Transboundary Extraction of Groundwater in the Presence of Hydraulic Fracturing

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We studied transboundary ground water management problems in the presence of hydraulic fracturing (fracking). We found that the presence of risk suggests a need to exercise caution in fracking. We also found that a cooperative outcome implies the decrease in fracking and the increase in steady state survival rate of groundwater. However, water extraction rates remained the same in both cooperative and noncooperative solutions. We also argue that a Pigouvian type tax could be imposed on the natural gas developers.

Key words: cooperative and noncooperative games, groundwater, hydraulic fracturing,

JEL Codes: C7, Q25, Q40, Q53

Transboundary Extraction of Groundwater in the Presence of Hydraulic Fracturing

A vast literature in the economics of transboundary resource sharing already exists (examples include Munro, 1979; Lange, Mungatana, and Hassan 2007; Calvo and Rubio, 2012; Long, 2012). However, this literature is void of explicit incorporation of endogenous risk in a dynamic game setting because of the technical difficulty. Many such analyses are done with the simplifying assumption that control is linear in state variables or by imposing other similar constraints on control or state space. Nonlinear strategies that approach optimal cooperative solutions have been suggested (Tsutsui and Mino, 1990). The sub-optimality of linear methods is well documented (Shimomura and Xie, 2008). Other issues such as time inconsistency (Calvo, 1978) and nonuniqueness of solutions (Tsutsui and Mino, 1990) are also well known. This often leads to an analysis of transboundary resource sharing using the Cournot Nash setting, in a steady state situation. Unfortunately, this approach can be of little assistance when the number of state variables increases. Since risk is generally modeled as a state variable in an optimal control method, particularly because control variables affect its evolution, including risk is similar to having an additional state variable. All of these issues have contributed to difficulty in solving a transboundary resource problem in the presence of risk. Recent papers have addressed this issue, but only in the context where risk is not a state variable (Antoniadou et al., 2013). We expand this literature by focusing on groundwater extraction under the transboundary situation when there is a risk of water quality deterioration.

Natural gas production through hydraulic fracturing or fracking (specifically horizontal slickwater fracking)[1] has brought or is likely to bring economic development into many parts of the U.S. Examples include: Marcellus Shale in New York, Barnett Shale in Texas, Eagle Ford Shale in Texas, Haynesville Shale in Louisiana, Arkansas and Texas, Bakken Shale in North Dakota and Montana, Niobrara shale in the Great Plains of U.S., and Utica shale in the northeastern part of the U.S. Hydraulic fracturing has been the subject of much controversy and discussion due to its impact on groundwater quality, groundwater quantity, environmental quality, and health. Recent work suggests that fracking could contaminate groundwater, either by helping expediate the salination process or by chemical or methane intrusion. Entrekin et al. (2011), Warner et al. (2012), Olmstead et al. (2013), and Vidic et al. (2013) provide excellent summaries of current technical issues surrounding fracking.

In general, fracking has been commended for reducing natural gas prices and providing economic opportunities. But its relationship with nearby aquifers is complicated. For example, the Carrizo-Wilcox aquifer, which lies beneath Haynesville-Bossier shale and is shared by Louisiana and Texas, provides 7.49 million gallon (mgal)/day of water for public supply, 2.29 mgal/day for industry use, 4.60 mgal/day for rural domestic use and approximately 2 mgal/day for agricultural use. If this water source is used to supply water for fracking, it will put enormous pressure on the aquifer. As an alternative, the Louisiana Department of Natural Resources has suggested withdrawing water from the Red River Alluvial Aquifer, if continuous and rapid growth of the energy sector in the region is desired. It is possible that the increase in fracking will be accompanied by increasing shadow prices for water resources, along with health

[1] The use of hydraulic fracturing dates back to the 1940s. However, it was not widely used until 2003. One of the reasons why fracking has been widely used in the U.S. is the EPA's 2005 announcement that hydraulic fracturing does not violate the Safe Drinking Water Act of 1996. This also led to the development of Energy Policy Act of 2005. The EPA is reconsidering this statement and is planning to release detailed findings on the relationship between hydraulic fracturing and water quality in 2014 (USEPA, 2013).

risks. Precise economic studies of the impact of these new developments have been rare, but are slowly emerging. Muehlenbachs et al. (2012) found that housing prices near the shale gas sites were increasing because of gains in commercial values, even though there was a significant price decrease due to water contamination. Gopalakrishnan and Klaiber (2013) provided empirical evidence of the negative, but temporary, impact of fracking on nearby houses. Our goal in this paper is to provide a theoretical economic framework for this issue.

Vidic et al. (2013) list many issues surrounding fracking, but in our opinion, there are two ways through which fracking and groundwater extraction interact. Fracking can cause negative impacts on groundwater, due to chemical or saltwater intrusion, if aquifers are intensively extracted. Among these, saltwater intrusion problems are not specific to fracking, and can occur due to excessive extraction anywhere. Such intrusion into drinking water aquifers has been reported in southern California, southern Florida, and in the Gulf of Mexico coastal region. Economic modeling of saltwater intrusion in aquifers owned by one entity has been done (for example, Tsur and Zamel, 1995), but such analyses are rare in the framework of a transboundary resource allocation problem. In our opinion, this is mainly due to the intractability of these models, especially when the search is confined to analytical solutions. This paper incorporates transboundary and economic issues, relating to groundwater withdrawal under saltwater and chemical intrusion risk due to fracking, and takes small steps towards analytical solutions to such problems.

Our major result is that the presence of risk implies caution is needed in fracking. This is a consistent result in all of our models, and it is consistent with the so called “precautionary principle” (Polasky et al., 2011). We also find that states “over-frack” when they don’t cooperate. However, water extraction rates remain same for both cooperative and noncooperative solutions. We also find that policy makers in principle can impose Pigouvian type tax on natural gas developers which would make the developers operate at the socially optimal level.

Model

(I) Basic Model

There are two states which share a source of water: a groundwater aquifer. The states both have another useful resource, natural gas reserves, which they can mine. The benefit from mining the aquifer is given by $u_i(w_i), i=1,2$, where the subscript i indicates the state, and similarly, the benefits from the natural gas mining, i.e. fracking, is given by $v_i(f_i), i=1,2$. The fracking may, however, pose a risk to the water resources. Any such risk will take a form of chemical spill, methane release or salt water intrusion into the aquifer, after which the aquifer will be useless for both states. However, an intrusion into the aquifer won't affect the use of fracking itself, i.e. even after the chemical or salt water intrusion, the states may continue fracking. Let τ indicate the time at which aquifer will be useless due to the intrusion. This is a random variable and will depend on the amount of fracking. In particular, let F(t) indicate the probability that $\tau \leq t$. Fracking activities change the probability in the following way:

(1) $\quad F^{'}(t) = (f_1 + f_2)(1 - F(t))$

Before solving this problem, we conduct the following transformations of our risk representation. We first change (1) into a more manageable form as follows. Define the survival function $S(t) = 1 - F(t)$. It is clear that (1) can be written in terms of S(t) as follows:

(1') $\quad S^{'}(t) = -S(t)(f_1 + f_2).$

Before proceeding, we provide the following parametric specification for the benefit functions from water and fracking. Let U(w)=$\psi_1 w - \psi_2 w^2$ and v(f)=$\psi_3 f - \psi_4 f^2$. The functional specifications imply that we assume these benefits are symmetric for both states. Clearly, in the absence of risk to aquifer , both States will extract aquifer at the level $w^* = \frac{\psi_1}{2\psi_2}$ and $f^* = \frac{\psi_3}{2\psi_4}$ forever. Our main interest now is to identify impact of the presence of risk on the extraction behavior.

We look for the Markovian strategies of the players. The objective function is given as follows.

$$\max_{w_1, f_1} \int_0^\infty [U(w_1) + v(f_1)](1 - F(t) + v(f_1)F(t))e^{-rt}dt$$

Given that the state variable is as in (1). This can be simplified and written in terms of S(t) as follows:

$$(2) \qquad \max_{w_1, f_1} \int_0^\infty (U(w_1)S(t) + v(f_1))e^{-rt}dt$$
$$s.t. \quad S' = -S(f_1 + f_2)$$

Assuming the value function of this problem to be V(S), where again we have removed player specific subscripts to indicate that we will be largely operating under the assumption that the players will be symmetric, and hence their value function will be the same, and exploiting the autonomous nature of the problem, we note that (2) can be written as follows:

$$(3) \qquad rV(S)= \max_{w_1, f_1} \{U(w_1)S + v(f_1) - V'(S)(S(f_1 + f_2)\}$$

Solving for optimal extraction of water and fracking, we get

$w_1^* = \frac{\psi_1}{2\psi_2}$; $f_1^* = \frac{\psi_3 - SV^{'}(S)}{2\psi_4}$. Assuming symmetry, the fracking and water extraction decisions of player 2 are also going to be the same. In particular, $f_2 = \frac{\psi_3 - SV^{'}(S)}{2\psi_4}$.

This leads to the following simplification of (3):

(4) $$rV(s)= \frac{\psi_1^2}{4\psi_2} S + \frac{\psi_3^2}{4\psi_4} - \frac{\psi_3 SV^{'}(S)}{\psi_4} + \frac{3S^2(V^{'}(S))^2}{4\psi_4}$$

This is a nonlinear differential equation of V(S), with no known analytical solution. Even for the parametric forms given above, numerical methods must be implemented to understand what happens further. We make two small modificationsto equation (4) to progress. The first is what we call a log approximation method. In this method, the variables are used in such a unit that they will be small enough to justify the relationship log(1+x)=x for some x. The second method is more exact, but requires us to constrain our benefit functions for fracking slightly by assuming that its benefit function is purely quadratic. In many case, this would not be a too outrageous an assumption.

To solve equation (4) using the log approximation method, we begin with the ansatz that V(S) is a function given by V(S)= $A\log S + B$. Putting this in (4), and using the first order approximation for log S=S-1 with the appropriate assumption for S, we get an approximate solution as follows: $A = \frac{\psi_1^2}{4r\psi_2}; B = \frac{\psi_1^2}{4\psi_2} + \frac{\psi_3^2 - 4\psi_3}{4\psi_4} + \frac{3\psi_1^4}{64r^2\psi_2^2\psi_4}$. This indicates that

$$f_1 = \begin{cases} \frac{\psi_3 - A}{2\psi_4} & if \ \ \psi_3 \geq A \\ 0 & otherwise \end{cases}.$$

The value of A is equal to the discounted net present value of the aquifer. While the presence of risk doesn't affect the optimal extraction of the aquifer directly, it decreases the optimal fracking by an amount which is the ratio of A to $2\psi_4$. The decrease in fracking also, predictably, depends on the discount rate: the more the discount rate, the less the decrease in fracking. If the value of the aquifer is negligent compared to the value of the fracking, then the fracking will be the same as if there were no aquifer (or no threat to aquifer). Note that this solution is not very reliable for cases when $r \to 0$. The method we talk next is more appropriate to understand what happens when r is close to 0.

The second method is a bit more involved, and is attributable to Tsutsui et al.(1990). The maximization problem again leads to the relationship given in equation (3). Let $U^* = \frac{\psi_1^2}{4\psi_2}$ be the value of $U(w_1)$ when the maximizing value of w_1 is used. Notice that this value doesn't depend on anything other than the parameters. Furthermore, the optimality condition for fracking is given by

$$SV'(S) = \psi_3 - 2f\psi_4$$

Plugging the solutions for w_1 and f_1 into equation (3), considering a symmetric situation (i.e. $f_1 = f_2 = f$) and temporarily assuming $r = 0$, we get

$$0 = U^*S + \psi_3 f - \psi_4 f^2 - 2f(\psi_3 - 2f\psi_4)$$

Which upon simplification give us

$$f = \frac{\psi_3 \pm \sqrt{\psi_3^2 - 12\psi_4 U^* S}}{6\psi_4}$$

To determine whether a + or – sign should be used we make the following observations. Notice that when S=0, there is no reason why fracking should be zero which is what would result if the sign is negative. Therefore, the sign above must be positive and the solution to f must be

(5) $$f = \frac{\psi_3 + \sqrt{\psi_3^2 - 12\psi_4 U^* S}}{6\psi_4} \text{ if } r = 0$$

Two things can be noted immediately from (5). When $S \to 0, f \to \frac{\psi_3}{3\psi_4}$. But when S=0, the problem is that of unconstrained problem , and , as equation (3) indicates, $f = \frac{\psi_3}{2\psi_4}$, indicating a discontinuous jump at the boundary. It should be clear that fracking continuously decreases with S and reaches the value for unconstrained maximization (i.e. $\frac{\psi_3}{2\psi_4}$) at $S = \frac{\psi_3^2}{4\psi_4 U^*}$. Hence, when r=0, as the value of the survival function decreases the optimal fracking will overshoot in the sense that fracking will be less than what the limiting value of fracking is at S=0. Once S=0, fracking jump up to its static maximization value.

This discussion also provides us the guidance on the nature of nonlinearity in f when r $\neq 0$. Let us guess that the solution is

$f = \frac{\psi_3}{6\psi_4} + \theta(S)$, where $\theta(S)$ needs to be determined.

Since $rV(S) = \frac{\psi_1^2}{4\psi_2} S - \psi_3 f + 3\psi_4 f^2$, taking the derivative of both sides with respect to S gives,

$$\frac{2r\psi_3}{3} - 2r\psi_4\theta(S) = S\,U^* + 6\psi_4 S\theta(S)\theta'(S)$$

Upon simplifying, we get, for $S \neq 0, \theta(S) \neq 0$,

(6) $$\theta^{'}(S) = \frac{\frac{2r\psi_3}{3} - U^{*}S - 2r\psi_4\theta(S)}{6\psi_4 S\theta(S)}$$

Let $\Omega_1 = \frac{2r\psi_3}{3}; \Omega_2 = -U^{*}; \Omega_3 = -2r\psi_4; \Omega_4 = 6\psi_4$. Then, we can rewrite equation (6) as

(6') $$\theta^{'}(S) = \frac{\Omega_1}{\Omega_4 S\theta(S)} + \frac{\Omega_2}{\Omega_4\theta(S)} + \frac{\Omega_3}{\Omega_4 S}$$

with $\theta(0) = \frac{\psi_3}{3\psi_4}$, this initial condition is derived with the assumption that as S=0, the optimal fracking will be maximized $f = \frac{\psi_3}{2\psi_4}$ .Understandably (6') is hard to solve analytically and has to be solved numerically.

For our further discussion, we just look at the equation (5), which is nonlinear in S , even though r is still assumed to be 0. From equation (5), clearly, fracking is reduced compared to no restriction case by $\frac{2\psi_3 - \sqrt{\psi_3^2 - 12\psi_4 U^{*}S}}{6\psi_4}$. Initially, as S is large, this difference is also large as states would want to save the aquifer. However, as S becomes smaller and smaller, the difference also becomes smaller, and when S=0, the fracking converges to its value (i.e. $\frac{\psi_3}{3\psi_4}$) which is smaller than the value when no constraint regarding risk exists. This is understandable, since once states realize that survival function is too low, they would be less inhibited to extract the benefit from fracking.

In our opinion, these results conform to our a priori intuition. We now turn to situations where modifications are made to our basic model.

II. Model with limited aquifer capacity

Our first modification is the inclusion of aquifer as a stock variable, effectively indicating that it is a renewable resource, that has a finite stock at any given time. We indicate the evolution of aquifer stock level as follows:

(7) $$A' = \delta A - w_1 - w_2$$

We also explicitly include the technological innovations in fracking. In particular, we assume that the industry will dynamically improve the safety of its operations over time, and let it be denoted by α. The survival function thus evolves as follows:

(8) $$S' = -(f_1 + f_2)S + \alpha$$

Where S is bounded between 0 and 1. Notice that when $\alpha = 0$, we will have the evolution of the survival function as dealt in the simple model, so (8) is a more generalized setting.

We now solve the problem for an open loop Cournot Nash equilibrium in the steady state. We will continue to assume that the States are symmetric, in that they have identical utility function for water and fracking. State 1 will be maximizing the following problem:

$$\max_{w_1,\, f_1} \int_0^\infty (U(w_1)S(t) + v(f_1))e^{-rt}dt$$

$$s.t. \quad S' = -S(f_1 + f_2) + \alpha$$

$$A' = \delta A - w_1 - w_2$$

The Hamiltonian for this problem is

$$\tilde{H} = U(w_1)S(t) + v(f_1) + \lambda(\alpha - S(f_1 + f_2)) + \mu(\delta A - w_1 - w_2)$$

Note that because we look for an open loop solution, both f_2 and w_2 are assumed to be fixed. The first order necessary conditions are (for interior solutions)

(9) $U^{'}(w_1)S - \mu = 0$

(10) $v^{'}(f_1) - \lambda S = 0$

(11) $\lambda^{'} = \lambda(r + f_1 + f_2) - U(w_1)$

(12) $\mu^{'} = (r - \delta)\mu$

With associated transversality conditions, $\lim_{t \to \infty} e^{-rt}\lambda(t)S(t) = 0; \lim_{t \to \infty} e^{-rt}\mu(t)A(t) = 0$.

Assuming $r \neq \delta,$ and given w_2 *and* f_2 , at the steady state, the following must hold:

(13) $$\begin{bmatrix} \lambda^* \\ \mu^* \\ S^* \\ A^* \end{bmatrix} = \begin{bmatrix} \frac{U(w_1)}{r + f_1 + f_2} \\ 0 \\ \frac{\alpha}{f_1 + f_2} \\ \frac{w_1 + w_2}{\delta} \end{bmatrix},$$

where $w_1 = \frac{\psi_1 - \left(\frac{\mu}{S}\right)}{2\psi_2}$ and $f_1 = \frac{\psi_3 - \lambda S}{2\psi_4}$.

Using the fact that $f_1 = f_2; w_1 = w_2$ for symmetric players, and using the steady state values given above, we can solve for the optimal solutions at the steady state as follows:

$$w_1 = w_2 = w = \frac{\psi_1}{2\psi_2}$$

$$f = \{\Delta_1 + \sqrt{\Delta_1^2 + \Delta_2^3}\}^{\frac{1}{3}} + \{\Delta_1 - \sqrt{\Delta_1^2 + \Delta_2^3}\}^{\frac{1}{3}} + \Delta_3$$

Where $\Delta_1 = -\left[\frac{\left(8r\psi_4^2 - 4\psi_3\right)^3}{110592\psi_4^6} + \frac{2r\psi_3(8r\psi_4^2 - 4\psi_3)}{1536\psi_4^4} + \frac{\alpha U^*}{32\psi_4^2}\right]$

$$\Delta_2 = -\left[\frac{2r\psi_3}{48\psi_4^2} + \frac{\left(8r\psi_4^2 - 4\psi_3\right)^2}{2304\psi_4^4}\right]$$

and $\Delta_3 = -\frac{(8r\psi_4^2 - 4\psi_3)}{48\psi_4^2}$

where $U^* = \frac{\psi_1^2}{4\psi_2}$. Equation(13) now becomes:

$$\text{(13')} \quad \begin{bmatrix} \lambda^* \\ \mu^* \\ S^* \\ A^* \end{bmatrix} = \begin{bmatrix} \frac{\psi_1^2}{4\psi_2(r + 2f)} \\ 0 \\ \frac{\alpha}{2f} \\ \frac{2w}{\delta} \end{bmatrix}$$

Again, in the steady state, the water extraction rate,w, is same as when there were no constraints. However, the fracking is reduced by $\frac{\lambda^* S^*}{2\psi_4}$. Since λ^* is the marginal contribution of the survival function to the welfare, λS is the total effect of risk (i.e. survival function) to welfare. Fracking is therefore downward adjusted by the number that represents the impact of risk on welfare. It is as if the fracking benefit has been transformed from $v(f) = \psi_3 f - \psi_4 f^2$ to $v(f) = (\psi_3 - \lambda^* S^*) f - \psi_4 f^2$ in an unconstrained problem. Expressed differently, if there exists some "supra-state" authority with the ability to tax the fracking wells, then it could impose a linear tax of λS per unit onto the fracking developers and let the states act as if there was no risk at all[2]. The steady state water extraction should not be a surprise as in the steady state, the costate variable associated with aquifer is also zero.

We now look at the optimal rate of water extraction during the transitional phase before the system reaches the steady state. The water extraction rate, as given by (9), is

$$w_1(t) = \frac{\psi_1 - \left(\frac{\mu(t)}{S(t)}\right)}{2\psi_2}, \ when\ \psi_1 - \left(\frac{\mu(t)}{S(t)}\right) > 0..$$
$$0 \qquad\qquad otherwise$$

Since $\mu' = (r - \delta)\mu$, we have $\mu(t) = \mu_0 e^{(r-\delta)t}$. Furthermore, as $\lim_{t \to \infty} e^{-rt}\mu(t)A(t) = 0$, we can get $\lim_{t \to \infty} e^{-\delta t}\mu_0 A(t) = 0$. If A(t) is strictly positive, then the transversality condition is satisfied with $\mu_0 = \frac{\frac{\psi_1}{\delta\psi_2} - A_0}{\int_0^\infty \frac{2e^{(r-2\delta)\tau}}{S(\tau)} d\tau} \neq 0$.

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[2] Some of the States such as Pennsylvania and Texas are proposing to charge fee per well during the entire working life of the well. In Pennsylvania, Act 13 allows impact fee to charge on natural gas wells.

Since $w_1 = \frac{\psi_1 - \left(\frac{\mu}{S}\right)}{2\psi_2}$, in case where $r < \delta$, this implies $\mu \to 0$ as $t \to \infty$. Assuming that $\mu \to 0$ faster than $S \to 0$ as the system approaches steady state, water extraction increases. This is clearlythe case since S approaches a nonzero value (i.e. $\frac{\alpha}{2f}$).

The situation is different when $r > \delta$. In this case, the numerator becomes negative at some finite time t for large enough steady state value of S. In such a case, water extraction will be zero.

When $r = \delta$, $\mu(t) = \mu_0 \ \forall t$. This implies that $w = \frac{\psi_1}{2\psi_2} - \frac{\mu_0}{2S\psi_2}$ and

$w' = \frac{\mu_0}{2\psi_2 S^2}\left[\alpha - 2fS\right]$. This further suggests that if survival function and fracking are such that $Sf(S) \le \frac{\alpha}{2}$ (at least one of them is small enough) and $S \ge \frac{\mu_0}{\psi_1}$, water extraction increases over time. Hence when fracking is small and risk is low, the water extraction slowly increases before it converges to its steady state value.

III. Model with the cooperation among states

We now analyze the situation in which states cooperate. We look at the model of section II, where the aquifer stock is considered to be of limited quantity. Assume the states are symmetric and that $f_i = f, w_i = w$. This leads to the following problem:

$$\max_{w,\, f} \int_0^\infty \left[2U(w)S + 2v(f)\right]e^{-rt}dt$$

s.t. $S^{'} = -2Sf + \alpha$

$A^{'} = \delta A - 2w$

The first order conditions for this problem will be:

(14) $w = \dfrac{\psi_1 - (\frac{\mu}{S})}{2\psi_2}$;

(15) $f = \dfrac{\psi_3 - \lambda S}{2\psi_4}$

(16) $\lambda^{'} = (r + 2f)\lambda - 2U(w)$

(17) $\mu^{'} = (r - \delta)\mu$

When compared to the noncooperative problem, we note that the only difference is the speed at which marginal contribution of survival rate(λ) evolves. In particular, the steady state value of λ is different in cooperative scenario ($\dfrac{2U^*}{r+2f^*}$) than in noncooperative scenario ($\dfrac{U}{r+2f}$). The steady state solutions are given as follows:

(18) $$\begin{bmatrix} \lambda^* \\ \mu^* \\ S^* \\ A^* \end{bmatrix} = \begin{bmatrix} \dfrac{\psi_1^2}{2\psi_2(r+2f)} \\ 0 \\ \dfrac{\alpha}{2f} \\ \dfrac{2w}{\delta} \end{bmatrix},$$

where

$$w = \frac{\psi_1}{2\psi_2}$$

$$f = \{\Delta_1 + \sqrt{\Delta_1^2 + \Delta_2^3}\}^{\frac{1}{3}} + \{\Delta_1 - \sqrt{\Delta_1^2 + \Delta_2^3}\}^{\frac{1}{3}} + \Delta_3$$

Where $\Delta_1 = -\left[\frac{(8r\psi_4^2 - 4\psi_3)^3}{110592\psi_4^6} + \frac{2r\psi_3(8r\psi_4^2 - 4\psi_3)}{1536\psi_4^4} + \frac{\alpha U^*}{16\psi_4^2}\right]$

$$\Delta_2 = -\left[\frac{2r\psi_3}{48\psi_4^2} + \frac{(8r\psi_4^2 - 4\psi_3)^2}{2304\psi_4^4}\right]$$

and $\Delta_3 = -\frac{(8r\psi_4^2 - 4\psi_3)}{48\psi_4^2}$

where $U^* = \frac{\psi_1^2}{4\psi_2}$.

The steady state values of the aquifer remains the same in symmetric noncooperative and cooperative steady states when $r \neq \delta$.

To compare the fracking under noncooperative and under cooperative, in appendix, we show that fracking under noncooperative (f^{NC}) is greater than the fracking under cooperative f^C. This implies that under the cooperative outcome, the survival function ($\frac{\alpha}{2f}$) is maintained at the higher level compared to the noncooperative outcome. By acting independently, the states are thus more likely to endanger the aquifers. Cooperation, on the other hand, is more likely to save the aquifer.In a noncooperative case, if r= δ , we note that water extraction will be same as in a noncooperative case, i.e. will decrease over time before converging to the steady state value.

Conclusions

We provided the model of the impact of fracking on transboundary aquifers. Major findings are: i. presence of risk implies caution in fracking activities in the steady state; ii. optimal fracking is higher in the noncooperative symmetric setting than in the cooperative outcome; and iii. a steady state survival function level will be higher under cooperation than the noncooperative situation. The water extraction function will be the similar in both cases in the steady state situation. The water extraction steadily increases and converges to its steady state value.

Further efforts in the modeling of fracking should focus on providing explicit representation for amenity values, water contamination and the population dynamics of the area where fracking wells are located. Since fracking sites are also exhaustible resources, a more realistic model would have fracking sites as a state variable. Extraction and well set up in these sites will extend over time.

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Appendix

This appendix compares the fracking quantity under cooperative and noncooperative decision making. Notice that the cooperative's fracking must satisfy the following cubic polynomial:

(A.1) $16\psi_2\psi_4 f^3 + 8\psi_2(r\psi_4 - \psi_3) f^2 - 4\psi_2\psi_3 rf + \alpha\psi_1^2 = 0$

(To derive this use equations (15) and steady state values of λ and S from (18)).

For the noncooperative problem, the corresponding equation to solve would be

(A.2) $32\psi_2\psi_4 f^3 + 16\psi_2(r\psi_4 - \psi_3) f^2 - 8\psi_2\psi_3 rf + \alpha\psi_1^2 = 0.$

Since the analytical solution of an equation

$ax^3 + bx^2 + cx + d = 0$

is given by

$$\left\{\left(\frac{-b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a}\right) + \sqrt{\left(\frac{-b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a}\right)^2 - \left(\frac{c}{3a} - \frac{b^2}{9a^2}\right)^3}\right\}^{\frac{1}{3}} + \left\{\left(\frac{-b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a}\right) - \sqrt{\left(\frac{-b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a}\right)^2 - \left(\frac{c}{3a} - \frac{b^2}{9a^2}\right)^3}\right\}^{\frac{1}{3}} - \frac{b}{3a}$$

we can compare the roots of A.1 and A.2 directly. First, let's write A.1 and A.2 in the following format:

(A.3) $a^C x^3 + b^C x^2 + c^C x + d^C = 0$

(A.4) $a^{NC} x^3 + b^{NC} x^2 + c^{NC} x + d^{NC} = 0$

where

(A.5) $a^{NC} = 2a^C; b^{NC} = 2b^C; c^{NC} = 2c^C; d^{NC} = d^C$

For simplicity, from now on we omit superscript of a, b,c and d when they indicate noncooperative solution. i.e . $a^{NC} = a$, and so on.

Let $\tilde{x} = \frac{-b^3}{27a^3} + \frac{bc}{6a^2}; y = \frac{c}{3a} - \frac{b^2}{9a^2}; \Delta = \frac{d}{a}$

Define $G(\tau) = \{\tau + \sqrt{\tau^2 - k}\}^{\frac{1}{3}} + \{\tau - \sqrt{\tau^2 - k}\}^{\frac{1}{3}}$, where

$\tau = \tilde{x} - \Delta; k = y^3$. Here $\tau < 0,$ and we assume that $\tau + \sqrt{\tau^2 - k}$ and $\tau - \sqrt{\tau^2 - k}$ are positive so that we get a positive fracking amount.

Then $f^{NC} - f^{C} > 0$ if $G'(\tau) > 0$

Since

$$G'(\tau) = \frac{1}{3}\left\{\left(\tau + \sqrt{\tau^2 - k}\right)^{\frac{-2}{3}} + \left(\tau - \sqrt{\tau^2 - k}\right)^{-\frac{-2}{3}}\right\} + \frac{\tau}{3\sqrt{\tau^2 - k}}\left\{\frac{1}{\left(\tau + \sqrt{\tau^2 - k}\right)^{\frac{2}{3}}} - \frac{1}{\left(\tau - \sqrt{\tau^2 - k}\right)^{\frac{2}{3}}}\right\}$$

By assumption, the first expression inside curly bracket is positive, and $\frac{\tau}{3\sqrt{\tau^2 - k}}$ <0. Hence,

$G'(\tau) > 0$ if $\frac{1}{\left(\tau + \sqrt{\tau^2 - k}\right)^{\frac{2}{3}}} - \frac{1}{\left(\tau - \sqrt{\tau^2 - k}\right)^{\frac{2}{3}}}$ <0. But that is clearly the case.