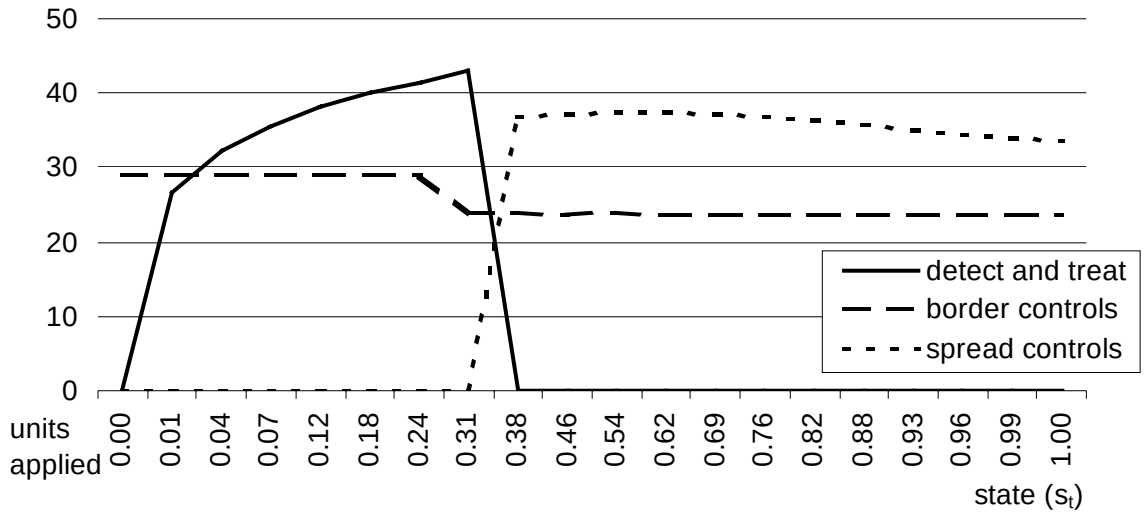


In the counter experiment where treatment costs are high, detection and treatment is employed only at low levels of infection (figure 2b). Detection and treatment control is then replaced by an effort to reduce the rate of spread of infection. This, along with the effort on border controls, eventually declines if the disease continues to spread through the population. It should be noted that given the functional form and random variation

in detection, some proportion of the infected population is detected even with no expenditure on detection. It is assumed that all detected members of the population are treated even if it is not cost effective.

When the efficacy of the controls is reduced all three control measures are again employed (figure 2c). The use of detection and treatment and border controls increase with the level of infection while the optimal use of border controls declines. The increased use of detection and treatment has been discussed previously. The increase use of spread reduction control reflects the fact that with the reduced effectiveness of detection and incursion controls, the rate at which the disease can be expected to spread has increased.

Figure 2b: Controls in high treatment cost scenario



The expected cost of a given level of disease incursion for each scenario is shown in figure 3. When compared to the baseline, the reduction in efficacy of the controls shifts the cost curve upward by a relatively constant amount. There is also a shift in the strategies employed. That is, spread controls are introduced when efficacy is reduced, unlike in the baseline scenario where the introduction of spread controls were not optimal at any population state. However, the net effect is roughly equivalent to an equal percentage increase in the cost of all controls. The high cost of treatment scenario produces a significant shift in the cost profile, with cost escalating as it is no longer efficient to fully control the spread of the disease.

Figure 2c: Controls in 90% efficacy scenario

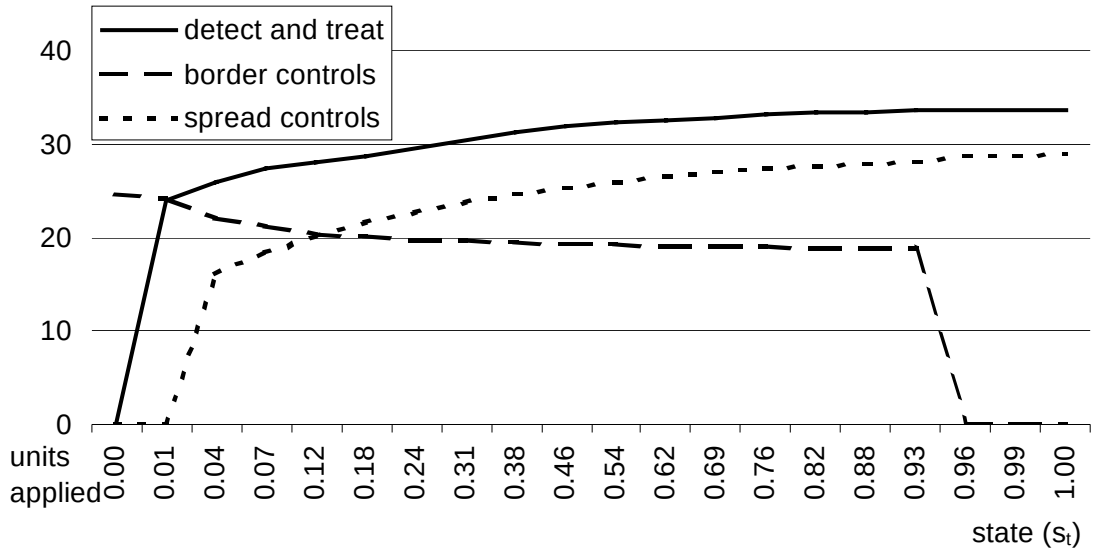
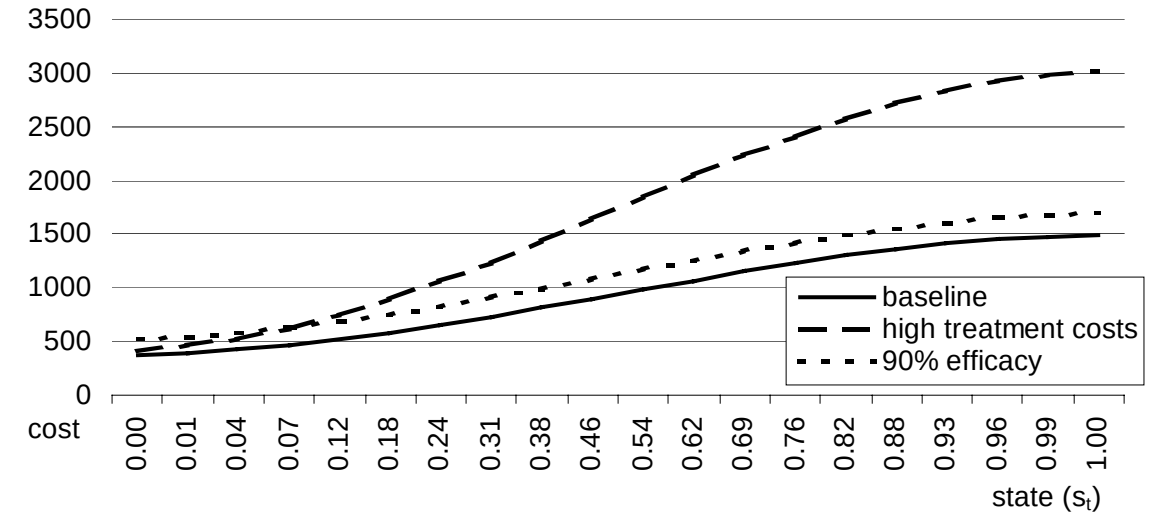
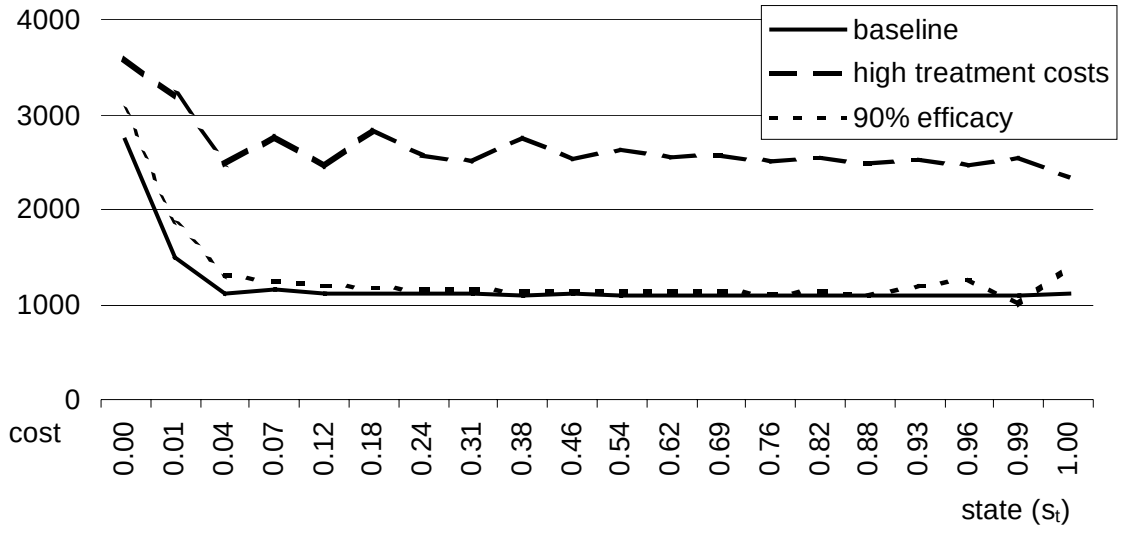


Figure 3: Expected costs of an optimally managed disease incursion



The shadow costs for each state are shown in figure 4. Shadow costs are high when the initial state of the population is relatively disease free. A small increase in the level of infection imposes a substantial increases in either disease or control costs. Under the assumed conditions of spread, once the diseases is established the shadow costs decline slowly.

Figure 4: Shadow costs of an optimally managed disease incursion



Concluding remarks

The problem of formulating an optimal response to a disease incursion is in theory well suited to stochastic optimal control. The use of numerical techniques such as collocation make such an approach operational. The results presented here suggest that relatively complex management models which integrate the dynamics of both epidemiology and economics can be specified and solved.

The results highlight the importance of the cost of disease management options in developing a disease incursion strategy. Further, it should be anticipated that strategies may need to change given the progression of an incursion because the cost effectiveness of treatment changes with the status of the disease. For example, in the scenario where detection and treatment costs were high it was only cost effective to use the control while the levels of infection are low. When the proportion infected in the initial period was 38 per cent or higher it was no longer cost effective to use this control. From a practical perspective, the usefulness of the framework will depend on how well the incursion, spread and control of a disease of actual interest can be specified.

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Appendix

The problem

A general state space of K dimensions is denoted by $\tilde{s} = (s^1, s^2, ..., s^K)$.

By choosing a 1 x D vector of control variables $\tilde{x} = (x^1, x^2, ..., x^D)$, a profit of $f(\tilde{x}, \tilde{s}, \tilde{\alpha})$ is achieved over the one time unit, where $\tilde{\alpha}$ is a vector of parameters subject to stochastic variation and f is known. This results in a state space one time unit later of $g(\tilde{x}, \tilde{s}, \tilde{\alpha})$ where g is known.

If a discount rate of r applies then the total future value of an initial state space s(0) is given by

$$V(\tilde{s}(0)) = \sum_{t=0}^{\infty} (1-r)^{-t} f(\tilde{x}(t), \tilde{s}(t), \tilde{\alpha}(t))$$

If we choose the control variables to always maximise the expected future value, then this can be expressed recursively by

$$V(\tilde{s}) = \max_{x} E_{\alpha}\{f(\tilde{x}, \tilde{s}, \tilde{\alpha}) + (1-r)V(g(\tilde{x}, \tilde{s}, \tilde{\alpha}))\} \qquad (1)$$

This needs to be solved for $\tilde{x}$ simultaneously on the K dimensional space of $\tilde{s}$.

The method is to solve simultaneously on a predefined set of nodes of $\tilde{s}$, and to approximate V by polynomials in $\tilde{s}$, that allow interpolated solutions to all other possible values of $\tilde{s}$.

The approximation

The *Chebychev polynomial* functions of scalar s are defined by:

$$\varphi_1(s) = 1$$
$$\varphi_2(s) = s$$
$$\varphi_j(s) = 2s\varphi_{j-1}(s) - \varphi_{j-2}(s) \qquad \text{for } j > 2$$

For scalar s define $\Phi(s, n_c)$ to be the $1 \times n_C$ vector of the first n_C Chebychev polynomials of s:

$$\Phi(s, n_C) = \left[\varphi_1(s), \varphi_2(s), ..., \varphi_{n_C}(s)\right]$$

For any $n_R \times 1$ vector $S = (s_1, s_2, ..., s_{n_R})'$ define $\Phi(S, n_C)$ to be the $n_R \times n_C$ matrix defined by

$$\Phi(S)_{ij} = \varphi_j(s_i)$$

ie the ith row of $\Phi(S)$ is $\Phi(s_i, n_C)$.

For any n_RxK matrix **S** and 1xK vector $\tilde{n} = (n_1, n_2, ..., n_K)$ define the n_R x N matrix

$$\Phi(\mathbf{S}, \tilde{n}) = \Phi(S_1, n_1) \otimes \Phi(S_2, n_2) \otimes ... \otimes \Phi(S_K, n_K)$$

$S_{.,2}$

where S_j is the jth column of **S**,

and $N = n_1.n_2...n_K$.

One dimensional case

An established approximation for solving the one dimensional case of (1), ie

$$V(s) = \max_x E_\alpha \{ f(\tilde{x}, s, \tilde{\alpha}) + (1-r)V(g(\tilde{x}, s, \tilde{\alpha})) \}$$

is to select a set of n *nodes* of s, $S = (s_1, s_2, ..., s_n)'$, and approximate the n by 1 vector $V(S) \equiv (V(s_1), V(s_2), ..., V(s_n))'$ by

$$V(S) = \Phi(S, n).\tilde{c} \qquad (2)$$

where $\tilde{c} = (c_1, c_2, ..., c_n)'$.

K–dimensional case

(2) is of the form

$$V(s_i) = \sum_{j=1}^{n} \varphi_j(s_i).c_j \qquad \text{for i = 1,...,n.}$$

We could extend to the K-dimensional case and assume the approximation to be multiplicative, ie for node $\tilde{s}_i = (s_{i_1}^1, s_{i_2}^2, ..., s_{i_K}^K)$:

$$V(\tilde{s}_i) = V(s_{i_1}^1).V(s_{i_2}^2)...V(s_{i_K}^K)$$

$$= \sum_{j_1=1}^{n_1} \sum_{j_2=1}^{n_2} ... \sum_{j_K=1}^{n_K} \varphi_{j_1}(s_{i_1}^1).\varphi_{j_2}(s_{i_2}^2)....\varphi_{j_K}(s_{i_K}^K).c_{j_1}^1.c_{j_2}^2...c_{j_K}^K$$

but we choose the more general form

$$V(\tilde{s}_i) = \sum_{j_1=1}^{n_1}\sum_{j_2=1}^{n_2}\ldots\sum_{j_K=1}^{n_K}\varphi_{j_1}(s_{i_1}^1).\varphi_{j_2}(s_{i_2}^2)....\varphi_{j_K}(s_{i_K}^K).c_{j_1 j_2 \ldots j_K} \quad (2.1)$$
$$= \left[\Phi(s_{i_1}^1, n_1)\otimes\Phi(s_{i_2}^2, n_2)\otimes\ldots\otimes\Phi(s_{i_K}^K, n_K)\right].\tilde{c}$$

where

$\tilde{c}$ is an N x 1 vector, where
$N = n_1.n_2...n_K$.

Define **S** to be the N x K matrix where the rows (i = 1,...N) consist of every possible combination of nodes $(s_{i_1}^1, s_{i_2}^2, ..., s_{i_K}^K)$ with the $s_{i_1}^1$ cycling slowest and $s_{i_K}^K$ cycling fastest:

$$\mathbf{S} = \begin{bmatrix} s_1^1 & s_1^2 & \ldots & s_1^{K-1} & s_1^K \\ s_1^1 & s_1^2 & \ldots & s_1^{K-1} & s_2^K \\ : & : & : & : & : \\ s_{N_1}^1 & s_{N_2}^2 & \ldots & s_{N_{K-1}}^{K-1} & s_{N_K}^K \end{bmatrix} \equiv \begin{bmatrix} \tilde{s}_1 \\ \tilde{s}_2 \\ : \\ \tilde{s}_N \end{bmatrix}$$

then it can be shown that the N by 1 vector of the $V(\tilde{s}_i)$'s, is given by

$$V(\mathbf{S}) = \Phi(\mathbf{S}, \tilde{n}).\tilde{c}$$

The solution

The set of *Chebychev nodes* of s_k on $[a_k, b_k]$ are defined by:

$$s_i^k = \frac{1}{2}\left[a_k + b_k + (b_k - a_k)\cos\left(\frac{n_k - i + .5}{n_k}\pi\right)\right]$$

for i = 1,..., n_k

Used in conjunction with the Chebychev polynomial approximation, these yield good solutions to (1).

(1) needs to be solved at the N nodes and can now be expressed in vector form:

$$\mathbf{V}(\mathbf{S}) = \max_{\mathbf{X}} E_\alpha\{\mathbf{f}(\mathbf{X}, \mathbf{S}, \tilde{\alpha}) + (1-r)\Phi(\mathbf{g}(\mathbf{X}, \mathbf{S}, \tilde{\alpha}), \tilde{n}).\tilde{c}\} \quad (3)$$

where

X is an N x D matrix, row i being the control solution of node i

$\mathbf{f}(\mathbf{X},\mathbf{S},\tilde{\alpha})$ is the vector of profits $f(\tilde{x}_i,\tilde{s}_i,\tilde{\alpha})$ for nodes i = 1,...,N

$\mathbf{g}(\mathbf{X},\mathbf{S},\tilde{\alpha})$ is the vector of state spaces $g(\tilde{x}_i,\tilde{s}_i,\tilde{\alpha})$ for nodes i = 1,...,N

Solutions need to be found for $\tilde{c}$, $\mathbf{X}$ and for $\mathbf{V}(\mathbf{S})$.

The iterations

The method is to begin with a guess at $\tilde{c}$ and then alternate between using $\tilde{c}$ to update $\mathbf{V}(\mathbf{S})$ and then using $\mathbf{V}(\mathbf{S})$ to update $\tilde{c}$.

1. $\mathbf{V}(\mathbf{S})$ (and $\mathbf{X}$) is updated by holding $\tilde{c}$ fixed and solving (3).

2. $\tilde{c}$ is updated by Newton's method:

$$\tilde{c} \leftarrow \tilde{c} - [\Phi(\mathbf{S},\tilde{n}) - \mathbf{V}'(\mathbf{S})]^{-1}[\Phi(\mathbf{S},\tilde{n})\tilde{c} - \mathbf{V}(\mathbf{S})]$$

where $\mathbf{V}'(\mathbf{S})$ is the N x N Jacobian defined by

$$\begin{aligned}\mathbf{V}'(\mathbf{S}) &= \frac{\partial}{\partial c}\mathbf{V}(\mathbf{S}) \\ &= \frac{\partial}{\partial c}\max_{\mathbf{X}} E_\alpha\{\mathbf{f}(\mathbf{X},\mathbf{S},\tilde{\alpha}) + (1-r)\Phi(\mathbf{g}(\mathbf{X},\mathbf{S},\tilde{\alpha}),\tilde{n})\tilde{c}\} \\ &= (1-r)E_\alpha\frac{\partial}{\partial c}\Phi(\mathbf{g}(\mathbf{X},\mathbf{S},\tilde{\alpha}),\tilde{n})\tilde{c} \\ &= (1-r)E_\alpha\Phi(\mathbf{g}(\mathbf{X},\mathbf{S},\tilde{\alpha}),\tilde{n})\end{aligned}$$

giving

$$\tilde{c} \leftarrow \tilde{c} - [\Phi(\mathbf{S},\tilde{n}) - (1-r)E_\alpha\Phi(\mathbf{g}(\mathbf{X},\mathbf{S},\tilde{\alpha}),\tilde{n})]^{-1}[\Phi(\mathbf{S},\tilde{n})\tilde{c} - \mathbf{V}(\mathbf{S})]$$

Stochastics

The expectation over the stochastic vector $\tilde{\alpha}$ is accounted for by giving it a known discrete distribution over m=1,...,M shocks. Equation (3) becomes:

$$\mathbf{V}(\mathbf{S}) = \max_{\mathbf{X}}\sum_{m=1}^{M} p(\tilde{\alpha}_m)\{\mathbf{f}(\mathbf{X},\mathbf{S},\tilde{\alpha}_m) + (1-r)\Phi(\mathbf{g}(\mathbf{X},\mathbf{S},\tilde{\alpha}_m),\tilde{n})\tilde{c}\}$$