DOES CONSISTENT AGGREGATION REALLY MATTER?

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Abstract

Consistent aggregation assures that behavioral properties which apply to disaggregate relationships also apply to aggregate relationships. The agricultural economics literature is reviewed which has tested for consistent aggregation or measured statistical bias and/or inferential errors due to aggregation. Tests for aggregation bias and errors of inference are conducted using indices previously tested for consistent aggregation. Failure to reject consistent aggregation in a partition did not entirely mitigate erroneous inference due to aggregation. However, inferential errors due to aggregation were small relative to errors due to incorrect functional form or failure to account for time series properties of data.

Keywords: agent, aggregation, bias, commodity, composite commodity, error, separability

JEL Listing: Q11, C51

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> "[I]t is no longer useful to assume that 'truth' exists at some level, and that an analogous system may be fitted at another level, followed by an inquiry into the connection between the fitted values of the analogous system and the underlying 'truth.' A seminal idea . . . suggests that there are different 'truths' at different levels of aggregation, and that they are connected by both the aggregation rules and the properties of the distribution of the microvariables. I think that when we come to know more, we shall find that good monthly and annual models do not really look alike, and that there is rhyme and reason for this difference." (Griliches, 1972, p. 37).

During the last two decades considerable attention has been given to the question of consistent aggregation of agricultural data. The primary goal is to facilitate analysis and inference with aggregate data and aggregate models. Mistakes due to aggregation are to be avoided, and consistent representative agent and/or multi-stage choice modeling is to be enabled. While few have any illusions that the true model structure can ever be identified, improved model specification is certainly sought in which the behavioral properties that apply to disaggregate relationships also apply to the aggregate relationships.

While the benefits of using aggregate data are often substantial, the costs can also be high and their magnitudes are generally unknown. One of the reasons the magnitudes are unknown is because of a disconnect in the literature. The literature reporting empirical testing for consistent aggregation with agricultural data has generally concentrated on commodity-wise aggregation while the literature focusing on the errors created by aggregation has primarily addressed aggregation across firms and individuals. In addition, the latter has seldom differentiated in their measurements between data sets that satisfy sufficient conditions for consistent aggregation and those which do not. Consequently, missing from both sets of literature is an explicit assessment of whether consistent aggregation really matters. What are the effects of inconsistent aggregation on econometric results and policy implications? Is it important whether individuals, firms, inputs, or outputs are grouped in ways that are consistent with the implications of empirical test results, whether they are disaggregated, or whether they are grouped for convenience or pragmatic reasons? What are the practical effects of inappropriate aggregation on economic inference?

We seek some preliminary answers to these questions in this paper. We first proceed by documenting the historical attention given to the issues of consistent aggregation, incorrect aggregation, and the problems of drawing policy-relevant inferences from analyses with aggregate data. We then introduce a testing procedure adapted from Lee, Pesaran, and Pierse (1990) to determine whether consistent commodity-wise aggregation really matters statistically and economically, conduct tests on U.S. agricultural production data for which consistent commodity-wise aggregation tests were previously conducted, draw inferences, and conclude.

Problems Due to Inappropriate Aggregation

The problem of aggregation has been explored from various viewpoints. They have included theoretical works that identified restrictions on either technology (preferences) or data which enable the representative agent framework to be applied to aggregate commodities. A few of the prominent authors in this area are Hicks (1936), Leontief (1936, 1947), Gorman (1964), Green (1964), Barnett(1979), and more recently Chambers and Pope (1996), and Lewbel (1993, 1996). They have also included a variety of empirical works. Some of the latter tested for satisfaction of the restrictions for consistent aggregation in various data sets. Others considered ways to empirically incorporate heterogeneity across individuals or commodities into the aggregate analysis or examine the effects of failing to do so. Some of the prominent authors pursuing this approach are Theil (1954), Grunfield and Griliches (1960), and more recently Stoker (1986), Pesaran, Pierse, and Kumar (1989), Hildenbrand (1998), and Just and Pope (1999). This literature has often split along two different objectives: aggregate prediction and aggregate parameter estimation.

In this section, we first identify sufficient conditions for aggregation that enable consistent multi-stage choice with aggregate commodities or representative agent representation of multiple firms or consumers. We then address two sets of empirical literature in agricultural economics. The first reports test results for consistent aggregation. The second measures mistakes from aggregation generally without regard to whether the aggregates provide empirical evidence that they satisfy sufficient conditions for consistent aggregation.

Theoretical Restrictions Enabling Consistent Aggregation

Sufficient conditions for commodity-wise aggregation. Commodity-wise aggregates exist and enable consistent two-stage choice models to be optimized if any one of four sufficient conditions is satisfied -- Hicks composite commodity theorem, Leontief composite commodity theorem, homothetically separable production or utility function, or generalized composite commodity theorem. The Hicks composite commodity theorem is satisfied for a commodity (output and/or input) subset if the prices of all items in the subset move in exact proportion over the data sample. The Leontief composite commodity theorem is satisfied for a commodity subset if the quantity ratios of all items in the subset move in exact proportion over the data sample. Homothetic separability is a structural property of the production (or utility) function and is satisfied for a subset if the marginal rate of substitution of all pairs of items within the subset are homogeneous of degree zero in the quantities of items within the subset (so the subset is homothetic in its quantities) and also independent of the quantities of all items outside the subset.[1] The generalized composite commodity theorem recently discovered by Lewbel (1996) relaxes the rigid conditions of the Hicks composite commodity theorem. To be consistent with this theorem, the price ratios may vary across observations as long as the distribution of the ratio of the commodity price to the group price is independent of the level of the group price. That is, the relative difference between the individual commodity price and the aggregate commodity price must be independent of the aggregate commodity price. The three composite commodity theorems impose alternative restrictions on all observations in the data series while homothetic separability imposes restrictions on the technology or utility. Satisfaction of any one of the four is a sufficient condition for consistent commodity-wise aggregation.

Sufficient conditions for agent-wise aggregation. A number of sufficient conditions exist for consistently aggregating across agents (firms, individuals). Chambers (1988) identifies sufficient conditions for both linear and nonlinear aggregation across firms.

Aggregation across firms is most often sought by linear aggregation such as summing output and/or input quantities and averaging prices across the firms. In this case, the sufficient conditions for consistent aggregation are highly restrictive. Consistent linear aggregation in the long run is assured only if each firm produces the same output level using a technology characterized by constant returns to scale.

Sufficient conditions for consistent nonlinear aggregation across firms are less stringent but still demanding. Each firm's cost function must be quasi-homothetic. Marginal cost does not have to be identical

[1] The subset is weakly separable if the second condition is satisfied but the first condition is not. A weakly separable production function is sufficient for the existence of a consistent quantity aggregate for the subset, but it does not imply the existence of a corresponding price aggregate which would be required to conduct consistent two-stage choice.

across firms or independent of firm-level output, but each firm-level production function must be a transform of a linear homogeneous function. Input requirement sets must be parallel across firms.

Identical technologies is treated in some empirical literature as a sufficient condition for consistent aggregation across firms. However, as Chambers notes, that alone is not sufficient for linear aggregation.

Empirical Tests for Consistent Aggregation in Agricultural Data

Considerable research has been devoted to testing whether any of the sufficient conditions for consistent commodity-wise aggregation hold in agricultural data. A wide variety of outputs and inputs have been included in these tests. Most tests used production data. Considerably less empirical testing has used food (or agricultural) consumption data. Even less empirical testing has been reported for consistent aggregation across agricultural firms or individuals.

A variety of procedures have also been used. For consistent commodity-wise aggregation, they include parametric and nonparametric testing for weak separability and homothetic separability and time series testing for generalized composite commodities.

Literature surveyed. A survey of 10 agricultural economics journals since 1984 was conducted to identify articles that conducted empirical tests for consistent aggregation. The journals included *Agricultural Economics, American Journal of Agricultural Economics, Australian Journal of Agricultural and Resource Economics, Canadian Journal of Agricultural Economics, European Review of Agricultural Economics, Journal of Agricultural Economics, Journal of Agricultural and Applied Economics, Journal of Agricultural and Resource Economics, Review of Agricultural Economics,* and *Review of Agricultural and Resource Economics*

Nineteen articles were found in the survey period that reported such tests. In addition, we are aware of three earlier articles. All 22 are listed in Table 1. They included 20 articles that tested for consistent commodity-wise aggregation, one that tested for consistent aggregation across units of production (actually across already aggregated units of production), and one that tested for both. Of those that tested for commodity-wise aggregation, eight tested for weak separability, 11 tested for homothetic separability, and two for generalized composite commodities; 17 conducted aggregation tests in agricultural production models and four in food demand models; 17 used parametric testing procedures and four used nonparametric procedures.

All tests in production models, including those for spatial aggregation, used data that was already highly aggregated across firms. The aggregation tests across production units were conducted using state-level data. Most of the commodity-wise aggregation studies used state-level or national data. Only two production studies conducted commodity-wise aggregation tests using data aggregated to less than state-level areas. One demand study conducted tests on fully disaggregated individual decision data.

Commodity-wise aggregation test results. Of individual studies that conducted several tests of consistent aggregation, almost all rejected the hypothesis of consistent aggregation of some categories and failed to reject the hypothesis for others. There was little evidence of a clear pattern concerning the types of categories that were not rejected for one data set and those that were not rejected in other data sets.

Examining the studies collectively, empirical tests of the hypothesis that all outputs could be consistently aggregated into a single index were reported with 61 data sets. Of these tests, 64% rejected the hypothesis, 33% failed to reject, and 3% were ambiguous. More than 550 separate empirical tests of the hypothesis that a subset of outputs could be consistently aggregated into an index were reported. Of these tests 58% rejected the hypothesis, 41% failed to reject, and 1% were ambiguous. Of 56 tests of the hypothesis that all inputs could be consistently aggregated, 46% rejected the hypothesis and 54% failed to reject. These same percentages of rejections and failures to reject also applied to more than 200 tests of the hypothesis that a subset of inputs could be consistently aggregated into an index. Although there were differences in rates of rejection among output and input aggregation and among total and subset aggregation, a substantial proportion of aggregates were rejected and a substantial proportion were not rejected in each.

More of a pattern emerged when the evidence was examined relative to test procedure. Of 132 parametric tests of weak or homothetic separability, 84% rejected the consistent aggregation hypothesis. Unfortunately, the parametric tests also maintained an auxiliary functional form hypothesis. Consequently, when the test rejected the hypothesis, it was not possible to know whether the hypothesis of homothetic separability was rejected or whether the specific functional form was rejected.

Since the nonparametric tests did not maintain specific functional forms, we would expect them to result in rejection less frequently. Our finding was consistent with that expectation. Of nearly 750 nonparametric tests of separability, only 52% rejected the consistent aggregation hypothesis.

Although we had no *a priori* basis for expecting a smaller or larger percent of rejections of homothetic separability than of the generalized composite commodity theory, the latter resulted in the smallest frequency of rejection. Of 30 time series tests of the generalized composite commodity theorem (GCCT), none rejected the consistent aggregation hypothesis, but only 2/3 of the tests resulted in a clear failure to reject. The rest of the GCCT test results were ambiguous. Thus, the parametric tests of separability led to the largest proportion of rejections, and the time series tests of the GCCT led to the smallest proportion of clear, and even ambiguous, rejections.

Finally, comparing test results for national and world boundaries vs. state and sub-state areas revealed a higher level of rejection (59%) for the states and sub-states than for the nations and world (34%). However, we should note that the distribution of tests varied greatly among geographic types. All of the GCCT tests were conducted on national and world data sets. Although a smaller share of the separability tests was nonparametric for national and world data than for state and sub-state data, the share of nonparametric plus GCCT tests was slightly larger for national and world data than for state and sub-state data.

While the percent of rejections varied most by test procedure, they also varied considerably between outputs and inputs and between national and state areas. These empirical findings consistently reflect one conclusion -- there is no obvious empirical generalization (or stylized fact) about consistent aggregation of agricultural data. Except for the GCCT tests, all of the above classifications included considerable proportions of both rejections and nonrejections of the consistent aggregation hypothesis.

Agent-wise aggregation results. Both studies that tested for consistent geographic aggregation rejected the hypothesis in each data set. Although they used different approaches, both tested for consistent aggregation across states. The hypothesis was rejected even for pairs of states.

Aggregation Mistakes

Other empirical literature has attempted to measure the mistakes made by aggregation. It appears that few have also conducted tests to determine whether the data satisfied sufficient conditions for consistent aggregation. In this section we note nine such articles from the agricultural and resource economics literature. Only one also conducts tests for consistent aggregation. Most use actual data observations, but a few of the applications are based on Monte Carlo simulation.

Buccola and Sil (1996) found evidence of substantial negative representative-agent aggregation bias in productivity growth. They conducted a Monte Carlo simulation based on data for four food manufacturing industries -- meat processing, dairy, baking, and beverage -- that operated with nonjoint technologies. Using base data, the aggregate estimate of productivity growth underestimated the true growth rate by 28%. For some other scenarios, the underestimate was as high as 88% and never lower than 21%.

Hellerstein (1995) found that the bias in consumer welfare measures from aggregation within travel cost models was frequently less than the bias from distributional errors in limited dependent variable models of travel cost which used individual observations. Like Buccola and Sil, his analysis was based on Monte Carlo simulations. He found that the aggregation bias was generally greatly reduced by including distributional information in the aggregate model. He included the distributional covariance matrix in the modified aggregate model such that the variances and covariances were included along with the aggregated means in the set of regressors.

Reed and Riggins (1981) reported that estimation of corn acreage response in Kentucky was improved by disaggregating the data into 14 sub-state regions. Statistical fit was greater, and parameter signs were more frequently consistent with expectations.

Park and Garcia (1994), on the other hand, found little loss in predictive accuracy by modeling Illinois corn and soybean acreage response at the state level rather than at the substate, crop reporting district, level. In addition, they observed that the state-level model provided estimates more consistent with expectations.

Like Park and Garcia, Arnade and Davison (1989) reported little adverse effects from aggregation bias in their analysis of U.S. soybean export data. Their aggregate model included worldwide demand for U.S. soybean exports while their disaggregate models were six major importing countries and regions. Although one of the conditions was violated for consistently aggregating the data to a single equation, the distortions from aggregation were smaller than the distortions from incorrect simultaneity assumptions.

Paul (1999) noted that aggregate time-series data told the same story as disaggregate cross-sectional data about the reasons for increased concentration in the meat packing industry. She found little evidence of excessive profits being generated by the meat packing plants and firms. Instead, both analyses revealed that cost economies, which were primarily transmitted to suppliers of cattle and demanders of meat products, appeared to be the primary cause of increased concentration.

Shumway, Saez, and Gottret (1988) also found little impact from aggregation bias in their analysis of U.S. agricultural production. When output supply and input demand estimates for 10 farm production regions were aggregated to the national level, few elasticities differed by more than a magnitude of 0.1 from those estimated using national data.

Although not statistically significant, Thomas and Tauer (1994) found evidence that linear aggregation across inputs impacted estimated technical efficiencies of New York dairy farms. However, their nonparametric procedure was considerably more sensitive to the number of input categories included in the analysis than to improper aggregation.

Davis (1997) found evidence of statistically significant commodity-wise aggregation bias in his study of the demand for cigarette leaf tobacco by the U.S. tobacco industry. The economic implication of the bias was to erroneously conclude that domestic and foreign tobaccos were substitutes rather than complements.

What implications can be drawn from these nine diverse studies? Of the cited studies, seven focused on representative agent or geographic aggregation and two considered commodity-wise aggregation. One found very large mistakes from representative agent aggregation (some of which approached 90 percent). Another found statistically significant evidence of commodity-wise aggregation bias that resulted in an important error of economic inference. A third found that estimation was improved by using agent-wise or geographically disaggregated data. The remaining six either found little error of inference created by the aggregations or they found that the error created by other common mispecifications exceeded those from aggregation. A majority of the studies found little inferential error because of the aggregations.

However, recognizing that some studies found substantial errors due to aggregation, it is important to also note the observation of one article that representative agent aggregation bias was generally greatly reduced by including distributional information about the individual agents in the aggregate model. This observation echoes earlier findings by Blundell, Pashardes, and Weber (1993), Stoker (1986), Simmons (1980), Blinder (1975), and even a 1937 article by Staehle. It is also consistent with the recent recommendations by Just and Pope (1999) for dealing in a practical way with the seemingly pervasive problem of inconsistent aggregation across firms. They develop theoretical insight as well as a call for minimal improvements in data collection procedures to make feasible the practical recommendation of including "second own- and cross-moments of producer characteristics" in aggregate supply and demand specifications.

Empirical Tests for Aggregation Bias

We now turn to our own empirical tests. We first introduce a procedure for testing for the presence of commodity-wise aggregation bias. It is adapted from Lee, Pesaran, and Pierse (1990). The test is applied to two aggregations in a data set that has been extensively tested for consistent aggregation. One of the aggregates received clear and unambiguous empirical support by the previous tests. The other aggregate was only partially (ambiguously) supported.

Theoretical Framework

At some low level of aggregation, the tenets of economic theory are presumed to be untainted by aggregation. At that level consider the netput share equations associated with an explicit functional form (translog) of the variable profit function,

$$y_i = \beta_{io} + \sum_{j=1}^{m} \beta_{ij} x_j + \sum_{j=1}^{n} \gamma_{ij} z_j \quad i = 1, \ldots, m \tag{1}$$

where y_i is the variable profit share of netput i (positive for an output, negative for an input), x_j is the log of the price of netput j, and z_j is the log of a fixed factor or other exogenous variable. The netputs are indexed by $i \in D = \{1, 2, \ldots, m\}$, so there are m disaggregate netputs. By assumption, equation (1) satisfies all the properties coming from a well-behaved translog variable profit function. These include the following restrictions on the parameters which result from linear homogeneity in prices of a twice continuously differentiable profit function: $\sum_{i=1}^{m} \beta_{io} = 1, \sum_{i=1}^{m} \beta_{ij} = 0, \sum_{i=1}^{m} \gamma_{ij} = 0, \beta_{ij} = \beta_{ji}$. In addition, equation (1) must be consistent with convexity and monotonicity of the profit function in prices. These latter conditions are only local properties of a translog profit function and are dependent on the magnitudes of x and z. Now the question of interest is what happens to this system if a subset of the equations in (1) is aggregated together? Specifically, what theoretical properties are retained? What econometric results are retained?

To answer the above questions requires additional notation. Let there be an aggregate indexing set I = $\{I_1, I_2, \ldots, I_m\} \subseteq D$, such that $I_r \subseteq D$ for any $r = 1, \ldots, M \subseteq m$. For example, the I set could be I = {{1,2}, {3,4}} so I_1 contains the first two netputs, I_2 contains the third and fourth netputs, M = 2, and m =4. When researchers consider aggregating quantities, it is common to also construct an aggregate price index to correspond to the aggregate quantity index. Operationally, what is done, perhaps tacitly, is that equations are aggregated together and the individual prices of the aggregate quantity are replaced with the aggregate price index. Ultimately it does not matter the order in which these operations are done, but in our case it is more enlightening for econometric reasons to first replace disaggregate prices with their associated aggregate price index and an explicit aggregation error and then aggregate over equations.

Following Lewbel (1993, 1996), let the log of the aggregate price index be X_r, then the deviation of the log of the disaggregate price from x_j can be defined as $\rho_j = x_j - X_r$ for $j \in I_r$, so

(2) $\quad x_j = X_r + \rho_j.$

The term ρ_j can be considered a measure of price aggregation error. Note that equation (1) can always be written equivalently using (2)

(3) $$y_i = \beta_{io} + \sum_{r=1}^{M} b_{ir} X_r + \sum_{j \notin I} \beta_{ij} x_j + \sum_{j=1}^{n} \gamma_{ij} z_j + \sum_{j \in I} \beta_{ij} \rho_j \quad i = 1,2,\ldots,m$$

where $b_{ir} = \sum_{j \in I_r} \beta_{ij}$, r = 1, 2, …, M. Aggregating the $i \in I$ equations in the system (3) gives the corresponding aggregate system

(4.1) $$Y_s = b_{so} + \sum_{r=1}^{M} B_{sr} X_r + \sum_{j \notin I} b_{sj} x_j + \sum_{j=1}^{n} \Gamma_{sj} z_j + \sum_{j \in I} b_{sj} \rho_j \quad s = 1,2,\ldots,M$$

(4.2) $$y_i = \beta_{io} + \sum_{r=1}^{M} b_{ir} X_r + \sum_{j \notin I} \beta_{ij} x_j + \sum_{j=1}^{n} \gamma_{ij} z_j + \sum_{j \in I} \beta_{ij} \rho_j \quad i \notin I.$$

where $b_{sj} = \sum_{i \in I_s} \beta_{ij}$, $B_{sr} = \sum_{i \in I_s} b_{ir} = \sum_{i \in I_s} \sum_{j \in I_r} \beta_{ij}$, and $\Gamma_{sj} = \sum_{i \in I_s} \gamma_{ij}$. Before turning to the theoretical properties of this system, it should be noted for estimation purposes that the parameters on the aggregation errors can be used to help identify, through parameter restrictions, components of the aggregate parameters. Furthermore, if these aggregation errors are omitted in the estimation then the components of the aggregate parameters cannot be restricted and identified.

<u>Theorem (Lewbel 1993)</u>: If the disaggregate system (3) satisfies symmetry ($\beta_{ij} = \beta_{ji}$) and homogeneity $\left(\sum_{i=1}^{m} \beta_{io} = 1, \sum_{i=1}^{m} \beta_{ij} = 0, \text{and } \sum_{i=1}^{m} \gamma_{ij} = 0\right)$, then the aggregate system (4) will also satisfy symmetry and

homogeneity: $B_{rs} = B_{sr}$, $b_{rj} = b_{jr}$, $\beta_{ij} = \beta_{ji}$, $\sum_{s=1}^{M} b_{so} + \sum_{i \notin I} b_{io} = 1$, $\sum_{s=1}^{M} B_{sr} + \sum_{i \notin I} b_{ir} = 0$,

$\sum_{s=1}^{M} b_{sj} + \sum_{i \notin I} \beta_{ij} = 0$, $\sum_{s=1}^{M} \Gamma_{sj} + \sum_{i \notin I} \gamma_{ij} = 0$.

<u>Proof</u>: The proofs are straightforward applications of the definitions of b_{rj} and B_{rs} using the rules of multiple summation so are omitted here.

Econometric Estimation and Testing Procedure

The theory presented in the previous section is deterministic and the relationship between the aggregate parameters B_{sr}, b_{sj}, and the disaggregate parameters β_{ij} are very simple linear functions. However, as Theil (1954) demonstrated in his seminal work on aggregation, the relationships between the estimated parameters are not the same as between the deterministic parameters. In this section the relationship between the estimators of the aggregate and disaggregate parameter estimates is presented in a framework similar to Theil, and the significance of aggregation bias in estimating aggregate parameters is tested using procedures developed by Lee, Pesaran, and Pierse (1990).

Using standard econometric notation, let the disaggregate system of share equations corresponding to equation (3) be written as

(5) $\mathbf{y}_d = \mathbf{X}_d \boldsymbol{\beta}_d + \mathbf{e}_d$

where $\mathbf{y}_d$ is a $(T \cdot m \times 1)$ vector, $\mathbf{X}_d = (I_m \otimes \underline{\mathbf{X}})$ is a $(T \cdot m \times m \cdot k_d)$ matrix, I_m is an $(m \times m)$ identity matrix, $\underline{\mathbf{X}}$ is a $(T \times k_d)$ matrix of regressors including a vector of ones. The matrix $\underline{\mathbf{X}}$ is the same in all equations and may be partitioned as $\underline{\mathbf{X}} = (\mathbf{1}, \mathbf{p}, \mathbf{z}, \boldsymbol{\rho})$ with $\mathbf{1}$ being a $(T \times 1)$ vector of ones, $\mathbf{p}$ the sub-matrix of prices, $\mathbf{z}$ the sub-matrix of other variables, and $\boldsymbol{\rho}$ the sub-matrix of aggregation errors. The parameter vector

$\boldsymbol{\beta}_d = (\boldsymbol{\beta}_1^d, \boldsymbol{\beta}_2^d, \boldsymbol{\beta}_m^d)$ is the $(m \cdot k_d \times 1)$ vector of disaggregate parameter vectors $\boldsymbol{\beta}_i^d = (\beta_{io}, \mathbf{b}_{ir}, \boldsymbol{\beta}_{ij}, \boldsymbol{\gamma}_{ij}, \boldsymbol{\beta}_{ij})$ associated with the m equations, and $\mathbf{e}_d$ is the $(T \cdot m \times 1)$ vector of error terms.

The "true" aggregate system based on equation (4) is just a linear transformation of (5) and can be obtained by premultiplying both sides of (5) by the transformation matrix $\boldsymbol{\varphi}$, which must be designed according to which equations are to be aggregated together. For example, if m = 3 and the first two equations were to be added together, leaving the last in disaggregate form, then

$$\varphi = \begin{bmatrix} \mathbf{I_T} & \mathbf{I_T} & \mathbf{O_T} \\ \mathbf{O_T} & \mathbf{O_T} & \mathbf{I_T} \end{bmatrix}$$

where $\mathbf{I_T}$ is the (T × T) identity matrix and $\mathbf{O_T}$ is a (T × T) matrix of zeros. So the "true" aggregate system becomes $\mathbf{y}_A = \boldsymbol{\varphi}\ \mathbf{y}_d = \boldsymbol{\varphi}\ \mathbf{X}_d\boldsymbol{\beta}_d + \boldsymbol{\varphi}\ \mathbf{e}_d$. However, because the $\underline{\mathbf{X}}$ matrix is the same for all equations and has been written in terms of aggregate prices and aggregation errors, an equivalent representation of the model $\boldsymbol{\varphi}\ \mathbf{X}_d\boldsymbol{\beta}_d$ is to define $\mathbf{X}_A = (\underline{\mathbf{X}} \otimes I_M)$, with I_M the (M × M) identity matrix and define a parameter aggregating matrix $\mathbf{A}$ such that $\boldsymbol{\varphi}\ \mathbf{X}_d\boldsymbol{\beta}_d = \mathbf{X}_A\mathbf{A}\boldsymbol{\beta}_d$. This is the matrix equivalent of writing an expression such as wa + wb as w(a + b) and amounts to collecting like terms. In the m = 3 example, where the first two equations are to be aggregated together, $\mathbf{A}$ would be defined as

$$\mathbf{A} = \begin{bmatrix} \mathbf{I_d} & \mathbf{I_d} & \mathbf{O_d} \\ \mathbf{O_d} & \mathbf{O_d} & \mathbf{I_d} \end{bmatrix}$$

where $\mathbf{I_d}$ is a ($k_d \times k_d$) identity matrix and $\mathbf{O_d}$ is a ($k_d \times k_d$) matrix of zeros. Therefore, the aggregate model can be written as

(6) $\mathbf{y}_A = \mathbf{X}_A\boldsymbol{\beta}_A + \mathbf{e}_A$.

Based on the economic theory and deterministic aggregation, then $\boldsymbol{\beta_A} = \mathbf{A}\boldsymbol{\beta_d}$. This suggest that there are two ways to estimate $\boldsymbol{\beta_A}$: a deterministic approach that just uses the matrix $\mathbf{A}$ and an estimate of the disaggregate vector, say $\mathbf{b_A} = \mathbf{A}\hat{\boldsymbol{\beta}}_\mathbf{d}$, and an econometric approach that estimates $\boldsymbol{\beta_A}$ directly. As previously indicated, including the aggregation errors in the design matrix allows the components of the aggregate parameters to be identified but only if the identification restrictions are imposed in the estimation. In this context an appropriate econometric estimator of $\boldsymbol{\beta_A}$ is the restricted seemingly unrelated regression estimator, or

(7) $\mathbf{b}_\mathbf{C}^\mathbf{r} = \mathbf{C_C}\mathbf{X}_\mathbf{A}^\mathbf{T}\boldsymbol{\Omega}_\mathbf{C}^{-1}\mathbf{y_A} + \mathbf{C_C}\mathbf{R}_\mathbf{C}^\mathbf{T}[\mathbf{R_C}\mathbf{C_C}\mathbf{R}_\mathbf{C}^\mathbf{T}]^{-1}(\mathbf{q} - \mathbf{R_C}\boldsymbol{\beta_A})$

where $\mathbf{C_C} = (\mathbf{X}_\mathbf{A}^\mathbf{T}\boldsymbol{\Omega}_\mathbf{C}^{-1}\mathbf{X_A})^{-1}$ is the $M \cdot k_d$ parameter covariance matrix, $\mathbf{R_C}$ is the J x $M \cdot k_d$ restriction matrix, $\mathbf{q}$ is the $M \cdot k_d$ x 1 restriction vector which for this paper is always zero, and the subscript C refers to the complete aggregate model which explicitly includes the aggregation errors $\boldsymbol{\rho}$. Assuming the disaggregate model is true, standard regularity conditions of the design matrix apply, and E($\mathbf{e}_A$) = 0, it is easy to show that

$$(8) \qquad E(b_C^r) = A\beta_d - E\left[C_C R_C^T \left(R_C C_C R_C^T\right)^{-1} R_C A\beta_d\right] .$$

If the restrictions are true, then the last term will be zero and the aggregate estimator will be unbiased. If the restrictions are not true, then the bias will be the last term.

In the incomplete aggregate model, the aggregation errors are not carried along and are omitted in the estimation. This leads first to a lack of identification of some of the parameters and second to a different design matrix that is a subset of $\mathbf{X}_A$. An easy way to handle the problem within the same estimation framework is to just redefine the restriction matrix **R** appropriately to include the zero restrictions on the aggregation error terms. In this case let $\mathbf{b}_\mathbf{I}^\mathbf{r}$ be the restricted estimator for the incomplete model. In a similar fashion to equation (8), then

$$(9) \qquad E(b_I^r) = A\beta_d - E\left[C_I R_I^T \left(R_I C_I R_I^T\right)^{-1} R_I A\beta_d\right]$$

with $\mathbf{C}_\mathbf{I}$ and $\mathbf{R}_\mathbf{I}$ appropriately redefined. As before, if the restrictions are true then the estimator is unbiased.

To test whether there is aggregation bias associated with either $\mathbf{b}_\mathbf{C}^\mathbf{r}$ **or** $\mathbf{b}_\mathbf{I}^\mathbf{r}$, the framework of Lee, Pesaran, and Pierse (1990) is trivially extended to the restricted systems estimator. Assuming the disaggregate model is correct, the null hypothesis is $H_o : \boldsymbol{\delta}_\mathbf{i} = \mathbf{b}_\mathbf{i}^\mathbf{r} - \mathbf{b}_\mathbf{A} = 0, \;\; i = C, I$ with $\mathbf{b}_\mathbf{A} = \mathbf{A}\hat{\boldsymbol{\beta}}_\mathbf{d}$ being the aggregate parameter vector constructed from the disaggregate parameter estimates. The relevant test statistic is then

$$(10) \qquad \boldsymbol{\delta}_i^T \Psi_i^{-1} \boldsymbol{\delta}_i \xrightarrow{a} \boldsymbol{\chi}^2_{M\cdot k_d}$$

and Ψ is the covariance of $\boldsymbol{\delta}_\mathbf{i}$ (Domowitz and White, 1982, Theorem 2). This is a generalized Durbin-Hausman test and the general formula for the covariance is given in the appendix. As is common for this type test, a generalized inverse must be computed.

Data

The annual price and quantity data used in the analysis are for the period 1950-1992. They come from Ball (1996). Research expenditure and price data for the period 1920-1992 are from Huffman and

Evenson (1993) and Huffman (1999). Except for an additional observation at the end of the series replacing two at the beginning, these are the same data used by Lim and Shumway (1997), and the research expenditure stock variables are constructed in the same way as in their paper. The disaggregate model consists of two outputs (livestock and crops), three inputs (hired labor, capital, and other purchased inputs), and four fixed factors (self-employed labor, real estate, private research expenditures, and public research expenditures). While we refer to this specification as the disaggregate model, it is admittedly already a highly aggregated model. However, the commodity-wise aggregations in this model are entirely consistent with the results of prior tests for consistent aggregation. Aggregation is accomplished using the Tornqvist index. The variable names and their definitions are given in table 2.

Estimation

Two aggregate models are considered. Based on work by Williams and Shumway (1998b) and Davis, Lin, and Shumway (2000), there is only partial (ambiguous) empirical support for aggregating the two outputs into one output based on the generalized composite commodity theorem tests while there is clear empirical support for aggregating all inputs into one input based on homothetic separability tests. The first aggregate model combines the two outputs from the disaggregate model into one output but leaves all other variables as in the disaggregate model. The second aggregate model combines the three variable inputs into one input and leaves all other variables as in the disaggregate model. Variable profit share equations of outputs and variable inputs are estimated for each model with one share equation omitted to avoid the singularity problem. Estimation is accomplished using the iterative seemingly unrelated regression method to achieve maximum likelihood estimates (assuming normally distributed error terms) with invariance to the equation deleted from the system. Symmetry and homogeneity are maintained in the estimation. Because the aggregate parameter estimates constructed from the disaggregate parameter estimates (i.e., $\mathbf{b_A} = A\hat{\boldsymbol{\beta}}_d$) are the primary concern, the disaggregate parameter estimates are not reported here but are available from the authors on request.

Results from Aggregating Outputs

Table 3 gives the parameter estimates associated with the aggregate output system constructed from the disaggregate model parameter estimates (i.e., $\mathbf{b_A} = A\hat{\boldsymbol{\beta}}_d$) along with the corresponding price elasticities matrix. As can be seen, 26 of the 36 parameters are significant at the 10% level or less. The main parameters that are insignificant are those associated with public and private research expenditures and self-employed labor. Nearly all other parameters and all the price elasticities are significant. All signs on the price elasticities appeal to intuition but their magnitudes are not consistent with a convex variable profit function in prices. Of course, a convex profit function is an implication of competitive behavior only for individual firms and not for an aggregate of firms.

Table 4 gives the parameters estimated from the completely specified aggregate model, $\mathbf{b}_{\mathbf{C}}^{\mathbf{r}}$, (i.e., it includes the aggregation errors) along with the corresponding price elasticity matrix. Overall, the parameter estimates are similar to those constructed from the disaggregate model. All have the same sign and similar magnitudes to those constructed from the disaggregate model. Of the 36 parameters, 23 are significant at the 10% level or less, which is three fewer than in the estimates from the disaggregate parameters. Most of the additional insignificant parameters are associated with the aggregation error of the livestock price.

The Chi-squared test statistic for aggregation bias, equation (10), is a quadratic form. The summary statistics on the square of the components of the difference vector $\boldsymbol{\delta}_{\mathbf{C}} = (\mathbf{b}_{\mathbf{C}}^{\mathbf{r}} - \mathbf{b}_{\mathbf{A}})$ indicate that the average squared difference between the estimated aggregate parameters from the complete aggregate model and the aggregate estimates based on the disaggregate parameters is .15, and the test statistic is 3.11. As is rather well known for this type of test, a generalized inverse must be used because the covariance matrix may not be of full rank and positive semi-definite. This in turn affects the degrees of freedom used in conducting the Chi-squared test. If the covariance matrix were of full rank, then the degrees of freedom would equal the number of parameters -- 36. In the present case the rank of the matrix and number of degrees of freedom is 27. Using degrees of freedom less than the desired degrees of freedom has the affect of reducing the size of the test, *ceteris paribus*. However, because the test statistic is so small in the present case, the p-value of the test statistic is 1.00 regardless of whether the degrees of freedom are 36 or 27, so the null of no aggregation bias clearly cannot be rejected. Thus, there is no statistically significant aggregation effect associated with aggregating together livestock and crops into one single aggregate and including the aggregation errors in the estimation.

With regard to the economic importance of using aggregate data in a completely specified aggregate model, consider the aggregate price elasticity matrices, tables 3b and 4b. The aggregate price elasticities in table 4b are very similar to those obtained by aggregating the disaggregate parameters. There are no sign changes between the two sets of elasticities, and only three of 16 elasticities differ by more than 10 percent. In addition, all elasticities in both tables are statistically significant.

Table 5a gives the aggregate parameters estimated from the incompletely specified aggregate model, that is, the model that ignores aggregation errors in the specification. Based on Williams and Shumway's (1998b) rejection of homothetic separability for this partition and Davis, Lin, and Shumway's (2000) finding of ambiguity with regard to the generalized composite commodity theorem, no clear support was previously found for consistent aggregation of all outputs into a single index. Thus, one might anticipate considerable difference in the parameter estimates. However, little difference is evident. These parameter estimates also appear similar to those constructed from the disaggregate model. All have the same sign and similar magnitudes to those constructed from the disaggregate model. Of the 30 estimated parameters (remember zero restrictions are imposed on the aggregation error terms), 21 are significant at the 10% level or less. The difference vector is $\boldsymbol{\delta}_{\mathbf{I}} = (\mathbf{b}_{\mathbf{I}}^{\mathbf{r}} - \mathbf{b}_{\mathbf{A}})$ and the average of the squared components of the difference vector is about twice as large as before, .27. However, the null hypothesis of no aggregation bias is still not rejected at any reasonable level because the test statistic is 32.69, which with 31 degrees of freedom (the rank of the covariance matrix) has a p-value of .38. With 36 degrees of freedom, the p-value is .62. Thus, there is no statistically significant aggregation effect associated with aggregating livestock and crops into one single aggregate and ignoring aggregation errors in the estimation. This result would tend to suggest that the ambiguous result found by Davis, Lin, and Shumway (2000) for aggregating the two outputs into one output is of no concern for estimating aggregate parameters.

Table 5b gives the aggregate price elasticities based on parameter estimates obtained from the incompletely specified aggregate model. They are again very similar to those obtained by aggregating the disaggregate parameters. None change sign and again only three elasticities differ by more than 10 percent. In addition, all elasticities are statistically significant.

Results from Aggregating Inputs

Table 6 gives the parameter estimates associated with the aggregate input system constructed from the disaggregate model parameter estimates (i.e., $\mathbf{b_A} = A\hat{\boldsymbol{\beta}}_d$, with the aggregating matrix **A** redefined appropriately) along with the corresponding price elasticities matrix. Sixteen of the 24 parameters are significant at the 10% level or less. The main parameters that are insignificant are again those associated with public and private research expenditures, self-employed labor, and the aggregation errors in the crop equation. Other parameters are significant, including all the price elasticities. As with the aggregate output system, all signs of price elasticities in the aggregate input system appeal to intuition but magnitudes are not consistent with a convex variable profit function.

Table 7 gives the parameters estimated from the completely specified aggregate model (i.e., including the aggregation errors) along with the corresponding price elasticity matrix. There are much larger discrepancies between the parameter estimates in tables 7a and 6a than between the corresponding parameters for the output aggregate system. Many of the parameter estimates have clear differences in terms of signs and magnitudes. Of the 24 parameter estimates, only nine are significant. The main difference is that none of the price parameters are significant in the estimated aggregate model while all are significant when computed from the estimated disaggregate system. Although only one parameter that changes sign is significant in both systems, more than half of the parameters have a different sign in the aggregate model than when derived from the disaggregate system. However, all but one price elasticities are significant and have the same signs as in table 6b.

The difference vector is defined as before $\boldsymbol{\delta}_{\mathbf{C}} = (\mathbf{b}_{\mathbf{C}}^{\mathbf{r}} - \mathbf{b}_{\mathbf{A}})$ and the average squared difference of the components is large -- 46.75. Most of this difference is due to different estimates of the intercepts. The Chi-squared statistic for aggregation bias is 36.16. The rank of the covariance matrix is 20. However, regardless of whether one uses the correct degrees of freedom or the desired 24, the null hypothesis of no aggregation bias is rejected at the 5% level. The p-values are .01 and .05 for 20 and 24 degrees of freedom respectively. Thus, there is a statistically significant aggregation effect associated with aggregating hired labor, capital, and other purchased inputs together into one input aggregate, even if the aggregation error is left in the model.

With regard to the economic importance of using aggregate data in a completely specified aggregate model, the price elasticities all have the same sign and all but one are significant. However, their magnitudes differ substantially from those derived from the disaggregate system. There is no obvious pattern. Some elasticity estimates are more elastic while some are less elastic. All differ by more than 10% and some by more than 100%. All except the livestock own-price elasticity are significant in table 7b.

Table 8a gives the aggregate parameters estimated from the incompletely specified aggregate model, which ignores the aggregation errors in the specification, and table 8b gives the corresponding price elasticity matrix. Based on Williams and Shumway's (1998b) clear failure to reject homothetic separability in this input partition and Davis, Lin, and Shumway's (2000) clear failure to reject the generalized composite commodity theorem in this output partition, one might not anticipate much difference in the parameter estimates. Indeed there is much less of a discrepancy between the parameter estimates in table 8a and 6a than between tables 7a and 6a. Overall the parameter estimates appear similar to those constructed from the disaggregate model. All have the same sign and similar magnitudes to those constructed from the disaggregate model. Of the common estimated parameters (remember zero restrictions are imposed on the aggregation error terms), nearly as many are significant at the 10% level or less in this model as in the input aggregate system derived from disaggregate model estimates. All of the price parameters are significant.

Again, the difference vector is $\boldsymbol{\delta}_{\mathbf{I}} = (\mathbf{b}_{\mathbf{I}}^{\mathbf{r}} - \mathbf{b}_{\mathbf{A}})$ and not surprisingly, the average squared difference of the components is small (.65) relative to the completely specified aggregate model, which was 46.75. The Chi-squared statistic for aggregation bias from the incomplete model is 29.70. With 22 (24) degrees of freedom the p-value of the statistic is .13 (.19), so the null hypothesis of no aggregation bias is not rejected at any reasonable level of significance. The fact that the complete aggregate model is rejected while the incomplete aggregate model is not rejected highlights an important aspect of consistent aggregation. Recall in estimating the completely specified aggregate model, it explicitly included the aggregation errors and allowed for the identification and imposition of some within- and cross-equation restrictions which cannot be imposed in the incompletely specified model. What these results tend to suggest is that the implicit zero restrictions associated with the incomplete aggregate model may be less binding than the restrictions associated with the complete aggregate model.

With regard to economic implications, all price elasticities in table 8b have the expected signs and are statistically significant. All have the same signs as those constructed from the disaggregated model parameters, and most are closer to the elasticity estimates constructed from the disaggregate parameters than were the completely specified aggregate model elasticities. However, only two are within 10% of the magnitudes of those elasticities.

Assessment of Empirical Findings

Aggregating all outputs from two output categories or all variable inputs from three input categories failed to produce statistically significant aggregation bias when aggregation errors were not explicitly included in the model. Thus, the statistical tests for aggregation bias also failed to reject the hypothesis that outputs and inputs could be consistently aggregated to such a high level of aggregation. This conclusion complements previous findings of Williams and Shumway (1998b) based on nonparametric tests of homothetic separability and the cointegration tests for generalized composite commodities conducted by Davis, Lin, and Shumway (2000). We note only two departures from this conclusion – (a) the bias test for input aggregation when aggregation errors were explicitly included, and (b) Davis, Lin, and Shumway's correlation tests for a generalized composite output commodity.

The economic significance of aggregation errors, however, was not trivial and did not reflect a high level of consistency with the statistical and related tests. Previous tests found less empirical support for aggregating all outputs than for aggregating all variable inputs. However, aggregating data for all outputs prior to estimation did not have an appreciable impact on price elasticities while aggregating data for all inputs prior to estimation had a very important impact. In the former case only two of 16 pairs of elasticities differed by more than 10 percent (and the largest differences were about 30 percent). In the latter case only two of nine pairs of elasticities differed by less than 10 percent and some differed by more than 100 percent.

As one possible explanation for this difference, it seems reasonable to conjecture that the more commodities that are aggregated together the more likely there will be significant differences due to aggregation. This would not be surprising and is just a corollary of Griliches' observation that there are "different truths at different levels of aggregation, and they are connected by both the aggregation rules and the properties of the distribution of the microvariable." The mixture distribution of the macrovariable, formed from aggregating the microvariables, will likely continue to lose its resemblance to any subset of microvariable distributions as more and more microvariables are aggregated together. This suggests there probably exists a *neighborhood aggregation invariance principle* that is a decreasing function of the number of commodities aggregated together. Since more input categories than output categories were aggregated here into individual indices, it is possible that the larger number of input categories included in the aggregate adversely affected the economic consistency of relationships between the macro model and the macro system constructed from micro model parameter estimates.

Even when considerable empirical support exists for consistent aggregation, it is apparent that aggregating data can lead to serious errors in policy recommendations. Thus, one might appropriately ask how such errors compare with other types of specification error. For comparison, consider two other common specification errors – incorrect choice of functional form and failure to properly account for time series properties of the data. Considering three functional forms in their analysis of Canadian consumer demands, Berndt, Darrough, and Diewert (1977) found that own-price elasticities varied by 15-50 percent when symmetry was maintained in their models and up to several thousand percent with frequent sign changes when symmetry was not maintained. Shumway and Lim (1993) also found similarly large differences among elasticities and among policy inferences as well as sign changes in elasticities when they estimated three functional forms for U.S. agricultural production. In both studies, all functional forms were second-order Taylor series expansions and seemingly equally suitable *a priori* for the analysis of production or consumption relationships. Lim and Shumway (1997) found that failure to properly account for the time series properties of the data in their analysis of U.S. agricultural production produced differences in the magnitudes and signs of estimated price elasticities comparable to those observed among functional forms. Consequently, the elasticity differences observed here from a possible input aggregation specification error are small relative to the differences previously observed from specification errors due to incorrect choice of functional form or failure to account for the time series properties of data.

Conclusions

Few have any illusions that the "true" model structure can ever be identified. Nevertheless, improved model specification is sought in this as well as other papers to assure that behavioral properties which apply to disaggregate relationships also apply to the aggregate relationships. We have documented a wide variety of commodity-wise aggregation test conclusions in the empirical agricultural economics literature. We have also documented considerable variation in measured errors of inference in related literature because of inappropriate or imprecise aggregation. Through our own empirical testing with two aggregations and alternative model specifications, we determined that failure to empirically reject consistent aggregation in a partition was insufficient to totally mitigate erroneous inference due to the aggregation. In one of the cases, considerable elasticity differences were observed when aggregate data were used in analysis. However, the elasticity differences observed here from the possible aggregation specification error were small relative to the differences previously observed from specification errors due to incorrect choice of functional form or failure to account for the time series properties of data.

It is also important to emphasize and warn that any effort to decrease specification error cannot be taken to an extreme. It is useful here to think in terms of a "neighborhood aggregation invariance principle" because the level of aggregation ultimately should be dictated by the question of interest. Even if some inferential errors occur because of aggregation, one cannot expect very disaggregate firm level data or commodity categories to be useful in analyzing what are often industry level concerns such as supply and demand.

We conclude with an excerpt from Davis (1999, pp. 478-79) a statement based on Mill (1844):

> "The theory of the firm is an inductive theory that came from observing the behavior of *many* firms and distilling from those observations the basic elements common to all of those firms. It does not actually describe the objective function and constraints of any particular firm, but only what all firms have in common as a 'tendency.' . . . A theory is like an inductive causal averaging procedure that ignores individual differences and concentrates only on similar tendencies. While highlighting a few common factors many more individual idiosyncrasies and factors are ignored. It is a theory of 'the' firm — the abstract firm. It is not a theory of 'a' firm, an individual firm. This simple but important distinction means the theory of the firm cannot be taken off the economics theory shelf and directly applied to some industry or firms without modification. A theory must be tailored to the market under study. A theory only provides a foundation for developing a more realistic account of the firm or industry under consideration. Thus when a researcher prepares to study a particular firm, adjustments, additions, and allowances must be made to the theory to take into account what Mill calls "disturbing causes." Alternatively stated, chopping off relevant aspects of markets (firms) or stretching other irrelevant aspects of markets (firms) so that they fit the Procrustean bed of a theory is poor applied economics."

References

Arnade, C.A. and C.W. Davison. "Testing for Aggregation and Simultaneous Bias in U.S. Soybean Export Equations." *Southern Journal of Agricultural Economics* 21(December 1989):129-37.

Asche, F., H. Bremnes, and C.R. Wessells. "Product Aggregation, Market Integration, and Relationships between Prices: An Application to World Salmon Markets." *American Journal of Agricultural Economics* 81(August 1999):568-81.

Ball, V. E. "U.S. Agricultural Data Set." Unpublished data, U.S. Department of Agriculture, Washington DC, 1996.

______ "Modeling Supply Response in a Multiproduct Framework." *American Journal of Agricultural Economics* 70(November 1988):813-25.

Barnett, W. A. "Theoretical Foundations for the Rotterdam Model." *Review of Economic Studies*. 46(1979):109-130.

Berndt, E.R., M.N. Darrough, and W.E. Diewert. "Flexible Functional Forms and Expenditure Distributions: An Application to Canadian Consumer Demand Functions." *International Economic Review* 18(October 1977): 651-75.

Blinder, A.S. "Distribution Effects and the Aggregate Consumption Function." *Journal of Political Economy* 83(1975):447-75.

Blundell, R. P. Pashardes, and G. Weber. "What Do We Learn About Consumer Demand patterns from Micro Data?" *American Economic Review* 83(June 1993):570-97.

Bonnieux, F. "Estimating Regional-Level Input Demand for French Agriculture Using a Translog Production Function." *European Review of Agricultural Economics* 16(1989):229-41.

Buccola, S. and J. Sil. "Productivity Measurement in the Agricultural Marketing Sector." *American Journal of Agricultural Economics* 78(December 1996):1366-71.

Capalbo, S.M. and M.G.S. Denny. "Testing Long-Run Productivity Models for the Canadian and U.S. Agricultural Sectors." *American Journal of Agricultural Economics* 68(August 1986):615-25.

Chambers, R.G. *Applied Production Analysis: A Dual Approach*. New York: Cambridge University Press, 1988.

Chambers, R.G. and R.D. Pope. "Aggregable Price-taking Firms." *European Economic Review* 40(1996):417-

428.

_____. “Testing for Consistent Aggregation.” *American Journal of Agricultural Economics* 73(August 1991):808-18.

Chavas, J.P. and T.L. Cox. “A Nonparametric Analysis of Agricultural Technology.” *American Journal of Agricultural Economics* 70(May 1988):303-10.

Davis, G.C. “Product Aggregation Bias as a Specification Error in Demand Systems.” *American Journal of Agricultural Economics* 79(February 1997):100-09.

_____. “The Science and Art of Promotion Evaluation.” *Agribusiness*: *An International Journal.* vol. 15. 4(1999):465-483.

Davis, G.C., N. Lin, and C.R. Shumway. “Aggregation without Separability: Tests of U.S. and Mexican Agricultural Production Data.” *American Journal of Agricultural Economics* 82(February 2000): forthcoming.

Domowitz, I. and H. White. “Misspecified Models with Dependent Observations,” *Journal of Econometrics*. 20(1982): 35-58.

Eales, J.S. and L.J. Unnevehr. “Demand for Beef and Chicken Products: Separability and Structural Change.” *American Journal of Agricultural Economics* 70(August 1988):521-32.

Green, H.A.J. *Aggregation in Economic Analysis: An Introductory Survey*. Princeton: Princeton University Press. 1964.

Gorman, W. M. “Community Preference Fields.” *Econometrica*. vol. 21(1953):63-80.

_____. “Separable Utility and Aggregation.” *Econometrica*. vol. 27(1959):469-81.

Griliches, Z. “Effects of Aggregation Over Time: Comments,” in *Econometric Models of Cyclical Behavior*, Vol. II. Bert G. Hickman, ed. New York: Columbia University Press. 1972.

Grunfield, Y and Z. Griliches. “Is Aggregation Necessarily Bad?” *Review of Economics and Statistics*. vol XLII. 1(February 1960): 1-13.

Hellerstein, D. “Welfare Estimation Using Aggregate and Individual-Observation Models: A Comparison Using Monte Carlo Techniques.” *American Journal of Agricultural Economics* 77(August 1995):620-30.

Hicks, J. R. *Value and Capital*. Oxford: Oxford University Press. 1936.

Hildenbrand, W. "How Relevant are Specifications of Behavioral Relations on the Micro-Level for Modelling the Time Path of Population Aggregates." *European Economic Review* 42(1998):437-458.

Huffman, W.E. and R.E. Evenson. *Science for Agriculture: A Long-Term Perspective*. Ames: Iowa State University Press, 1993.

Huffman, W.E. Personal communication to C.R. Shumway, Feb. 2, 1999.

Jegasothy, K., C.R. Shumway, and H. Lim. "Production Technology and Input Allocations in Sri Lankan Multicrop Farming." *Journal of Agricultural Economics* 41(January 1990):33-46.

Just, R.E. and R.D. Pope. "Implications of Heterogeneity for Theory and Practice in Production Economics." *Journal of Agricultural Economics* 81(August 1999):711-19.

Kuroda, Y. "The Output Bias of Technological Change in Postwar Japanese Agriculture." *American Journal of Agricultural Economics* 70(August 1988):663-73.

Lee, K.C., M.H. Pesaran, and R.G. Pierse "Testing for Aggregation Bias in Linear Models." *The Economic Journal.* 100 (Conference 1990): 137-150.

Leontief, W. "Composite Commodities and the Problem of Index Numbers." *Econometrica*. vol. 4(1936):39-59.

_____. "Introduction to a Theory of the Internal Structure of Functional Relationships." *Econometrica*. vol. 15(1947):361-73.

Lewbel, A. "Aggregation Without Separability: A Generalized Composite Commodity Theorem." *American Economic Review* 86(June 1996): 524-43.

_____. "Stochastic Hicksian Aggregation With an Application to Grouping Goods Without Separable Utility. *Annales d'Economie et de Statistiques*. 29(1993):17-42.

Lim, H. and C.R. Shumway. "Separability in State-Level Agricultural Technology." *American Journal of Agricultural Economics* 74(February 1992):120-31.

_____. "Technical Change and Model Specification: U.S. Agricultural Production." *American Journal of Agricultural Economics* 79(May 1997):543-54.

Mill, J.S. "On the Definition of Political Economy and on the Method of Investigating Proper to It." Reprinted in *John Stuart Mill's Philosophy of Scientific Method*. E. Nagel, Ed. New York. Hafner Publishing Co. 1950.

Nayga, R.M. and O. Capps. “Tests of Weak Separability in Disaggregated Meat Products.” *American Journal of Agricultural Economics* 76(November 1994):800-08.

Park, W.I. and P. Garcia. “Aggregate versus Disaggregate Analysis: Corn and Soybean Acreage Response in Illinois.” *Review of Agricultural Economics* 16(January 1994):17-26.

Paul, C.J.M. “Aggregation and the Measurement of Technological and Market Structure: The Case of the U.S. Meatpacking Industry. *American Journal of Agricultural Economics* 81(August 1999):624-29.

Pesaran, M. H., R.G. Pierse, and M. S. Kumar. “Econometric Analysis of Aggregation in the Context of Linear Prediction Models.” *Econometrica*. vol. 57. 4(July1989):861-888.

Polson, R.A. and C.R. Shumway. “Structure of South Central Agricultural Production.” *Southern Journal of Agricultural Economics* 22(December 1990):153-63.

Pope, R.D. and A. Hallam. “Separability Testing in Production Economics.” *American Journal of Agricultural Economics* 70(February 1988):142-52.

Ray, S.C. “A Translog Cost Function Analysis of U.S. Agriculture, 1939-77.” *American Journal of Agricultural Economics* 64(August 1982):490-98.

Reed, M.R. and S.K. Riggins. “A Disaggregated Analysis of Corn Acreage Response in Kentucky.” *American Journal of Agricultural Economics* 63(November 1981):708-11.

Sckokai, P. and D. Moro. “Direct Separability in Multi-Output Technologies: An Application to the Italian Agricultural Sector.” *European Review of Agricultural Economics* 23(1996):95-116.

Sellen, D. and E. Goddard. “Weak Separability in Coffee Demand Systems.” *European Review of Agricultural Economics* 24(1997):133-44.

Shumway, C.R. “Supply, Demand, and Technology in a Multiproduct Industry: Texas Field Crops. *American Journal of Agricultural Economics* 65(November 1983):748-60.

Shumway, C.R. and H. Lim. “Functional Form and U.S. Agricultural Production Elasticities.” *Journal of Agricultural and Resource Economics* 18(December 1993):266-76.

Shumway, C.R., R.R. Saez, and P.E. Gottret. “Multiproduct Supply and Input Demand in U.S. Agriculture.” *American Journal of Agricultural Economics* 70(May 1988):330-37.

Simmons, P. “Evidence on the Impact of Income Distribution on Consumer Demand in the UK 1955-68.” *Review* *of Economic Studies* 47(1980):893-906.

Staehle, H. “Short-Period Variations in the Distribution of Incomes.” *Review of Economic Statistics* 19(1937):133-43.

Stoker, T.M. “Simple Tests of Distributional Effects on Macroeconomic Equations.” *Journal of Political Economy* 94(1986):763-95.

Theil, H. *Linear Aggregation of Economic Relations*. Amsterdam: North-Holland. 1954.

Thomas, A.C. and L.W. Tauer. “Linear Input Aggregation Bias in Nonparametric Technical Efficiency Measurement.” *Canadian Journal of Agricultural Economics* 42(1994):77-86.

Turner, R. W. and M. L. Rockel. “Estimating Covariances of Parameter Estimates from Different Models.” *Economics Letters*. 26(1988):137-140.

Villezca, P.A. and C.R. Shumway. “Multiple-Output Production Modeled with Three Functional Forms.” *Journal of Agricultural and Resource Economics* 17(July 1992):13-28.

Weaver, R.D. *The Theory and Measurement of Provisional Agricultural Production Decisions*. Ph.D. Thesis, University of Wisconsin, Madison, 1977.

Williams, S.P. and C.R. Shumway. “Aggregation of Data and Profit Maximization in Mexican Agriculture.” *Applied Economics* 30(1998a):235-44.

_____. “Testing for Behavioral Objective and Aggregation Opportunities in U.S. Agricultural Data.” *American Journal of Agricultural Economics* 80(February 1998b):195-207.

Table 1. Articles in Ten Agricultural Economics Journals Reporting Tests for Consistent Aggregation, 1984-99

Author(s)	Year	Hypothesized Aggregates	Test
Weaver	1977	Production outputs and inputs	Homothetic separability
Ray	1982	Production outputs and inputs	Homothetic separability
Shumway	1983	Production outputs and inputs	Homothetic separability
Capalbo and Denny	1986	Production inputs	Separability
Pope and Hallam	1988	Production inputs	Homothetic separability
Chavas and Cox	1988	Production outputs and inputs	Nonparametric weak separability
Eales and Unnevehr	1988	Consumption goods	Weak separability
Kuroda	1988	Production outputs and inputs	Homothetic separability
Ball	1988	Production outputs	Weak separability
Bonnieux	1989	Production inputs	Weak separability
Jegasothy, Shumway, and Lim	1990	Production inputs	Homothetic separability
Polson and Shumway	1990	Production outputs and inputs	Homothetic separability
		States	Identical technologies
Chambers and Pope	1991	States	Laspeyres-form aggregation
Lim and Shumway	1992	Production outputs and inputs	Nonparametric weak separability
Villezca and Shumway	1992	Production outputs	Homothetic separability
Nayga and Capps	1994	Consumption goods	Weak separability
Sckokai and Moro	1996	Production outputs and inputs	Direct weak separability
Sellen and Goddard	1997	Consumption goods	Homothetic separability
Williams and Shumway	1998a	Production outputs and inputs	Nonparametric homothetic separability
Williams and Shumway	1998b	Production outputs and inputs	Nonparametric homothetic separability
Ashe, Bremnes, and Wessells	1999	Consumption goods	Generalized composite commodity
Davis, Lin, and Shumway	2000	Production outputs	Generalized composite commodity

Table 2. Variable Names and Definitions

Variable Name	Definition
q_1	Livestock quantity
q_2	Crop quantity
q_3	Hired labor quantity
q_4	Capital quantity
q_5	Other purchased input quantity
Q	Aggregate netput (output or input) quantity
y_1	Livestock share of profit
y_2	Crop share of profit
y_3	Hired labor share of profit
y_4	Capital share of profit
y_5	Other purchased input share of profit
Y	Aggregate netput (output or input) share of profit
p_1	Log of livestock price
p_2	Log of crop price
p_3	Log of hired labor price
p_4	Log of capital price
p_5	Log of other purchased input price
P	Log of aggregate netput (output or input) price
z_1	Log of public research expenditures
z_2	Log of private research expenditures
z_3	Log of self-employed labor quantity
z_4	Log of real estate quantity
z_5	Dummy variable (1 for 1983, 0 otherwise)

Table 3a. Output Aggregation Model Parameters Constructed from Disaggregate Parameter Estimates[a]

Share	intercept	P	p_3	p_4	p_5	z_1	z_2	z_3	z_4	z_5	ρ_1	ρ_2
Y	**-141.49**	**-3.33**	**.28**	**.63**	**2.42**	.36	-.95	-1.07	**14.06**	**6.83**	**-1.37**	**-1.95**
y_3	**8.88**	**.28**	**-.18**	**.05**	**-.15**	-.05	**.19**	.10	**-.95**	**-.51**	**.13**	**.39**
y_4	**49.74**	**.62**	**.05**	**-.59**	-.08	-.26	-.12	-.33	**-3.97**	**-1.85**	**.39**	.23

[a]All numbers in bold are significant at the 10% level or smaller.

Table 3b. Output Aggregation Price Elasticities Constructed from Disaggregate Parameter Estimates[a]

Netput	P	p_3	p_4	p_5
Q	**1.48**	**-.16**	**-.31**	**-1.01**
q_3	**2.28**	**-.48**	**-.69**	**-1.11**
q_4	**2.19**	**-.34**	**-.31**	**-1.54**
q_5	**2.04**	**-.16**	**-.45**	**-3.99**

[a]All numbers in bold are significant at the 10% level or smaller. Elasticities evaluated at sample means.

Table 4a. Complete Output Aggregation Model Parameter Estimates[a]

Share	Intercept	P	p_3	p_4	p_5	z_1	z_2	z_3	z_4	z_5	ρ_1	ρ_2
Y	**-140.19**	**-2.95**	**.26**	**.57**	**2.12**	.13	-1.01	-1.49	**14.49**	**6.81**	-.29	**-2.64**
y_3	**8.71**	**.26**	**-.18**	**.06**	-.13	-.02	**.19**	.14	**-.99**	**-.51**	.03	**.23**
y_4	**48.67**	**.57**	**.06**	**-.54**	-.08	-.18	-.07	-.17	**-4.09**	**-1.84**	.08	**.49**

[a]All numbers in bold are significant at the 10% level or smaller.

Table 4b. Complete Output Aggregation Price Elasticities[a]

Netput	P	p_3	p_4	p_5
Q	**1.59**	**-.17**	**-.33**	**-1.10**
q_3	**2.38**	**-.48**	**-.74**	**-1.16**
q_4	**2.30**	**-.36**	**-.40**	**-1.88**
q_5	**2.21**	**-.16**	**-.54**	**-3.83**

[a]All numbers in bold are significant at the 10% level or smaller. Elasticities evaluated at sample means.

Table 5a. Incomplete Output Aggregation Model Parameter Estimates[a]

Share	intercept	P	p_3	p_4	p_5	z_1	z_2	z_3	z_4	z_5	ρ_1	ρ_2
Y	**-139.96**	**-3.26**	**.27**	**.65**	**2.33**	.25	-.88	-1.03	**13.93**	**6.83**	0	0
y_3	**8.76**	**.27**	**-.19**	**.05**	**-.14**	-.04	**.18**	.10	**-.95**	**-.51**	0	0
y_4	**48.56**	**.65**	**.05**	**-.55**	-.16	-.21	-.09	-.25	**-3.99**	**-1.84**	0	0

[a]All numbers in bold are significant at the 10% level or smaller.

Table 5b. Incomplete Output Aggregation Price Elasticities[a]

Netput	P	p_3	p_4	p_5
Q	**1.51**	**-.16**	**-.31**	**-1.04**
q_3	**2.30**	**-.47**	**-.71**	**-1.12**
q_4	**2.13**	**-.34**	**-.39**	**-2.03**
q_5	**2.09**	**-.16**	**-.59**	**-3.89**

[a]All numbers in bold are significant at the 10% level or smaller. Elasticities evaluated at sample means.

Table 6a. Input Aggregation Model Parameters Constructed from Disaggregate Parameter Estimates[a]

Share	intercept	p_1	p_2	P	z_1	z_2	z_3	z_4	z_5	ρ_3	ρ_4	ρ_5
y_2	**-105.87**	**-1.15**	**-.79**	**1.95**	**.57**	-.41	-.48	**9.86**	**3.84**	**.15**	.23	1.57
Y	**142.49**	**1.38**	**1.95**	**-3.33**	-.36	.95	1.07	**-14.07**	-6.83	**-.28**	**-.62**	**-2.42**

[a]All numbers in bold are significant at the 10% level or smaller.

Table 6b. Input Aggregation Price Elasticities Constructed from Disaggregate Parameter Estimates[a]

Netput	p_1	p_2	P
q_1	**.38**	**1.16**	**-4.63**
q_2	**2.55**	**.51**	**-1.43**
Q	**.96**	**1.12**	**-2.09**

[a]All numbers in bold are significant at the 10% level or smaller. Elasticities evaluated at sample means.

Table 7a. Complete Input Aggregation Model Parameter Estimates[a]

Share	intercept	p_1	p_2	P	z_1	z_2	z_3	z_4	z_5	ρ_3	ρ_4	ρ_5
y_2	**-82.18**	-.16	.42	-.26	-.25	.44	-.62	**8.03**	**3.63**	**-.83**	**1.39**	-.84
Y	**120.56**	-.46	-.26	.72	.72	-.53	.89	**-12.00**	**-6.62**	.97	**-2.50**	2.25

[a]All numbers in bold are significant at the 10% level or smaller.

Table 7b. Complete Input Aggregregation Price Elasticities[a]

Netput	p_1	p_2	P
q_1	.12	**1.82**	**-2.75**
q_2	**1.44**	**1.14**	**-2.58**
Q	**1.72**	**2.03**	**-3.74**

[a]All numbers in bold are significant at the 10% level or smaller. Elasticities evaluated at sample means.

Table 8a. Incomplete Input Aggregation Model Parameter Estimates[a]

Share	intercept	p_1	p_2	P	z_1	z_2	z_3	z_4	z_5	ρ_3	ρ_4	ρ_5
y_2	**-105.87**	**-1.13**	-.48	**1.62**	.55	-.33	-.43	**9.79**	**3.85**	0	0	0
Y	**139.97**	**1.54**	**1.62**	**-3.16**	-.36	.96	1.10	**-13.87**	**-6.85**	0	0	0

[a]All numbers in bold are significant at the 10% level or smaller.

Table 8b. Incomplete Input Aggregation Price Elasticities[a]

Netput	p_1	p_2	P
q_1	**.79**	**1.18**	**-1.44**
q_2	**.94**	**.67**	**-1.61**
Q	**.89**	**1.26**	**-2.61**

[a]All numbers in bold are significant at the 10% level or smaller. Elasticities evaluated at sample means.

Appendix

The covariance of $\mathbf{\delta_i = (b_i^r - b_A) = (b_i^r - A\hat{\beta}_d)}$ is denoted by ψ_i. The general covariance and its asymptotic properties can be found in several places (see Turner and Rockel, or within a GMM framework see Domowitz and White). The derivations for the restricted seemingly unrelated regression estimator are rather tedious but straightforward and only the results are given here. The **r** superscript will be dropped here for simplicity. The general formula for the covariance is

(A.1) $\boldsymbol{\psi_i} = \mathrm{Cov}(\mathbf{b_i}) + \mathrm{Cov}(\mathbf{b_A}) - \mathrm{Cov}(\mathbf{b_i}, \mathbf{b_A}) - \mathrm{Cov}(\mathbf{b_i}, \mathbf{b_A})^T$.

The components of this general formula in the restricted seemingly unrelated regression case are as follows. Define in general the matrices $\mathbf{M_i} = \mathbf{I_i - C_i R_i^T(R_iC_iR_i^T)^{-1}R_i}$ and $\mathbf{C_i = (X_i^T\Omega_i^{-1}X_i)^{-1}}$ and the residual vector $\mathbf{e_i}$. By altering the **i** subscript, these matrices will alter accordingly as in the text where all matrices except $\mathbf{\Omega_i}$ are defined. The matrix $\mathbf{\Omega_i}$ is the covariance distribution for the system disturbance vector $\mathbf{e_i}$. Using this notation, it can be shown that the components of (A.1) are

(A.2) $\mathrm{Cov}(\mathbf{b_i}) = E[\mathbf{M_iC_i}]$

(A.3) $\mathrm{Cov}(\mathbf{b_A}) = E[\mathbf{AM_iC_iA^T}]$

(A.4) $\mathrm{Cov}(\mathbf{b_i}, \mathbf{b_A}) = E[(\mathbf{M_iC_iX_A^T\Omega_i^{-1}e_i})(\mathbf{AM_dC_dX_d^T\Omega_d^{-1}e_d})^\mathbf{T}]$.

As is standard practice, the residuals are used to estimate $\mathbf{\Omega_i}$ and the formulas (A.2), (A.3), and (A.4) are used without the expectation sign in (A.1) to form the covariance $\boldsymbol{\psi_i}$, which is used in the test statistic given by equation (10) in the text.