Farming Systems Planning Under Uncertainty: Mathematical Programming and Stochastic Efficiency Analysis

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FARMING SYSTEMS PLANNING UNDER UNCERTAINTY: MATHEMATICAL PROGRAMMING AND STOCHASTIC EFFICIENCY ANALYSIS

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The complexity of modelling risk in farming systems is explained and the artistic nature of the task noted. A brief outline is presented of an appropriate conceptual framework, drawing attention to the merits of stochastic efficiency criteria for analysis of systems when risk preferences of individual farmers are unavailable. A distinction is drawn between planning problems with and without embedded risk. For the non embedded risk case some risk programming methods are reviewed, drawing attention to those that generate stochastically efficient solutions. The merits of 'utility efficient' (UE) programming are explained. Some ways of using UE programming to generate solutions that are stochastically dominant with respect to a function are noted. Extensions of programming models, including UE formulations, to embedded risk using discrete stochastic programming are reviewed, noting the problems of dimensionality and means by which they can be minimised. The paper concludes with a discussion of the importance of correctly understanding the way risk impacts upon the target farming system, and then of formulating a programming model appropriate to the case. No longer does lack of computing capacity justify use of crude but easily solved a, ximate forms of model. Rather the task now facing analysts is to develop m. els that are sufficiently well formulated to generate helpful insights about the target farming systems

Introduction

All planning, including planning farming systems, involves uncertainty. Plans have their outcomes in the future and we can never be absolutely sure what the future will bring. Uncertainty is important because it affects the consequences of decisions in ways that decision makers are not indifferent about. Such uncertainty in consequences is called risk, and most people are averse to risk. In complex, non-linear (concave) systems, such as farming systems, uncertainty works to both reduce the expected value of consequences - downside risk - and to create deviations in consequences from their expected values - pure risk. Both types of risk may need to be accounted for when planning .ch syste .

Yet risk and uncertainty, by their very nature are difficult to deal with. Because uncertainty is widespread in its origins and pervasive in its impacts, it cannot be fully accommodated in any planning model. The analyst must always simplify, so that modelling becomes an artistic process, depending on the perceptions of reality of the analyst and on his or her ability to convert those perceptions into an 'appropriate' planning model. This paper is motivated by the belief that too often the kinds of mathematical programming (MP) models that are built to represent farming systems show a sorry lack of artistry. Analysts too often seem to allow themselves to be governed by outmoded ideas of what is computationally feasible. Therefore, this paper is more expository than novel, with the aim of redressing to some extent this situation.

Conceptual Background

What follows is based on the proposition that the subjective expected utility (SEU) hypothesis provides the best operational basis for structuring risky choice.[1] The SEU hypothesis involves disaggregating risky decision problems into separate assessments of the decision maker's beliefs about the uncertainty, captured via subjective probabilities, and his or her preferences for consequences, captured via a utility function, with the two parts then recombined to select as optimal the decision which yields the highest expected utility.

The implications for modelling decisions about farming systems seem clear, the individual farmer's beliefs and preferences are vital inputs to the planning process. But in practice things are not hat simple. Experience shows that there may be considerable problems of elicitation, especially of utilities. Also, in many planning studies it may be far from obvious whose beliefs and preferences are relevant. The analysis may be being performed to generate recommendations for numbers of farmers (constituting a 'target group' or 'recommendation domain'), each of whom may be supposed to have different beliefs and preferences. Such will often be the case in less developed countries where agriculture is typically composed of many small farm units, each alone too small to justify the expense of an individual planning study.

Approaches to such difficulties may start with the adoption of 'public' probabilities, that are based on the best available data or expert opinions. Such probabilities may be viewed as the beliefs towards which farmers' opinions may be expected to converge as extension programs are developed to inform them of such things as new technologies, improved market opportunities, better outlook information etc.

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[1] It must be admitted that the SEU hypothesis has come under increasing criticism on the grounds of accumulating evidence of frequent breakdown of the so called independence axiom (see, for example, Machina 1981). Yet it seems that no better operational framework has yet found wide acceptance.

So far as farmers' preferences are concerned, no such convergence may be expected, regardless of extension efforts. However, something may be known, or may be able to be inferred, about the range of risk attitudes among the target population of farmers. In this case, the methods of stochastic efficiency analysis provide a means of partitioning decision strategies into efficient and dominated sets. Any individual farmer whose risk averse behaviour is consistent with the assumptions made will find his or her optimal strategy among the efficient set. The task for the analyst is to make this set as small as possible without excluding from the set the strategies that would actually be preferred by any appreciable number of farmers in the target population. This is usually done by setting bounds on the degree of risk aversion anticipated, using the method of stochastic dominance analysis with respect to a function, also known as generalised stochastic efficiency analysis (Meyer 1977a, b). Clearly, it will be desirable to incorporate the same principles into MP models of farming systems.

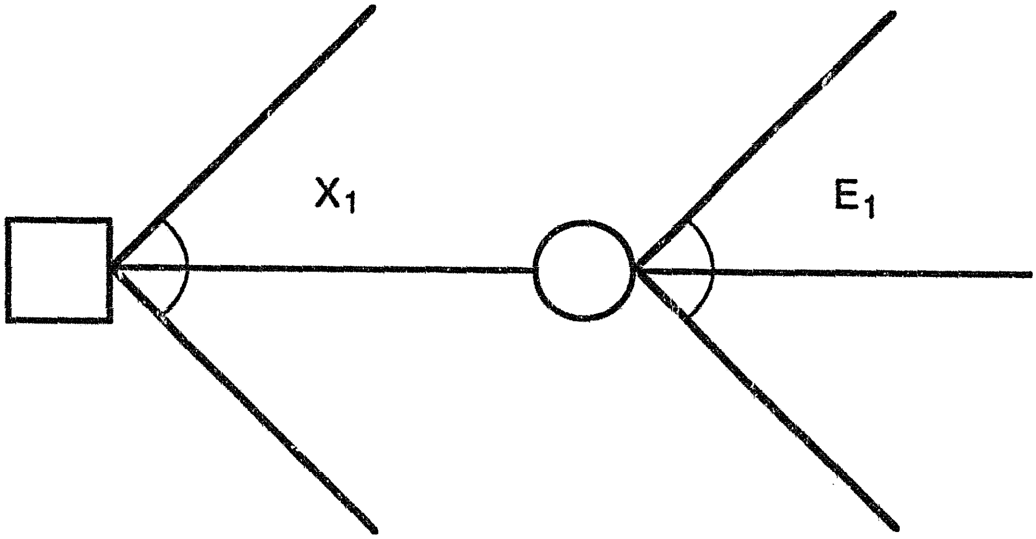
Structuring Risk Problems

The modelling of any risky farming system must start with an understanding of the way uncertainty impacts on that system. An outline decision tree provides a good means for capturing in a simple diagram the principal kinds of decision that the farmer must make and the main sources of uncertainty impinging on those choices. As noted, the focus should be on representing the essential features of the system rather than on the impossible task of reflecting all the detail and complexity of the real system.

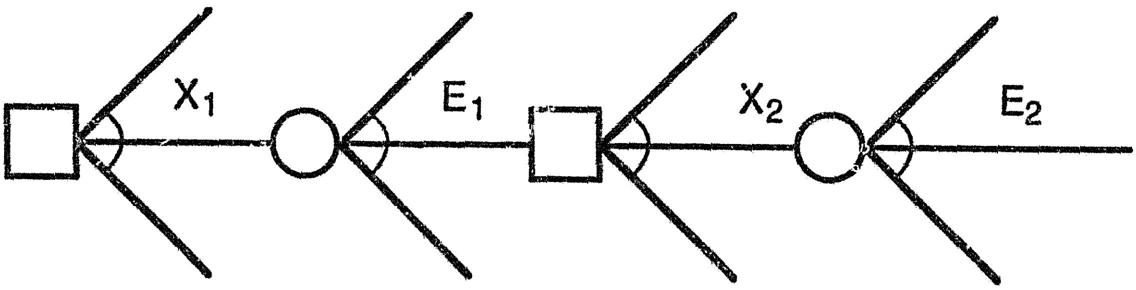
Outline decision trees for two basic cases are illustrated in Figure 1. In this figure the convention is followed of representing decisions with multiple options by decision fans, shown with small squares for their nodes, and uncertain events with many possible outcomes by event fans, represented with small circles as their nodes. At each node, the tree is continued for only one of the many possible branches.

The first case shown in the figure is based on the assumption that it is realistic to model the system as if all decisions are made initially and then the uncertainty unfolds subsequently in terms of risky consequences of the choice taken, i.e. non-embedded risk. In the second case some embedded risk is recognised, in that the decisions are segregated into those taken initially and those taken at a later stage when some uncertainty has unfolded. The second stage decisions will be conditioned by both the initial choices and the revealed uncertain outcomes. The final outcomes of the whole process are regarded as still uncertain, represented by the right-most event fan.

Most real decisions about farming system have the characteristics of the second case, rather than the first. Indeed, farm decision making involves a continuous sequence through time of decision and events, involving many stages, not just two as shown in the figure. Yet most MP



The simplest case



Embedded risk (two stage)

Figure 1: Outline Decision Trees

studies of farming systems have either ignored risk, or have treated it as not embedded. The reasons are not hard to find Any accounting for risk in MP models complicates the task, and accounting for embedded risk is especially difficult Risk programming methods are reasonably well developed to handle the non-embedded case, but less progress has been made in the development of stochastic programming approaches to handle embedded risk.

Risk Programming Approaches

To provide a basis for what follows, the notation for the standard linear programming (LP) model is introduced first, then some of the more widely used risk programming models are briefly reviewed

Linear programming

In a risk programming context, LP can be used to represent the maximisation of expected profit, as follows

$$\text{maximise } E = c'x - f$$

subject to

$$Ax \leq b$$

$$\text{and } x \geq 0.$$

where E is expected profit,

c is an n by 1 vector of activity expected net revenues;

x is an n by 1 vector of activity levels,

f is fixed costs;

A is an m by n matrix of technical coefficients, and

b is an m by 1 vector of resource stocks

Define $c = p'C$, where

p is an s by 1 vector of state probabilities, and

C is an s by n matrix of activity net revenues by state (row) and activity (column)

This formulation differs from the conventional one in that fixed costs are recognised (although their level does not affect the solution in this linear case) and by the explicit accounting for risk in activity net revenues across p states of nature The matrix C may be based on historical data, corrected for inflation and trends, or may be partially or wholly subjective In either case, there is no reason why the (subjective) probabilities, p, should necessarily be equal for all states

In the above LP model the stochastic nature of the activity net revenues is recognised, but risk aversion on the part of the farmer is ignored. Perhaps the best-known extension of the model to account for risk aversion is quadratic risk programming (QRP).

Quadratic risk programming

The QRP model may be formulated as follows:

$$\begin{aligned} &\text{maximise } E = c'x - f \\ &\text{subject to} \\ &\qquad Ax \leq b \\ &\qquad x'Qx = V, \quad V \text{ parametric} \\ &\qquad \text{and } x \geq 0 \end{aligned}$$

where Q is an n by n activity net revenue variance-covariance matrix; and V is the variance of the net income of the farm plan.

Note that $Q = (p'D)'(p'D)$, where $D = C - uc'$, i.e. an s by n matrix of deviations of activity net revenues from respective means, with u defined as an s by 1 vector of ones.

The formulation above generates the so-called (E, V)-efficient set of solutions. It is equivalent to the more usual formulation where variance is minimised subject to a parametric constraint on expected income but is preferred on grounds of consistency with what follows. In computation, however, it is usually easier to minimise the quadratic function for variance subject to a parametric constraint on expected income.

QRP is easy to use given access to a suitable computer program. However, the generated (E, V)-efficient set is stochastically efficient only under the strong assumptions that either the distribution of total net revenue is normal, or the farmer's utility function is quadratic (Levy and Hanoch 1970). The quadratic utility function has the unfortunate properties of not being everywhere increasing and of implying increasing risk aversion, so is generally not regarded as acceptable. Approximate normality in the distribution of total net revenue may be reasonable, but the question is really an empirical one and the form of distribution will vary from case to case. Moreover, conventional statistical tests of the adequacy of the normal approximation are not appropriate in the assumed presence of non-linear utility for income - the issue is whether the deviations from the normal distribution matter to the decision maker, not whether they satisfy some arbitrary statistical criterion.

In the days when quadratic programming computer codes were less available and less reliable than they are today, many efforts were made to find LP approximations to the QRP formulation. By far the most successful was Hazell's MOTAD programming (Hazell 1971).

MOTAD programming

The MOTAD model is:

$$\text{maximise } E = c'x - f$$

subject to

$$Ax \leq b$$
$$Dx + Iy \geq u0$$
$$p'y \leq M, \quad M \text{ parametric}$$
$$\text{and } x, y \geq 0$$

where I is an s by s identity matrix;

y is an s by 1 vector of activity levels measuring negative income deviations by state; and

M is mean absolute deviation of total net revenue.

An outline of the matrix for MOTAD programming is shown in Figure 2. As for QRP, alternative formulations exist that generate the same solution set.

Although rationalisations of MOTAD programming have been proposed in terms of the 'reasonableness' of a focus of concern on negative rather than positive deviations of income, in fact the approach can be justified in terms of the SEU hypothesis only in terms of it being an approximation to QRP The (E, M)-efficient frontier approximates the (E, V) frontier but, as noted, the latter is generally not stochastically efficient and therefore the (E, M) frontier is even less likely to contain the utility-maximising solution for a given farmer. Though undoubtedly valuable at the time it was developed, it is surprising in these circumstances that MOTAD programming is still so widely used A development of the MOTAD model by Tauer (1983) known as target MOTAD appears to have considerably more merit.

Target MOTAD

This model may be formulated as:

$$\text{maximise } E = c'x - f$$

subject to

$$Ax \leq b$$
$$Cx + Iy \geq uT$$
$$p'y \leq D, \quad D \text{ parametric}$$
$$\text{and } x, y \geq 0$$

where T is target level of total net revenue, and

D is deviation from target.

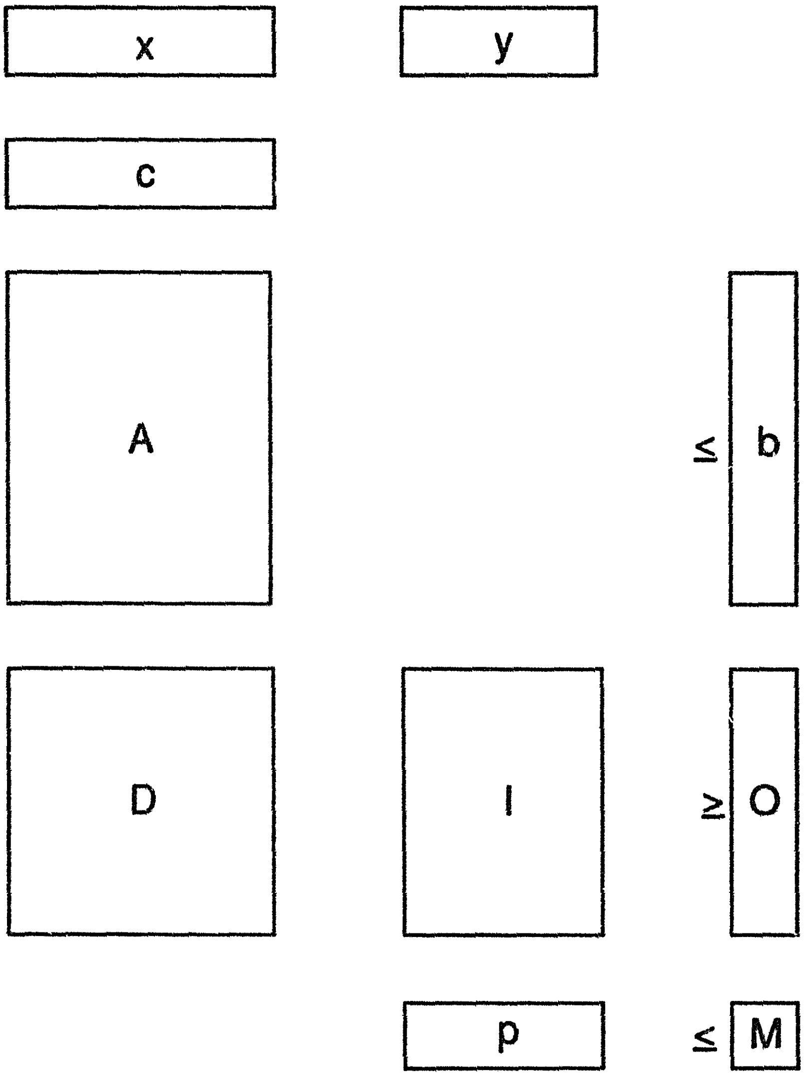


Figure 2: Outline of MOTAD Matrix

The formulation generates the (E, D)-efficient set of solutions for a given value of T. An outline of the matrix is given in Figure 3. The similarities with, and differences from, the standard MOTAD model are apparent

Target MOTAD has the important advantage that the solutions are second-degree stochastically dominant, meaning that they are stochastically efficient for risk-averse decision makers However, the approach has the disadvantage that values of both T and D to be specified. It is possible, but messy, to generate the full solution set for all possible values of these two parameters (McCamley and Kliebenstein 1987) Moreover, no means is provided within the model of discriminating amongst the large range of stochastically efficient solutions that in most cases would thereby be generated

Mean-Gini programming

The mean Gini approach suggested by Yitzhaki (1982), and illustrated in a farm planning context by Okunev and Dillon (1988), can be formulated as

$$\text{maximise } E = c'x - f$$

subject to

$$Ax \leq b$$

$$Bx - Iy^{+} + Iy^{-} = u0$$

$$q'y^{+} + q'y^{-} = G, \; G \text{ parametric}$$

$$\text{and } x, y^{+}, y^{-} \geq 0$$

where B is an h by n matrix of net revenue differences for n activities and all h possible discrete pairs of states, $h = s(s - 1)/2$,

y^{+} and y^{-} are h by 1 vectors of total positive and negative net revenue differences summed across activities for each discrete pair of states;

q is an h by 1 vector of probabilities of these pairs, found as the product of the probabilities of the corresponding two states; and

G is the total Gini difference

An outline for this form of model is given in Figure 4

The mean-Gini programming approach is general in the sense that it is applicable to any monotonic concave utility function and probability distribution Because (E, G)-efficient sets are always second-degree stochastically efficient (though the reverse is not always true) the method is superior to quadratic risk programming and MOTAD The main advantage of the approach is that it is relatively easy to use, being based on only a two-parameter model A possible limitation is that some stochastically efficient solutions that would be preferred by strongly risk-averse decision makers may be excluded from the efficient set However, this limitation may not be

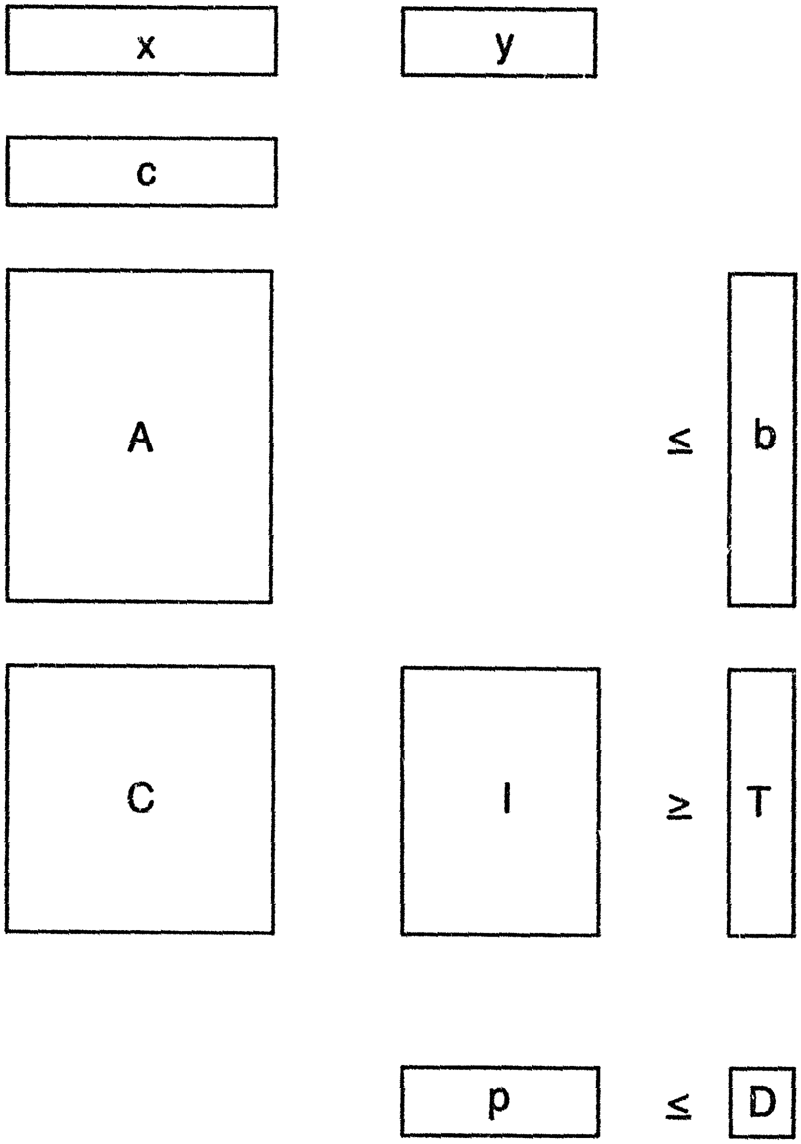


Figure 3: Outline of Target MOTAD Matrix

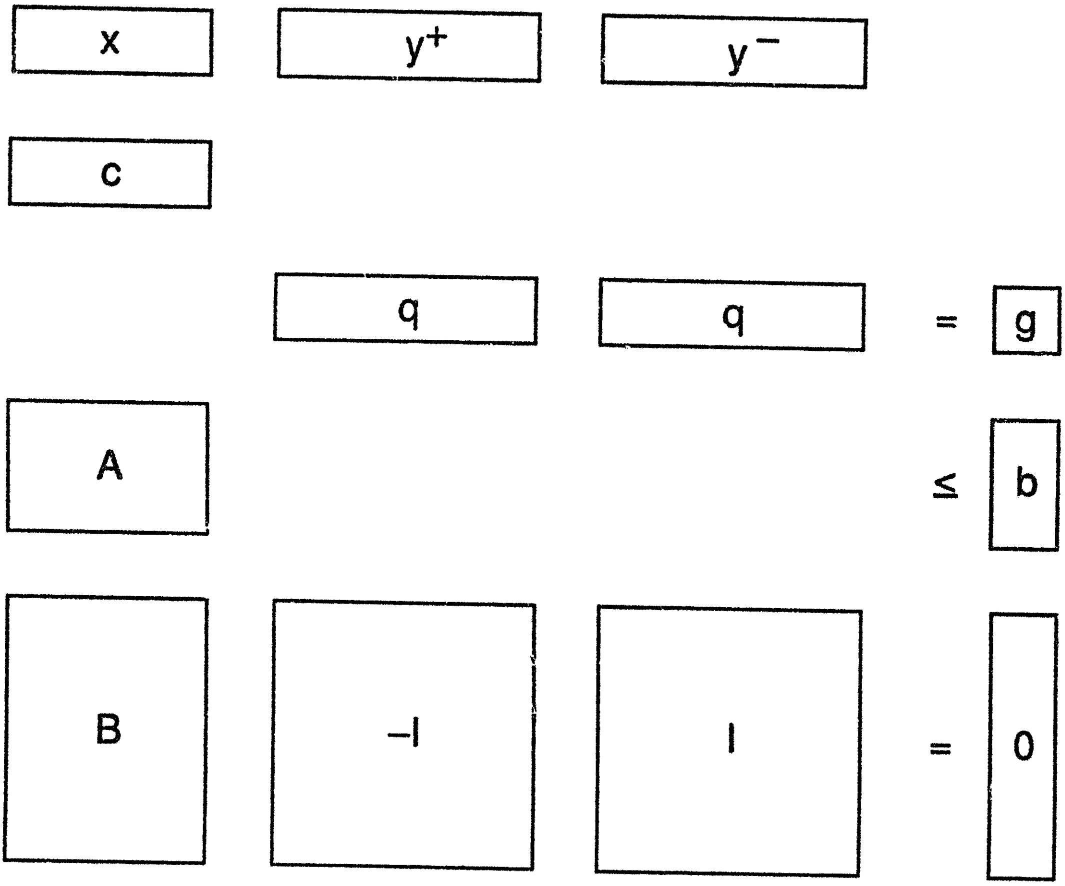


Figure 4: Outline of Mean-Gini Matrix

serious and may in fact be an advantage if decision makers are only weakly risk averse (Buccola and Subaei 1984)

As with target MOTAD. mean-Gini programming will generate a large set of solutions. To narrow down the range of solutions evidently requires some knowledge of the attitude(s) to risk of the farmer(s). One extreme possibility exists if an individual farmer is identified whose utility function can be elicited. In this case direct utility maximisation is appropriate

Utility maximisation

Lambert and McCarl (1985) have illustrated MP models involving direct maximisation of expected utility. The implied generally nonlinear programming model is of the form:

$$\text{maximise } E[U] = pU(z)$$

subject to

$$Ax \leq b$$
$$Cx - Iz = uf$$
$$\text{and } x \geq 0$$

where U( ) is a monotonic concave utility function,
z is an s by 1 vector of net incomes by state, and
U(z) is an s by 1 vector of utilities of net income by state

Because U( ) is monotonic and concave. nonlinear algorithms such as MINOS (Murtagh and Saunders 1977) will find the global optimum. Alternatively approximation on U( ) by linear segmentation is straightforward (Duloy and Norton 1975). A method of progressive improvement of the linear approximation can be used by adding additional linear segments in the region of the initially determined values of z

As discussed above. it will often be inappropriate or impossible to elicit an individual farmer's utility function for direct incorporation into a utility maximising risk programming model. Patten. Hardaker and Pannell (1988) have proposed a means of generating a set of solutions of wider interest when less than complete information is available about farmers' risk attitudes. They called their approach utility efficient (UE) programming

Utility efficient programming

The method proposed by Patten et al. depends on the definition of a separable utility function of the form $U = G(z) + \beta\{H(z)\}$ where variation in the parameter β can be interpreted as variation in risk preference

The UE programming model takes the form

$$\text{maximise } E[U] = p'G(z) + \beta\{p'H(z)\},\ \beta \text{ parametric}$$

subject to

$$Ax \leq b$$
$$Cx - Iz = uf$$
$$\text{and } x \geq 0$$

An outline of the matrix for this formulation in given in Figure 5

Patten et al emphasised the so called 'sumex' function,

$$U = -\exp(-gz) - \beta\{\exp(-hz)\},\ \beta, g, h \geq 0.$$

which has a number of desirable properties The function implies decreasing risk aversion as z increases, in accord with expected 'normal' behaviour In addition, as β is varied the coefficient of absolute risk aversion also varies, ranging from g when β is zero to close to h when β is large They illustrated UE programming using linear segmentation of the utility function, permitting solution using parametric linear programming However, the model can also be solved using a nonlinear algorithm Moreover, although software such as MINOS does not include a parametric option, it is possible to use the software to generate a large number of solutions for a range of values of β with little trouble, approximating the full set obtainable by parametric programming

Although not mentioned by Patten et al, another form of UE programming could make use of the negative exponential utility function of the parametric form

$$U = -\exp[-\{(1-\beta)g + \beta h\}z],\ \beta \text{ parametric.}$$

which may be supposed to generate a set of solutions very similar to, if not identical with, those identified as efficient using stochastic dominance with respect to a function, with bounds set on the coefficient of absolute risk aversion of g and h

Patten et al derived efficient farm plans assuming a sumex utility function which was approximated by linear segments to facilitate the application of a linear programming algorithm For illustrative purposes the same problem was solved for both the negative exponential and the sumex utility functions using a nonlinear programming algorithm In each case, a range of solutions was obtained by setting g and h at the assumed lower and upper bounds of the coefficient of absolute risk aversion then solving for the relevant range of values of β Both specifications produced very similar results Further empirical studies with more realistic examples will be needed to establish the generality of this observation

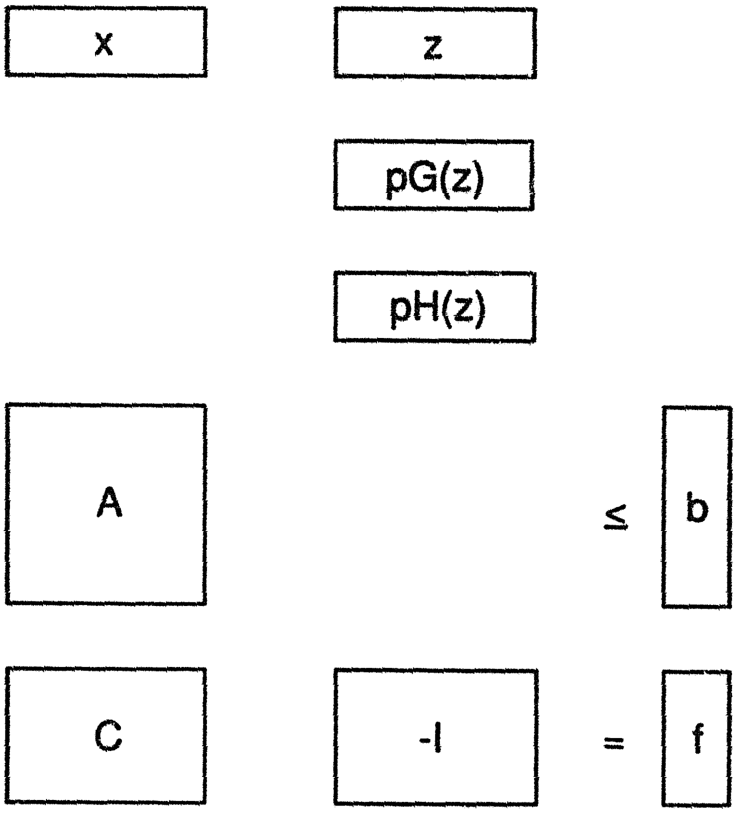


Figure 5: Outline of UE Matrix

Stochastic Programming

Various approaches to the solution of problems with embedded risk have been proposed. In methods such as chance-constrained programming, risk in the constraints is dealt with indirectly by setting a probability with which the constraints, individually or collectively, must be satisfied. However, such methods are less than ideal because the choice of the critical level of probability is itself a part of the decision problem, with associated payoffs and risks. To sweep this part away by means of an arbitrary judgment seems unsatisfactory.

Except for a few special cases, the best approach to problems with embedded risk appears to be via discrete stochastic programming (DSP) (Cocks 1968, Rae 1971a, b). A DSP model for the simplest two-stage problem may be formulated as

$$\text{maximise } E[U] = p'U(z_{2t})$$

subject to

$$A_1 x_1 \leq b_1$$
$$-L_{1t} x_1 + A_{2t} x_{2t} \leq b_{2t}$$
$$C_{2t} x_{2t} - I_{2t} z_{2t} = f_{2t}$$
$$\text{and } x_1, x_{2t} \geq 0 \quad t = 1, \ldots, s$$

where subscripts 1 and 2 indicate first- and second-stage activities, respectively, and the subscript t indicates the state of nature, and L_{1t} is a set of s matrices linking first- and second-stage activities.

The matrix layout for a two-stage problem with only two states is given in Figure 6. The figure indicates the capacity for the overall matrix of a DSP problem to become very large, especially if more stages and many states are to be accommodated. However, the computational capacity of modern computer MP software is considerable, making it technically possible to solve large problems. Nevertheless the so-called 'curse of dimensionality' is a very real consideration since large problems, even if computable, imply minimally a time-consuming and tedious data handling task. On the positive side, use of computer software such as spreadsheets adapted for data input or computerised matrix generators can help. Moreover, the later stages in large problems can be abridged since they need be present only in sufficient detail to assure the 'correct' first stage decision. Actual later stage decisions can be resolved by running further models incorporating the outcomes of uncertain events as they unfold (Kaiser and Apland 1989). The problem for the analyst, of course, is to know what degree of abridgment is acceptable at any stage for a given model.

A major advantage of DSP is that the sequential nature of decision problem can be represented in the model. Due to this feature, risks in both the constraints and the input-output

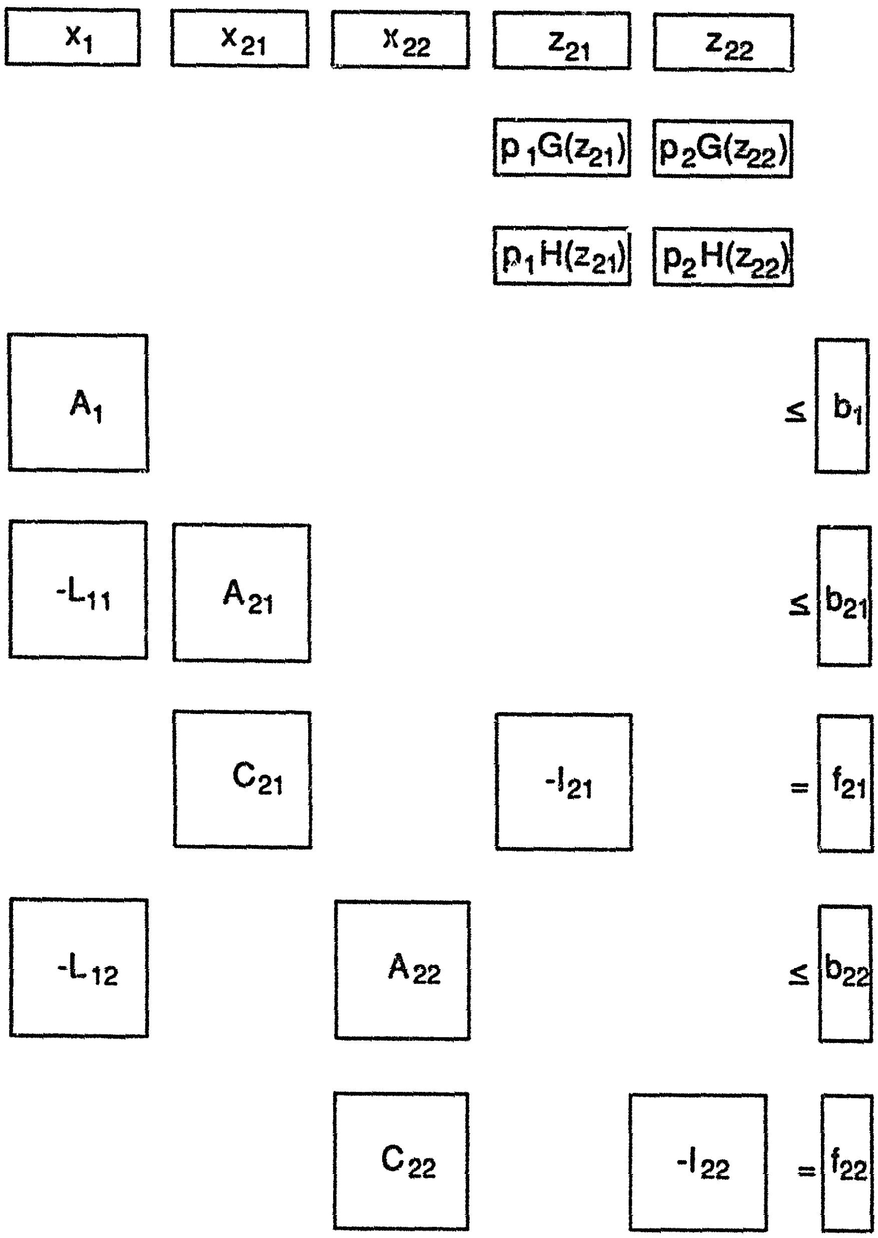


Figure 6: Outline of UE-DSP Matrix (nonlinear)
(Two states only)

coefficients can be modelled These are often more important sources of risks in farming. Whether or not a farmer is risk averse, the downside risk that is embedded in most farming systems can be captured at least in approximate fashion in DSP. Because in embedded risk cases the later stage decisions are not only dependent of the earlier decisions but also on the outcome of random events, DSP generates an optimal strategy, with recommended levels of some activities being conditional on uncertain outcomes that become known only with the passage of time.

Although the 'curse of dimensionality' can be tackled to a certain extent by the judicious formulation of the problem, DSP models often require more data and analyst's time than some of the models described earlier The extra insights that can be discerned from DSP need to be weighed against these costs.

Inspection of Figure 6 in comparison with earlier matrix layouts reveals that several of the risk programming models already discussed can best be viewed as special cases of DSP. In particular, it is evident that methods such as target MOTAD and UE programming extend readily to the full DSP case with embedded risk.

Overview and Prospect

Given the extra complexity of accounting for risk and risk aversion in MP models, the first issue to consider is whether risk matters in planning farming systems. Clearly, downside risk may be important in some cases, and should be accounted for. Too often, it seems, models are constructed using overly optimistic technical or economic planning coefficients. The phenomenon is not confined to MP studies, there is abundant evidence of widespread over-estimation in the formulation and appraisal of rural development projects (World Bank 1988) The causes appear to include a tendency to use modal or 'normal' values, rather than expected values, related to a lack of appreciation of the skewed nature of the distributions of many planning coefficients, especially on the output side.

Risk aversion may be less important than is commonly thought. For example, there is accumulating evidence about the levels of risk aversion in various farming communities. Certainly, farmers generally, and particularly poor farmers in LDCs, are risk averse, but not as markedly so as some literature has suggested Poor farmers cannot afford not to take some risks since risks are everywhere Moreover, they are highly constrained by their limited resources in what they can do. Although the importance of risk aversion will vary from situation to situation, Hardaker and Ghodake (1981) found little difference in predictive power for small-scale farmers in the semi-arid tropics of India between QRP models that accounted for the (measured) risk aversion and expected income maximising models However, where risk aversion is present and expected to be important, it clearly needs to be correctly accounted for in an MP model. If a utility function is available, the model should be formulated to maximise expected utility Where no such individual utility function can be used, it will be best to employ a parametric MP model that will generate the smallest possible subset of efficient solutions, while minimising the chance that the solution most preferred by any individual farmer is excluded. While target MOTAD and mean-Gini methods may do this job, UE programming appears to have most to offer, especially if something is known about the relevant form of utility function and range of risk aversion.

It is essential in designing MP models to judge whether important risks are embedded or non-embedded. Where embedded risk is present, recognising its impact via DSP will generally give much better solutions, irrespective of whether the utility function is linear or non-linear. Most farming systems tend to have embedded risk and, hence, efforts at modelling such risks via appropriate DSP formulations can be rewarding. Moreover, the methods for accounting for risk aversion discussed in relation to risk programming extend directly to the DSP case.

The expanding power of computers and the increasing availability and power of MP software appear to open the door for much bigger and better models of farming systems, including models accounting for risk and risk aversion. Of course, bigger models may not be better. In planning farming systems we are still a long way from the situation that prevails in the formulation of animal feeds where the output of a well-developed MP model can confidently be used as a plan of action. Farming systems, partly because of their human sub-systems, are not and will never be amenable to such treatment. Rather models of farming systems must be viewed as aids to decision making. The value of modelling comes first from the systems analysis implied in developing the model. It is necessary to find out many features of the real system before a plausible model can be built. But once built, it has to be recognised that the model is at best a caricature of the real system. The value in solving the model comes from understanding the cause and effect relationships at work within the model and then from noting the similarities and differences between these modelled features and the reality.

It follows that the value of a model in use depends not only on its size; the skill with which it has been constructed is also important. The challenge is to build better, not necessarily bigger models. Because model building is an artistic process, as discussed in the introduction, it is not surprising that some people do the job better than others, and that there is much to be learnt from experience. The unfortunate reality is that so many novices appear to ignore what is already known. Nowhere is this more true than in accounting for risk, as evidenced by the still widespread use of the inferior risk programming formulations that are inappropriate to accommodate both the type of risk and the preferences of the farmers. It has been argued in this paper that the challenge for the future in accounting for risk in modelling farming systems lies in (a) correctly accounting for embedded risk, and (b) finding the best way of generating smaller and yet more relevant stochastically efficient solution sets.

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