Escape from Third-Best: Rating Emissions for Intensity Standards

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An increasingly common type of environmental policy instrument limits the carbon intensity of transportation and electricity markets. In order to extend the policy's scope beyond point-of-use emissions, regulators assign each competing fuel an emission intensity rating for use in calculating compliance. I show that welfare-maximizing ratings do not generally coincide with the best estimates of actual emissions. In fact, the regulator can achieve a higher level of welfare by manipulating the emission ratings than by manipulating the level of the standard. Moreover, a fuel's optimal rating can actually decrease when its estimated emission intensity increases. Numerical simulations of the California Low-Carbon Fuel Standard suggest that when recent scientific information suggested greater emissions from conventional ethanol, regulators should have lowered ethanol's rating (making it appear less emission-intensive) so that the fuel market would clear with a lower quantity.

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Many environmental regulations take the form of an emission intensity standard. Rather than capping the quantity of emissions or pricing emissions at marginal damage, these regulations cap emissions per unit of output. For instance, the U.S. formerly regulated the emission of sulfur dioxide per unit of electricity generated, and new rules propose to do the same for carbon dioxide emissions from coal and natural gas power plants. Fuel economy standards regulate gasoline use per mile, and low-carbon fuel standards (LCFS) regulate greenhouse gas emissions per unit of transportation fuel consumed.[1] Such intensity standards are commonly understood to be second-best policies for correcting emission externalities because they subsidize production that is less emission-intensive than the standard.

Recently, emission intensity regulations have extended their reach beyond observable, point-of-use emissions to include "life-cycle" emissions. For instance, an LCFS encourages substitution away from gasoline and towards biofuels like ethanol. Because the carbon dioxide released from ethanol combustion was absorbed from the atmosphere when growing the source crop, the observable tailpipe emissions from biofuels are not the carbon emissions that generate externalities. Instead, the emissions of interest are generated further up the supply chain: when farming the crops, when transforming the crops into ethanol, and when converting land for agricultural production. To include these types of emissions, regulators assess compliance using predefined emission ratings rather than continuous measures of observed emissions.[2] These ratings are meant to include all relevant emissions from producing and consuming the fuel, which for gasoline is dominated by emissions from combustion but for biofuels is dominated by supply chain emissions and "indirect" land use emissions.

The high degree of uncertainty about the actual life-cycle emissions associated with various fuels has given regulators significant discretion in assigning emission ratings, but previous economic analyses have focused on a constrained form of the policy in which each fuel's rated intensity exactly matches its estimated life-cycle intensity. I demonstrate two results that are contrary to common assumptions. First, the optimal intensity policy does not directly couple emission ratings to emission estimates. When there are more than two regulated products, a regulator who can freely assign emission ratings can almost always achieve strictly greater welfare than a regulator constrained to rate products according to estimated emission intensities. Imagine that the optimal policy does happen to rate all products at their estimated emission intensities and consider a perturbation to some product's estimated emission intensity. When that product's rating changes to match its new emission estimate, the altered rating affects the market tradeoffs between every pair of products. Yet the change in estimated emissions did not affect welfare tradeoffs between other pairs of products. In order to reattain the policy optimum, the regulator must have the

[1] The state of California pioneered low-carbon fuel standards in 2007, with its goal to reduce average fuel carbon intensity by at least 10% by 2020. The policy innovation has since diffused to other U.S. states, Canadian provinces, the European Union, and proposed U.S. federal legislation (Andress et al., 2010; Yeh and Sperling, 2010).

[2] Ratings-based intensity regulations also feature prominently in banking, where capital requirements assign risk weights to classes of assets. In trade or industrial policy, domestic purchasing requirements must categorize products that might be sourced widely. Many policies assign binary weights to sets of bins that actually have fuzzy boundaries: affirmative action mandates must categorize students or employees, inclusionary zoning rules must describe which households qualify as low-income, and renewable portfolio standards must judge electricity sources to be renewable or not. As an extension to the latter policy, recently proposed clean energy standards aim to incentivize relatively clean non-renewable sources of electricity by including intermediate ratings.

flexibility to precisely control the policy along multiple dimensions. A policymaker who can adjust only the level of the standard lacks precise control and cannot usually attain the policy optimum.

Second, I show that when new scientific information suggests higher emissions from what was previously believed to be a low-emission product, a welfare-maximizing regulator should often *lower* the assigned rating to make the product look like it generated fewer emissions. The reason is that the new information makes a welfare-maximizing regulator want to obtain less of the product from the market. Raising the low-emission product's rating has conflicting effects on its market-clearing quantity. A higher rating increases the amount of low-emission product required to achieve compliance for a given quantity of high-emission product. But raising a low-emission product's rating also reduces the market-clearing quantity of high-emission product and increases the competitiveness of other low-emission products. The first effect dominates when other products are sufficiently inelastic with respect to the rating. In that case, a higher, "tougher" rating for the low-emission product would actually increase its market-clearing quantity. A regulator who wants to obtain less of the product should therefore lower its rating, even though that lower rating may appear to contradict scientific information about the product's actual emission intensity.

This perverse result is particularly likely in the case of low-carbon fuel standards because fuel demand is generally thought to be price inelastic and first-generation ethanol might be the only compliance pathway available at scale. Raising ethanol's rated emission intensity could plausibly increase ethanol consumption rather than strongly reducing total fuel consumption or strongly shifting compliance to next-generation biofuels. In the second half of the paper, I simulate fuel market responses to the California LCFS. The results suggest that welfare-maximizing emission ratings for ethanol might in fact move opposite to its expected life-cycle emissions: a higher estimated emission intensity makes the optimal policy use less ethanol, and the way to obtain less ethanol from fuel markets is often to rate it as generating fewer emissions. This result is contrary to the common supposition among policy analysts that recent increases in ethanol's estimated emission intensity should translate into higher ratings.

The environmental economics literature has previously considered the efficiency properties of pollution intensity standards when emissions can be controlled independently of production. In that setting, intensity standards tax emissions while subsidizing output (e.g., Fullerton and Heutel, 2010). The latter effect occurs because producing more output for a given level of emissions helps meet the standard via dilution. The tax-subsidy combination leads to an ambiguous effect on total emissions (Helfand, 1991) and prevents equivalence with an emission tax (e.g., Ebert, 1998; Hatcher, 2007). When the only market failure is an emission externality, intensity standards cannot generally induce the first-best outcome. However, in the presence of market power (Holland, 2009) or leakage beyond the regulated jurisdiction (Holland, 2012), the output subsidy embedded in an intensity standard can make it dominate an emission tax. Similarly, imperfectly appropriable learning-by-doing calls for an output subsidy and thereby increases the relative efficiency of intensity standards. Numerical models of climate policy find that learning-by-doing can result in a carbon intensity standard dominating a carbon tax (Gerlagh and van der Zwaan, 2006) unless decreasing returns to scale in the low-emission sector limit the payoff to learning (Fischer and Newell, 2008). Preexisting tax distortions might also favor an emission intensity standard over an emission pricing policy if the latter does not use its revenue to increase economic efficiency (Parry and Williams III, 2012). Finally, targeting the carbon intensity of economic output instead of absolute emissions (via a standard price or quantity instrument) generates flexibility that can decrease the cost of unexpected productivity shocks (Fischer and Springborn, 2011).

In contrast to this literature, many forms of pollution are not byproducts of production but are instead necessary features of consumption. For instance, burning gasoline implies releasing greenhouse gas emissions. Producing and consuming ethanol generates pollution through activity in unregulated sectors (e.g., fertilizer use) and through price effects in other markets (e.g., indirect land use change). In these cases, regulated firms do not control emissions per product but instead control the mix of products they sell. This mix is determined by the emission ratings assigned by the regulator, which substitute for the continuous monitoring of emissions assumed in more traditional settings. I extend consideration of pollution intensity standards to the case where pollution is embedded in the regulated sector's products and is therefore "measured" by a regulation's assignment of emission ratings.[3]

The most closely related work is by Holland et al. (2009): they consider the efficiency properties of several forms of LCFS when there is a single high-emission product and a single low-emission product.[4] While the first-best greenhouse gas regulation taxes all fuels in proportion to their emissions, an LCFS implicitly subsidizes fuels with emission intensities below the mandated average. This subsidy distorts the fuel mix. I extend their setting by allowing for an arbitrary number of products in the market and by analyzing the optimal emission ratings. Allowing for more than two products is the extension that prevents an intensity standard from attaining even the second-best outcome unless the regulator can adjust the emission ratings. However, optimal ratings can move opposite to estimated emissions even in a two-product setting.

The next section formally develops the market and regulatory setting. Section 2 then analyzes the welfare cost of constraining the regulator to set emission ratings equal to emission estimates. Section 3 analyzes how optimal emission ratings change with the estimated emission factors. Section 4 numerically simulates the California LCFS to assess how the rating for conventional (corn or sugarcane) ethanol should respond to changes in its estimated emissions. Section 5 discusses the effect of technology objectives as well as interactions with federal policies and international fuel markets. Section 6 concludes.

1 A model of intensity regulation

A representative, price-taking firm produces $N > 1$ products. The cost of producing goods of type i is an increasing, strictly convex, twice-differentiable function $C_i(q_i)$ of its quantity q_i. A representative consumer obtains utility $U(\boldsymbol{q})$ from consuming the N products, where bold script indicates column vectors. Utility is increasing, concave, and twice-differentiable.

An intensity standard constrains the average emission intensity of a firm's products to be no greater than a constant σ. To calculate compliance, the regulator assigns each product i an emission

[3]The rating selection problem is similar to the one faced by banking regulators who require banks to hold a minimum level of capital as a fraction of total assets in order to reduce the probability of bank failure (Koehn and Santomero, 1980; Kim and Santomero, 1988; Rochet, 1992). Whereas the rated attributes (assets' risk profiles) directly affect banks' payoffs independently of the policy, they are pure externalities in our emission setting.

[4]The present paper considers what Holland et al. (2009) call an energy-based LCFS, as this is the form usually discussed for actual policies.

rating α_i. In order for the standard to be feasible with positive production, some rating must be weakly less than σ. Assume the standard binds. In that case, some rating must be strictly greater than σ. The intensity regulation forces firms to sell more units of low-emission products for each unit of high-emission product. The N products are produced in strictly positive quantities in equilibrium.

The representative firm maximizes profit subject to the intensity standard and to market prices p_i:

$$\max_{\{q_1,...,q_N\}} \left\{ \sum_{i=1}^{N} [p_i q_i - C_i(q_i)] \right\} \tag{1}$$

$$s.t. \; \frac{\sum_{i=1}^{N} \alpha_i q_i}{\sum_{i=1}^{N} q_i} \leq \sigma. \tag{intensity constraint}$$

The profit-maximizing quantity of product i meets the following first-order condition:

$$C_i'(q_i) = p_i - \lambda[\alpha_i - \sigma], \tag{2}$$

where primes indicate derivatives and $\lambda \geq 0$ is the shadow cost of the intensity constraint. Markets clear with $p_i = \partial U(\boldsymbol{q})/\partial q_i$. The equilibrium outcome $\{q_1^e, ..., q_N^e, \lambda^e\}$ is defined by the intensity constraint and the following N conditions:

$$C_i'(q_i) = \frac{\partial U(\boldsymbol{q})}{\partial q_i} - \lambda[\alpha_i - \sigma] \quad \text{for } i \in \{1, ..., N\}. \tag{3}$$

For product types with $\alpha_i > \sigma$, the intensity standard acts like a tax proportional to the product's "excess" emissions and to the shadow cost of the constraint. For product types with $\alpha_i < \sigma$, the intensity standard acts like a subsidy proportional to the product's "unused" emissions and to the shadow cost of the constraint. These observations are familiar from Holland et al. (2009).

The regulator determines the size of the implicit taxes and subsidies by selecting the N ratings α_i and the standard's level σ. In fact, two of these $N + 1$ parameters are (nearly) redundant: by varying $N - 1$ of the parameters, the regulator can achieve every set of equilibrium quantities compatible with some intensity constraint that maintains the sign of the difference between the two fixed parameters. First, as is clear from the first-order conditions, market-clearing quantities depend only on intensities relative to some baseline, not on their absolute values. In particular, it will be convenient to work with deviations from the level of the standard: $\hat{\alpha}_i \equiv \alpha_i - \sigma$, $\hat{\sigma} \equiv 0$. Second, unit conversions do not affect compliance. If we scale each deviation by some factor $k > 0$, then we again have the same equations defining equilibrium outcomes, except with the equilibrium shadow cost λ rescaled by $1/k$. If we fix product N as the numeraire ($\hat{\alpha}_N \equiv 1$),[5] then market-clearing quantities are uniquely defined by $\{\hat{\alpha}_1, ..., \hat{\alpha}_{N-1}, 1, 0\}$. Any change in α_N or σ that maintains $\alpha_N > \sigma$ can be perfectly offset by rescaling or shifting the other $N - 1$ ratings.[6]

[5] This choice is without loss of generality because some product must be rated above σ by the assumption that the standard binds.

[6] Fixing the level of the standard and one of the high ratings roughly matches the way in which the California

Each product generates emissions at rate β_i.[7] Each unit of emissions causes damage $\tau > 0$. The welfare-maximizing regulator selects the ratings so as to maximize consumer utility net of production costs and damages:

$$\max_{\{\hat{\alpha}_1,...,\hat{\alpha}_{N-1}\}} U(\boldsymbol{q^e}(\hat{\boldsymbol{\alpha}})) - \sum_{i=1}^{N} C_i(q_i^e(\hat{\boldsymbol{\alpha}})) - \tau\, \boldsymbol{\beta}^T \boldsymbol{q^e}(\hat{\boldsymbol{\alpha}}), \tag{4}$$

where superscript T indicates transpose. The optimal regulation solves the first-order conditions:

$$0 = \sum_{i=1}^{N} \left(\frac{\partial U(\boldsymbol{q^e}(\hat{\boldsymbol{\alpha}}))}{\partial q_i} - C_i'(q_i^e(\hat{\boldsymbol{\alpha}})) - \tau\, \beta_i \right) \frac{\partial q_i^e(\hat{\boldsymbol{\alpha}})}{\partial \hat{\alpha}_j} \quad \text{for } j \in \{1, ..., N-1\}. \tag{5}$$

Substituting in the equations governing equilibrium outcomes, we obtain a different version of the regulator's first-order conditions:

$$\lambda^e(\hat{\boldsymbol{\alpha}}) \sum_{i=1}^{N} \hat{\alpha}_i \frac{\partial q_i^e(\hat{\boldsymbol{\alpha}})}{\partial \hat{\alpha}_j} = \tau \sum_{i=1}^{N} \beta_i \frac{\partial q_i^e(\hat{\boldsymbol{\alpha}})}{\partial \hat{\alpha}_j} \quad \text{for } j \in \{1, ..., N-1\}. \tag{6}$$

The left-hand side is the marginal private cost of a tougher rating,[8] and the right-hand side is the expected marginal emission benefit of a tougher rating. The optimal regulation equates the marginal private cost and marginal social benefit of each rating. For a policy taxing each quantity at rate $t\,\alpha_i$, each $\lambda\,\hat{\alpha}_i$ on the left-hand side is replaced by $t\,\alpha_i$. In that case, the first-order conditions require that $t = \tau$ and $\alpha_i = \beta_i$ for all i. All products would be taxed at a positive rate in this first-best policy. Because an intensity policy requires that some $\hat{\alpha}_i$ be negative, it cannot attain the first-best outcome.

2 Achieving the best possible intensity standard

In the above model, the regulator can select how it rates each fuel's emission intensity without reference to its actual emission intensity. But the common assumption is that the regulator sets a product's rating to match its estimated emission intensity. What if we constrained our regulator to follow this rule-of-thumb? Under an emission pricing instrument, this political constraint would not bind: the emission externality is internalized when the tax equals the marginal damage from

Low-Carbon Fuel Standard has been implemented. At the time the policy was announced, the emission intensity of conventional gasoline (α_N) was already well understood. Further, the initial executive order fixed the required average fuel carbon intensity (σ) relative to a gasoline-dominated market (ensuring $\alpha_N > \sigma$). The California Air Resources Board then spent years developing ratings for the other $N-1$ potential fuels.

[7]Modeling uncertain emission factors does not affect key results under linear damages: expected emission factors would be sufficient statistics for policy. Under nonlinear damages, modeling uncertainty makes the effect of any regulatory variable on expected damages depend not just on how expected total emissions respond but also on how the quantities of products with especially uncertain emission factors respond. Uncertainty would matter in a more interesting way if emission factors were correlated with production cost, but that would change the regulatory problem to one of mechanism design under asymmetric information.

[8]This marginal private cost arises from the loss in marginal total surplus across products due to a tougher rating.

emissions and the emission ratings equal the emissions per unit of product. However, this section shows that the political constraint would in fact generally bind under an intensity instrument.

Define $V_N^{\alpha}(\sigma)$ as the highest level of welfare attainable in a market with N products if the regulator is free to adjust the emission ratings $\boldsymbol{\alpha}$ for given level σ of the standard, and define $V_N^{\sigma}(\boldsymbol{\beta})$ as the highest level of welfare attainable in a market with N products if the regulator is free to adjust the level σ of the standard but each rating α_i is constrained to equal estimated emissions β_i. The following proposition establishes that an intensity standard constrained to set $\boldsymbol{\alpha} = \boldsymbol{\beta}$ cannot generally attain the second-best outcome, defined as the best possible intensity standard:

Proposition 1 (Third-best outcomes) *The maximal level of welfare is weakly greater when the regulator can manipulate the ratings:* $V_N^{\alpha}(\sigma) \geq V_N^{\sigma}(\boldsymbol{\beta})$. *When* $N > 2$, *the inequality is strict for all* $\boldsymbol{\beta} \notin \mathcal{A}$, *where* $\mathcal{A}$ *is a set of measure zero. When* $N = 2$ *and the optimal intensity policy taxes the higher-emission product,* $V_2^{\alpha}(\sigma) = V_2^{\sigma}(\boldsymbol{\beta})$.

Proof See appendix.

There are worlds in which the regulator can achieve the best possible intensity standard by adjusting the level σ of the standard and keeping each intensity rating α_i fixed to equal the product's estimated emission intensity β_i. In particular, if there are only two products in the regulated market, then varying any one parameter is equivalent to varying any other. Constraining the regulator to only control σ does not actually constrain the regulator at all. However, when there are more than two products in the market, the world is almost never such that the regulator can achieve the best possible intensity standard only by varying σ: there are vectors $\boldsymbol{\beta}$ such that this outcome is possible, but if any β_i is varied by even an infinitesimal amount, then the regulator must set some $\alpha_i \neq \beta_i$ in order to achieve the best possible intensity standard. Constraining the regulator to only control σ generally means constraining the regulator to implement a third-best solution. To reach the best solution conditional on using an intensity standard, the regulator must be able to also control the emission ratings.

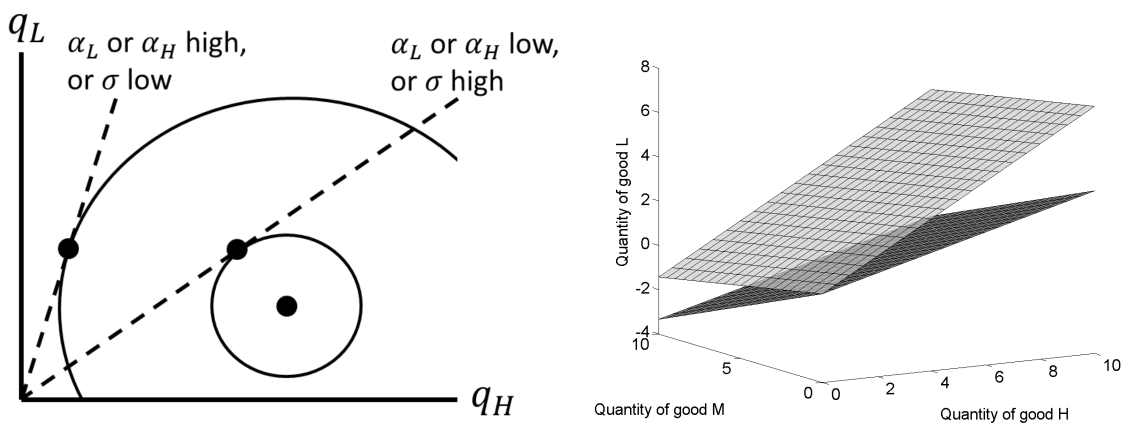
The intuition is in two parts. First, it is crucial to recognize how the regulator's options are limited by only being able to choose σ. Begin by considering the setting with only two products ($N = 2$), indexed by H for high-emission and L for low-emission. In Figure 1a, the market-clearing quantities occur at the highest iso-private-surplus curve (solid lines) that meets the intensity constraint (dashed lines). The intensity constraint is a ray from the origin:

$$q_L \geq \frac{\alpha_H - \sigma}{\sigma - \alpha_L}\, q_H.$$

The regulator can achieve any average emission intensity by varying only one of the three parameters. If it wants to increase the slope of the constraint, it can do so by increasing α_H, increasing α_L, or decreasing σ. Fixing two parameters does not handicap the regulator, assuming they are fixed such that $\alpha_H > \sigma > \alpha_L$.

Now consider a setting with three products ($N = 3$), where we index the middle-intensity product with M. Now the iso-private-surplus curves become spheres, and the intensity constraint becomes a plane:

$$q_L \geq \frac{\alpha_H - \sigma}{\sigma - \alpha_L}\, q_H + \frac{\alpha_M - \sigma}{\sigma - \alpha_L}\, q_M,$$



(a) Two-product setting ($N = 2$) (b) Three-product setting ($N = 3$)

Figure 1: In a two-product setting (left), the intensity constraint (dashed line) is a ray from the origin and equilibrium outcomes are determined by tangency between that ray and iso-private-surplus curves (solid lines). Increasing α_L or α_H or decreasing σ raises the slope of the constraint. In a three-product setting (right), the intensity constraint is a plane. Decreasing σ raises the plane from the lower, darker version to the upper, lighter version.

where α_M could be above or below σ. Figure 1b plots the minimal required q_L for $\alpha_M < \sigma$. Decreasing σ rotates the plane in both the q_H and q_M dimensions simultaneously, changing it from the lower, darker plane to the higher, lighter plane. In contrast, changing the rating α_H rotates the plane only in the q_H direction, and similarly for the rating α_M. A regulator who can control two parameters can turn the intensity constraint into any plane having a vertex at the origin, but a regulator who can only adjust a single parameter is limited to a subset of these possible planes. This limitation becomes more acute as N grows.

It turns out that the regulator needs all of its degrees of freedom. It is not the case that the planes traced out by varying any single parameter happen to include all the planes that would be chosen by a welfare-maximizing regulator constrained only to using an intensity instrument. The reason why is the second step in the chain of intuition.

Assume that all parameters are such that the optimal intensity standard sets $\boldsymbol{\alpha} = \boldsymbol{\beta}$ with some σ^*. In Figure 2, the solid ovals depict iso-private-surplus and the dotted ovals depict iso-welfare. The left-hand panel shows these relationships for products L and H, and the right-hand panel shows these relationships for products L and M. The initially optimal policy is the solid line that induces market-clearing quantities of products L, M, and H defined by point A.[9] Consider perturbing some β_k to a slightly greater value. This change in β_k has two effects. First, it increases the damages per unit of product k, which makes welfare more sensitive to the quantity of product k. In the left-hand panel of Figure 2, increasing β_H squeezes the iso-welfare ovals to the dashed lines. Second, by the

[9] Unlike in the two-good case, the intensity constraint generally has a nonzero intercept in each two-dimensional plot. For instance, when $q_H = 0$ in the left-hand panel, the level of q_L that makes the standard bind depends on $\beta_M q_M$.

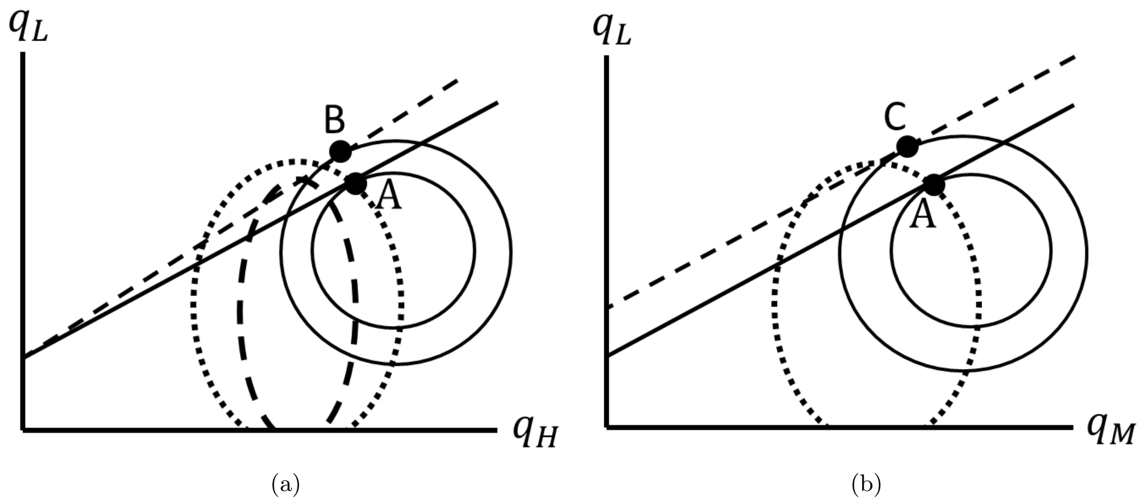


Figure 2: Assume that the intensity policy is initially optimal in a three-product setting with $\boldsymbol{\alpha} = \boldsymbol{\beta}$ and consider the effects of increasing β_H. First, the mapping from quantities to welfare changes in the q_H dimension (dashed ovals in left-hand plot). Second, the mapping from a choice of σ to market-clearing quantities changes, as illustrated by the shift from points A to points B and C for constant σ.

assumption that ratings are fixed at the estimated emission intensity, the increase in β_k induces an increase in α_k. This changes the market-clearing outcome for the regulator's choice of σ. In the left-hand panel of Figure 2, greater α_H increases the slope of the intensity constraint (dashed line), changing the market-clearing outcome to point B. While the intensity constraint's slope does not change in the right-hand panel, changing to point B in the left-hand panel while increasing β_H changes the intensity constraint's intercept in the right-hand panel. For instance, the constraint may shift to the dashed line, and market-clearing outcomes may shift to point C.[10]

The regulator should adjust σ to account both for the new mapping from σ to equilibrium outcomes and the new mapping from equilibrium outcomes to welfare. However, any change in σ affects the intensity constraint in multiple directions at once. In Figure 2, raising σ decreases both slopes and also changes both intercepts. By assumption, the regulator was doing as well as it could along all dimensions prior to any change in β_H. Changing β_H did not affect social tradeoffs between products L and M, but no single change in σ can offset the effects of greater α_H on these other products. In a two-good model, no such tension exists because there is no other dimension; there would be only a single panel in Figure 2.

But why is the best possible emission pricing policy achievable with $\boldsymbol{\alpha} = \boldsymbol{\beta}$? The optimal emission charge for product i is $\tau\,\beta_i$, where τ is both the emission price and the marginal damage from emissions. Upon increasing some product k's estimated emissions β_k and so its rating α_k, its emission charge increases proportionally. No change in τ is necessary to restore its optimal emission charge. And as long as τ does not change, production incentives for other products are

[10] And any such change affects the intercept of the constraint in the left-hand panel, and so on.

unaffected and the first-order conditions for each rating's optimality continue to hold. In contrast, the intensity standard implicitly prices emissions from product i at $\lambda(\alpha_i - \sigma)$. Changing some α_k changes the implicit emission charges for all other products via λ, the shadow cost of the constraint. In particular, the charge for product i ($\neq k$) changes by

$$\frac{\partial \lambda^e}{\partial \alpha_k}[\alpha_i - \sigma].$$

To restore the original implicit emission charge for product i, σ must change as:[11]

$$\frac{\partial \sigma}{\partial \alpha_k} = \frac{-\left[\frac{\partial^2 \lambda^e}{\partial \alpha_k \partial \sigma} + \frac{\partial^2 \lambda^e}{\partial \alpha_k^2}\right][\alpha_i - \sigma] + \frac{\partial \lambda^e}{\partial \alpha_k}}{\left[\frac{\partial^2 \lambda^e}{\partial \alpha_k \partial \sigma} + \frac{\partial^2 \lambda^e}{\partial \sigma^2}\right][\alpha_i - \sigma] - \frac{\partial \lambda^e}{\partial \alpha_k} - 2\frac{\partial \lambda^e}{\partial \sigma}}.$$

To restore the original optimal emission charges for all products other than k, this expression must hold for all $i \neq k$. But the expression depends on i, whereas there is only one σ. To remain at the best possible intensity standard, the regulator must be able to adjust $N-1$ parameters in a way that accounts for how each parameter affects the implicit emission charges for all product types via the shadow cost λ.

3 How scientific information affects the optimal policy

Having seen that the optimal regulation does not generally rate fuels at their estimated emission intensities, we now analyze the relationship between estimated emissions and optimal ratings. The conventional wisdom has been that when new scientific information raises some product's estimated emission intensity, the regulator should respond by raising the product's emission intensity rating accordingly. For instance, scientific concerns about emissions from indirect land use change recently increased estimated emissions from corn ethanol (Searchinger et al., 2008). The California regulator responded by adjusting the Low-Carbon Fuel Standard in the natural way: it rated corn ethanol as being more emission-intensive than before. However, this section shows that the common assumption about how information affects policy is incorrect in a broad range of intuitive cases.

For analytic tractability, I now assume that utility is separable in each product. The following proposition shows that the regulator's optimal response to new information about emissions may in fact be to change emission ratings in the opposite direction:

Proposition 2 (Comparative statics of the optimal ratings) *Assume either that the regulator can only adjust rating j, or that there are only two products in the regulated market ($N = 2$). The optimal response to altered emission and damage estimates is as follows:*

[11] The altered charge for product i from incrementally higher α_k is offset by an incremental change in σ when

$$0 = \frac{\partial \lambda^e}{\partial \alpha_k}[\alpha_i - \sigma] + \frac{\partial \lambda^e}{\partial \sigma}[\alpha_i - \sigma] - \lambda^e.$$

This expression implicitly defines σ as a function of α_k, for given values of the other regulatory parameters. The expression in the text follows by the Implicit Function Theorem.

(i) *There exists* $x > 0$ *such that* $\partial\alpha_j^*/\partial\beta_j > 0$ *if and only if* $\alpha_j > \sigma - x$.

(ii) *If* $\alpha_j^* < \sigma$, *then* $\partial\alpha_j^*/\partial\beta_i > 0$ *if and only if* $\alpha_i > \sigma$.

(iii) *If* $\alpha_j^* > \sigma$, *then there exists* $x > 0$ *such that* $\partial\alpha_j^*/\partial\beta_i > 0$ *if and only if either* $\alpha_j^* > \sigma + x$ *with* $\alpha_i < \sigma$ *or* $\alpha_j^* < \sigma + x$ *with* $\alpha_i > \sigma$.

(iv) $\partial\alpha_j^*/\partial\tau > 0$.

Proof See appendix.

The first result says that a product's optimal emission rating moves opposite to its estimated emissions if and only if the intensity standard subsidizes that product by a sufficiently large amount. The next two results describe how one product's optimal rating responds to a change in another product's estimated emission intensity. The final result says that increasing estimated marginal damage from emissions raises optimal ratings.

All of these results are driven by how market-clearing quantities respond to a change in the emission rating for product j. When scientific information increases estimated emissions from some product i, the regulator wants to adjust product j's emission rating to obtain less of product i from the market. However, the combination of implicit taxes and subsidies inherent to intensity regulation means that market-clearing quantities can respond in unexpected ways to changes in ratings.

We can think of low-emission products as generating compliance credits and high-emission products as generating compliance debits. The intensity regulation requires the total credits to be at least as great as the total debits. Changing some product j's rating affects production decisions by changing the value of the debits or credits generated by each product. And the value of the credits or debits changes through two channels: first, changing the rating alters the price of a credit or debit generated by any regulated product, and second, changing the rating affects the quantity of credits or debits generated by product j.

Recall that the implicit emission charge imposed on product i is $\lambda [\alpha_i - \sigma]$. When $\alpha_i > \sigma$, this charge is a tax; when $\alpha_i < \sigma$, this charge is a subsidy. Raising some rating α_j always increases the shadow cost λ of the intensity standard: it is more costly to achieve a given constraint when each unit of a high-emission j must be offset by more units of low-emission product, or when each unit of a low-emission j enables fewer units of high-emission product. This change in shadow cost increases the tax imposed on high-emission products, tending to decrease the quantities of high-emission products. This change in shadow cost also increases the subsidy provided to low-emission products by increasing the value of the compliance credits they generate, tending to increase the quantities of low-emission products.

This shadow cost effect operates on all product types simultaneously, but there is also a more direct effect on product j: raising α_j decreases the number of compliance credits, or increases the number of compliance debits, that it generates. For a high-emission product ($\alpha_j > \sigma$), the increase in compliance debits increases the implicit tax. This effect works along with the shadow cost effect to unambiguously decrease the market-clearing quantity of that product ($\partial q_j^e/\partial\alpha_j < 0$). Therefore, a greater emission estimate for product j raises its optimal emission rating so that the market provides less of it ($\partial\alpha_j/\partial\beta_j > 0$, result i).

For a low-emission product ($\alpha_j < \sigma$), the reduction in compliance credits decreases the implicit subsidy, which would reduce the market-clearing quantity. However, the shadow cost effect from a higher rating would increase the market-clearing quantity. The net effect is ambiguous. When the direct effect dominates, greater estimated emissions raise the optimal rating so as to obtain less of product j from the market ($\partial\alpha_j/\partial\beta_j > 0$), but when the shadow cost effect dominates, a higher rating would actually increase the market-clearing quantity of product j by increasing the total implicit subsidy. In that case, the optimal response to greater estimated emissions is actually to lower the product's emission rating ($\partial\alpha_j/\partial\beta_j < 0$). Because the shadow cost effect is proportional to $\sigma - \alpha_j$, it tends to dominate when α_j is far below σ (result i).

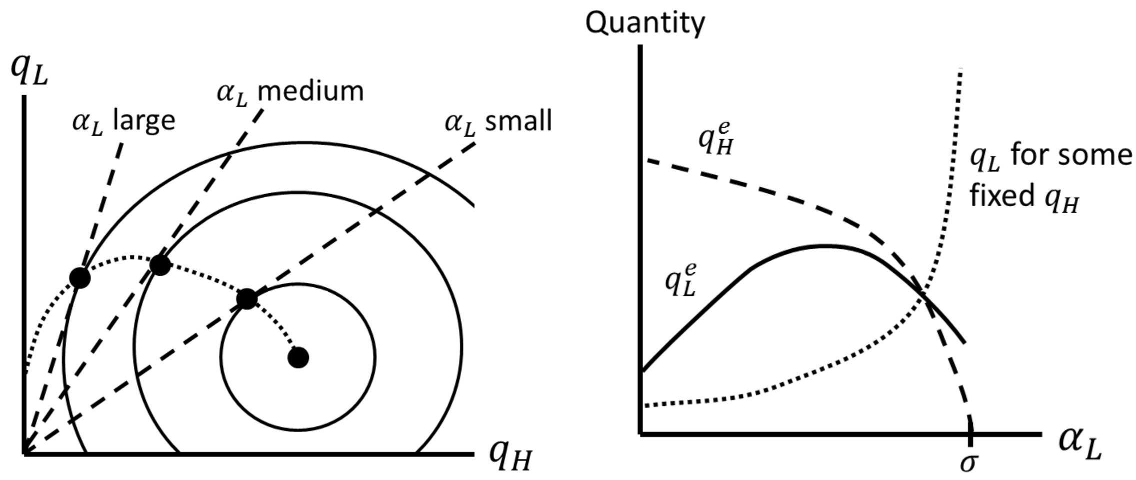
A two-product setting provides further intuition, with H indexing the high-emission product and L indexing the low-emission product. Figure 3a plots q_L as a function of q_H by using the intensity constraint (dashed lines) and iso-private surplus curves (solid lines). Market-clearing quantities occur where an iso-private-surplus curve is tangent to an intensity constraint. Raising the rating for the low-emission product increases the slope of the intensity constraint. The dotted line traces out the equilibrium quantities corresponding to possible intensity constraints. The quantity of high-emission product decreases as greater α_L makes the constraint steeper, but the quantity of low-emission product first increases then decreases. Along a given iso-private-surplus curve, a steeper constraint reduces q_L (the direct effect described above), but raising α_L also reduces private surplus by making the regulation tougher (the shadow cost effect). This shift in the constraint towards a curve representing less private surplus increases q_L. As α_L approaches σ, the firm's production of H goes to zero. Its profits become less sensitive to the rating, and the ("direct") effect of shifting along a given iso-private-surplus curve begins to dominate.

Differentiating the intensity constraint with respect to α_L yields additional insight in the two-product example:

$$\frac{dq_L^e(\alpha_L)}{d\alpha_L} = \frac{q_L^e(\alpha_L) + (\alpha_H - \sigma)\frac{dq_H^e(\alpha_L)}{d\alpha_L}}{\sigma - \alpha_L},$$
$$<0 \text{ iff } -\frac{dq_H^e(\alpha_L)}{d\alpha_L} > \frac{q_L^e(\alpha_L)}{\alpha_H - \sigma}.$$

For a given quantity of high-emission product, increasing α_L by one more unit requires $q_L^e/(\sigma - \alpha_L) > 0$ more units of low-emission product to maintain compliance with the intensity standard. The dotted line in Figure 3b demonstrates how a higher rating requires more low-emission product to offset a fixed, strictly positive quantity of high-emission product. As the low-emission product's rating approaches σ, its necessary quantity goes to infinity. However, the high-emission product also responds to the altered rating. It is easy to show in a two-product setting that the equilibrium quantity of high-emission product (dashed line) must decrease as the low-emission product's rating increases: $dq_H^e(\alpha_L)/d\alpha_L < 0$.[12] Each unit decrease requires $(\alpha_H - \sigma)/(\sigma - \alpha_L)$ fewer units of low-emission product. If the high-emission product decreases strongly enough, then the equilibrium quantity of low-emission product (solid line) also falls. However, if the high-emission product is relatively unresponsive to the change in rating, the first effect dominates and the quantity of low-emission product increases.

[12]Indeed, the proof of part ii of Proposition 2 shows this this to be true in a more general, N-product setting.



(a) Varying α_L changes the intensity constraint (b) Quantities as a function of α_L

Figure 3: Varying the low-emission product's rating (α_L) changes market outcomes (left) by changing the intensity constraint. A higher rating for the low-emission product increases the quantity required to offset a given quantity of high-emission product (dotted line on right) while also decreasing the quantity of high-emission product (dashed line on right). The net effect on the quantity of low-emission product is ambiguous (solid line on right).

Now consider using rating j to respond to information about emissions from some other product i. We can decompose the effect of α_j on q_i^e into two effects. First, holding the quantity of product j fixed, raising its emission rating means that other products must adjust in order to restore compliance: some products rated below the standard must increase in quantity, and/or some products rated above the standard must decrease in quantity. This is a shadow cost effect. Second, as the quantity of product j responds to its direct effect, other products further adjust in order to restore binding compliance. The direct effect always reduces the quantity of product j. If product j is subsidized (result ii), then its decrease in quantity means that some subsidized products must increase in quantity to restore compliance or some taxed products must decrease in quantity. These effects reinforce the first effect. Raising a subsidized product's rating j therefore decreases the quantities of taxed products and increases the quantities of other subsidized products.

However, if product j is taxed (result iii), the effects are more complicated. A decrease in the quantity of a taxed product enables more use of other taxed products and requires less use of subsidized products. These effects oppose the first effect described above. We have already seen that the direct effect dominates product j's response when α_j is close to σ. We might therefore expect greater α_j to increase the quantities of other taxed products when α_j is close to σ and to decrease their quantities otherwise. In this reasoning, we might then expect the optimal α_j to increase in β_i if product i is taxed and α_j is far from σ. Similarly, we might expect greater α_j to decrease the quantities of subsidized products when α_j is close to σ and to increase their quantities otherwise, and we might therefore expect the optimal α_j to increase in β_i if product i is subsidized and α_j is close to σ. Indeed, the third part of Proposition 2 shows these conclusions to be correct.[13]

Finally, result iv says that increasing the damage from each unit of emissions increases any product's optimal rating. The intuition is that greater damage per unit of emissions decreases optimal emissions, and raising any rating must decrease emissions around an optimum. Recall that optimal ratings must balance the private costs of a marginally tougher standard and the social benefits of reduced emissions from a marginally tougher standard. Raising any rating makes the standard tougher. It is possible for emissions to increase in a rating, but not around an optimum: in such a case, the regulator could lower both private costs and emission damages by reducing the rating. Any properly set intensity standard should become more stringent when the estimated marginal damage from emissions increases.

Previous literature has tended to ignore the rating assignment problem in favor of considering the optimal standard σ^* conditional on a set of ratings. The following corollary explains how our results map into results about the optimal level of the standard:

Corollary 3 (Comparative statics of the optimal standard) *If the regulator is free to choose only the level of the standard, then the optimal response to altered emission and damage estimates is as follows:*

(i) $\partial\sigma^*/\partial\beta_i < 0$ *if and only if* $\partial q_i^e(\sigma;\boldsymbol{\alpha})/\partial\sigma > 0$. *If* $\alpha_i > \sigma^*$ *and,* $\forall j \neq i$, $\alpha_j < \sigma^*$, *then* $\partial\sigma^*/\partial\beta_i < 0$.

(ii) $\partial\sigma^*/\partial\tau < 0$.

(iii) Assume second-order responses of equilibrium quantities are negligible. Then there exists $x < 0$ *such that* $\partial\sigma^*/\partial\alpha_i > 0$ *if and only if* $\partial q_i^e(\sigma;\boldsymbol{\alpha})/\partial\sigma > x$. *And if* $\alpha_i > \sigma^*$ *and,* $\forall j \neq i$, $\alpha_j < \sigma^*$, *then* $\partial\sigma^*/\partial\alpha_i > 0$.

Proof See appendix.

When estimated emissions from some product i increase, the optimal change in σ reduces the quantity of product i. If the ratings are fixed exogenously, then lowering the standard acts like raising all of the ratings (see appendix). The effect on the market-clearing quantity of some product i therefore depends on a combination of all the effects previously described for a change in a single rating. First, raising product i's own rating directly decreases its quantity by raising its tax or decreasing its subsidy. Second, smaller σ increases the shadow cost of the standard, which reinforces the direct effect when product i is taxed but opposes the direct effect when product i is subsidized. Third, the quantities of other products decrease when their own effective ratings are increased. When those products are subsidized and product i is taxed, this channel decreases the allowed quantity of product i. In this case, all three effects work together. In any other case, some effects oppose each other. Result i follows. Nonetheless, by identical logic as before, a tougher standard must decrease emissions around an optimum. Therefore greater estimated marginal damage from emissions unambiguously reduces the optimal σ (result ii).

Finally, one might think that raising some product's emission rating increases the optimal standard (so that the policy loosens to offset the more challenging rating), but this is true only if

[13] The appendix develops graphical intuition for these results.

raising the optimal standard does not decrease that product's quantity too much (result iii). The direct effect of a higher rating for product i is to reduce its quantity. As just described, the effect of a change in σ depends on a combination of three channels with potentially conflicting signs. If higher σ decreases the quantity of product i by a sufficiently large amount, then raising its rating is best "offset" by further tightening the standard via lower σ. The incrementally higher rating cannot simply be offset by an incrementally looser standard because that looser standard also affects other products' effective ratings. Indeed, the lack of a clear mapping between each individual rating and the level of the standard is part of the reason why reaching the second-best outcome requires the freedom to adjust emission ratings when there are more than two products in the regulated market.

4 Simulating the California Low-Carbon Fuel Standard

We have learned that a regulator constrained from setting ratings generally achieves lower welfare than a regulator free to set ratings and that optimal emission ratings can move opposite to estimated emissions. We now numerically assess the magnitude of the first effect and the existence of the second in the California transportation fuel market.

In 2007, California Governor Arnold Schwarzenegger issued Executive Order S-01-07 directing the California Air Resources Board (CARB) to establish an LCFS that would reduce average fuel carbon intensity by at least 10% by 2020. This goal, which defined the level of the standard, was the only thing known about this novel policy proposal. CARB then faced the task of figuring out the many details of trading credits, monitoring compliance, accounting for policy overlap, and, most prominently, rating fuels. Carbon intensity requirements began in 2011 at a level very close to reformulated gasoline's rating of 97.51 g CO_2 MJ^{-1}, and the standard declines over time until reaching 87.65 g CO_2e MJ^{-1} in 2020.[14] Initially, one could have expected that, barring a shift to electrified vehicles, most compliance would come from corn ethanol and possibly next-generation cellulosic ethanol (e.g., Farrell and Sperling, 2007). I approximate this state of affairs as a two-product model with corn ethanol and gasoline in the case that advanced biofuels would not be available at scale, and as a three-product model in the case that advanced biofuels would be available at scale.

The initial technical analysis prepared for CARB estimated that corn ethanol's emission factor was around 58–75 g CO_2e MJ^{-1} (Farrell et al., 2007). The LCFS would therefore subsidize the growing corn ethanol industry if CARB chose its rating to correspond to estimated emissions. Since 2007, two policy events have changed the expected compliance pathway. First, scientific concerns arose that directing corn crops to biofuel production would cause emission-increasing "indirect" emissions through price effects in agricultural markets. Attributing these emissions to corn ethanol would significantly increase its emission factor. In 2009, CARB decided to rate the indirect emissions at 30 g CO_2e MJ^{-1}. With these effects included, the combined (direct plus indirect) rating for average Midwestern corn ethanol became 99 g CO_2e MJ^{-1}. This rating is above the standard and even above that of gasoline. Average Midwestern corn ethanol now generated compliance debits, not credits.

[14] In all cases, I use the standard and ratings resulting from the amendments proposed by CARB in October of 2011. I focus only on the gasoline market, ignoring diesel and its substitutes.

Second, in 2011 the U.S. Congress allowed the 54 cent per gallon tariff on most imported ethanol to expire. Brazil has a substantial sugarcane ethanol industry that had sought ways to sell to the U.S. market without a tariff. Cane's high sugar content and use of bagasse for process energy enhance the transportation energy obtained per unit of emissions, leading CARB to assign it a total (direct plus indirect) rating of around 73 g CO_2e MJ^{-1}.[15] The LCFS therefore does still subsidize cane ethanol even after accounting for indirect effects. The LCFS may now be approximated by a two- or three-product model with gasoline, sugarcane ethanol, and possibly cellulosic ethanol.

I parameterize the model so as to assess the results' robustness to plausible ranges of inputs (Table 1). I assume constant elasticity supply and demand functions, and I assume that energy from one fuel is a perfect substitute for energy from another.[16] The social cost of carbon defines the constant marginal damage from greenhouse gas emissions. I fix the emission rating for gasoline at its estimated emission factor and consider how the regulator optimally rates the other fuels. I also consider the best possible policy for a regulator constrained to rate fuels at their expected emission factors but free to adjust the level of the intensity standard.

I solve the model for each combination of parameter values. The model solution involves three nested procedures. In the innermost step, firms select their profit-maximizing quantities given an intensity constraint and a fuel price. In the middle step, the model seeks the price that equates supply and demand for a given intensity policy. The outermost step optimizes the intensity policy by adjusting biofuels' ratings or the level of the standard. Welfare under a given intensity policy is determined by the market-clearing quantities obtained in the middle step.

4.1 Results with only conventional ethanol

I now describe results with two categories of fuel: gasoline and conventional ethanol, which could be corn-based or sugarcane-based depending on the regulatory setting. Figure 4a plots the optimal rating against the fuel's actual emission factor for several social costs of carbon. Figure 4b demonstrates the effect of assuming different elasticities of demand and supply. It uses a high social cost of carbon ($100/t$CO_2$) because the elasticity of demand has only a small effect for lower values.

Five points emerge. First, the optimal rating almost never equals the actual emission factor (the curves do not follow the main diagonal from the origin). Even if the emission factor were known perfectly, the regulator could improve welfare by choosing a different rating.

Second, the optimal rating decreases strongly in the actual emission factor in every case. Introducing estimates of indirect emissions would therefore decrease the optimal rating. In fact, the optimal rating is negative for high emission factors in all specifications. As will be illustrated below, the way to use less ethanol in these simulations is to make each unit of ethanol generate more compliance credits.

[15]Some forms of corn ethanol can achieve a similar rating even with indirect emissions included.

[16]The assumption of perfect substitutability holds when ethanol is at a sufficiently small volume (below the regulatory "blend wall") that it can be mixed into standard gasoline supplies. For larger volumes of ethanol, it may still hold conditional on the availability of flex-fuel vehicles and of distribution infrastructure. If the blend wall, oxygenate requirements, and the vehicle mix together fix the fraction of ethanol in the fuel mix, then the LCFS primarily selects the type of ethanol and not the total quantity of ethanol. In this case, a higher estimated emission factor for conventional ethanol favors a higher rating so as to increase use of lower-carbon substitutes.

Table 1: Parameterization of the California Low-Carbon Fuel Standard simulation.

Variable	Value
Carbon intensity constraint[a]	87.65 g CO_2e MJ^{-1}
Emission factor and rating for gasoline[a]	97.51 g CO_2e MJ^{-1}
Emission factor for conventional ethanol	$[0, 80]$ g CO_2e MJ^{-1}
Emission factor and rating for cellulosic ethanol[b]	$[0, 80]$ g CO_2e MJ^{-1}
Social cost of carbon	$\{25, 50, 100\}$ \$ (t CO_2e)$^{-1}$
Elasticity of demand[c]	$\{-0.1, -1\}$
Elasticity of gasoline supply[d]	3
Elasticity of conventional ethanol supply[e]	$\{2.5, 5\}$
Elasticity of cellulosic ethanol supply	$\{2.5, 5\}$
Baseline gasoline price[f]	3.138 \$ gal^{-1}
Baseline gasoline consumption[g]	15 billion gal yr^{-1}
Baseline conventional ethanol consumption[h]	1.5 billion gal yr^{-1}
Baseline cellulosic ethanol consumption[i]	0.3 billion gal yr^{-1}
Fuel tax[j]	0.537 \$ gal-gasoline^{-1}
Energy density of gasoline[a]	119.53 MJ gal^{-1}
Energy density of ethanol[a]	80.53 MJ gal^{-1}

[a] From the California Air Resources Board's LCFS amendments proposed in October of 2011.

[b] Liska and Perrin (2009) estimate that switchgrass (cellulosic) ethanol produces 6 g CO_2e MJ^{-1} in direct emissions. CARB assigned a total rating of 73.4 g CO_2e MJ^{-1} to average Brazilian sugarcane ethanol.

[c] The range is consistent with Brons et al. (2008), Hughes et al. (2008), and Park and Zhao (2010).

[d] Results not sensitive to variations.

[e] Luchansky and Monks (2009) estimate an elasticity of around 0.25 for U.S. ethanol (primarily corn), while Lee and Sumner (2010) estimate an elasticity of around 3 for Brazilian (sugarcane) ethanol imports.

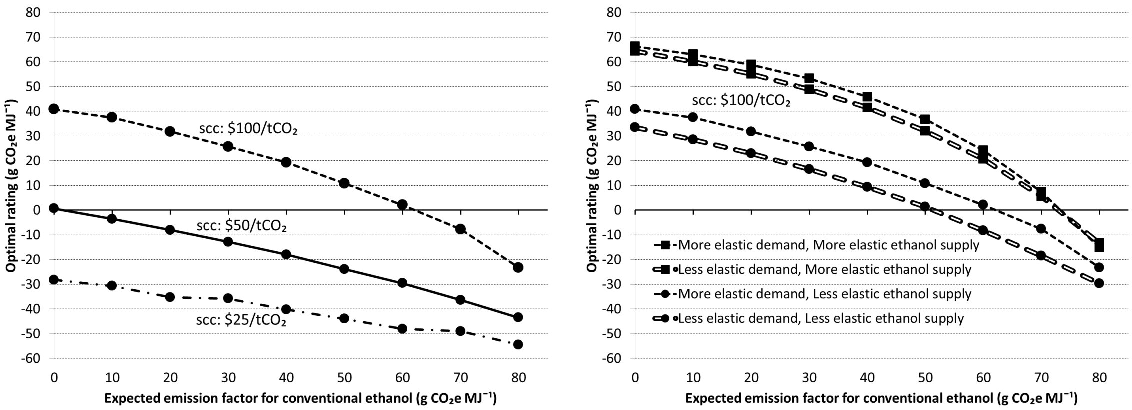
[f] Average tax-inclusive price in 2010 for all grades of gasoline in California (Energy Information Administration, from Petroleum Marketing Monthly).

[g] Motor gasoline sales volume (all grades) in 2010 in California by prime suppliers (Energy Information Administration, from Petroleum Marketing Monthly).

[h] For California in 2010, with 88% coming from Midwestern corn (California Energy Commission's Energy Almanac).

[i] Assumed to be 20% of conventional ethanol production.

[j] Total state and federal excise taxes on gasoline in California as of July 2010 (California Energy Commission's Energy Almanac). Taxes are assumed to apply to biofuels on an equivalent-gallon basis.



(a) Varying the social cost of carbon (b) Varying elasticities

Figure 4: The optimal rating for conventional (corn or cane) ethanol as a function of its expected emission factor when advanced biofuels are not available. The left-hand chart uses specifications with more elastic demand and less elastic supply.

Third, increasing the elasticity of ethanol supply increases the optimal rating (compare the squares and circles). This is because making ethanol more responsive to its price also makes it more responsive to the LCFS subsidy. The greater ease of obtaining ethanol reduces the private cost of the standard, which raises the optimal standard and so the optimal stringency of the LCFS.

Fourth, making demand more elastic also raises the optimal rating (compare the filled and hollow lines in Figure 4b). With more elastic demand, some compliance shifts to reduced fuel use rather than increased ethanol production. The marginal emission benefit of the policy increases, and the regulator therefore makes it more stringent by raising the rating for conventional ethanol.

Finally, as indicated in Proposition 2, the optimal rating increases in the social cost of carbon. The optimal rating is negative for all emission factors when the social cost of carbon is $\$25/tCO_2$ but is positive for most emission factors when the social cost of carbon is $\$100/tCO_2$. The U.S. government recently decided to use $\$21/tCO_2$ as its central value, with sensitivity analyses to be performed for values as high as $\$65/tCO_2$ (Greenstone et al., 2013). The negative ratings for the U.S. government's central value suggest that the current LCFS is difficult to justify on the basis of its emission reductions. If the rating for conventional ethanol were fixed at its expected emission factor, then the optimal standard would be closer to the emission factor for gasoline than is the current standard.

Figure 5a plots the market-clearing quantity of each fuel against the rating assigned to conventional ethanol. Figure 5b plots the market-clearing fuel price, the shadow cost of the standard, and total emissions for each possible rating. Both of these charts use the specifications with more elastic demand and less elastic ethanol supply. As the analytic model suggested, the reason why the optimal rating decreases in the actual emission factor is that ethanol consumption increases when it is rated as more emission-intensive. In fact, until the rating reaches high levels, fuel markets meet the intensity standard by only slightly reducing gasoline use. Total emissions do not change much, limiting the benefit of a higher rating. A very high rating does sharply reduce gasoline use, but the private cost of the standard also increases sharply.[17] With less elastic demand or more elastic ethanol supply, the compliance pathway shifts even more strongly to increasing ethanol consumption rather than decreasing gasoline consumption.

4.2 Results with both conventional and cellulosic ethanol

I next introduce cellulosic ethanol as a possible compliance pathway. By converting residues, waste, and woody biomass into liquid fuel, cellulosic technologies promise lower net emissions, more potential feedstocks, and the possibility of reduced land use effects. I calibrate cellulosic ethanol's supply function by charitably assuming its production volume is 20% of the conventional ethanol volume at 2010 prices. The rating for conventional ethanol now affects the relative competitiveness of cellulosic ethanol: a higher rating for conventional ethanol makes cellulosic ethanol a more attractive compliance option. Further, to attain the best possible intensity standard, the regulator now jointly optimizes the ratings for both types of ethanol.

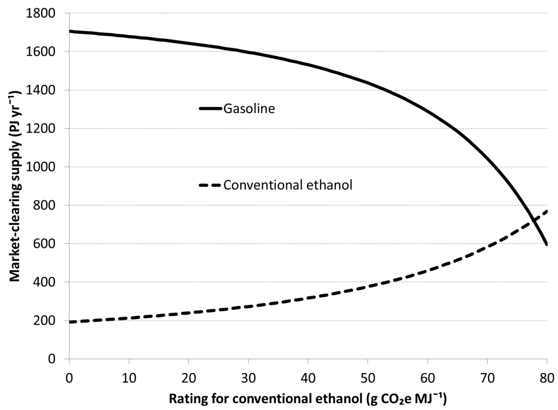
Figures 5c and 5d plot equilibrium quantities, prices, and emissions in the case with available cellulosic ethanol. The market-clearing quantity of conventional ethanol still increases over low ratings, but it eventually begins to decrease. This decrease happens when a high rating makes conventional ethanol much less competitive with respect to cellulosic ethanol. Compliance shifts to increasing the quantity of cellulosic ethanol. Emissions and the fuel price are less sensitive to conventional ethanol's rating than in the case without available cellulosic ethanol. When conventional ethanol receives a high rating, making cellulosic ethanol available increases the market-clearing quantity of gasoline and also total emissions relative to a case in which it is unavailable.

Freeing the regulator to select both fuels' ratings changes the relationship between optimal ratings and estimated emissions. Figure 6 plots the level curves of each optimal rating with a social cost of carbon of $\$50/tCO_2$ and the more elastic demand specification.[18] The top panel uses the smaller supply elasticities for both fuels, and the lower panel uses the larger supply elasticities. The optimal policy at each combination of emission factors is to rate conventional ethanol using the result in the left panel while rating cellulosic ethanol using the result in the right panel. As before, the optimal ratings are higher with more elastic supply because the cost of compliance is lower.

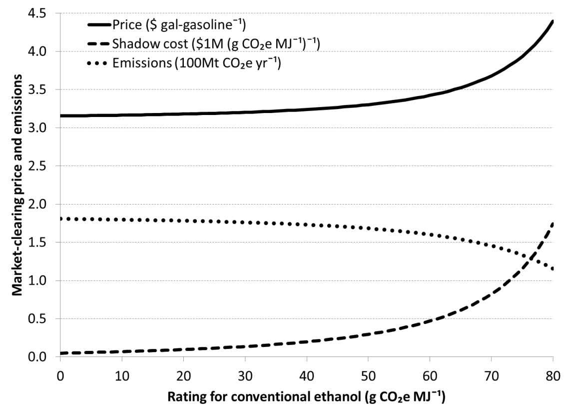
Four results stand out. First, the optimal rating for either fuel decreases as the other fuel becomes more emission-intensive (moving towards the top of the left panels or towards the right in the right panels). This occurs because increasing the emission intensity of fuel B makes the regulator want to shift compliance towards fuel A, which it achieves in part by lowering the rating for fuel A. However, the regulator will also adjust the rating for fuel B. The second result is that as cellulosic ethanol becomes more emission-intensive, the regulator raises its rating (moving towards the top of the right panels). Because cellulosic ethanol is not a dominant compliance pathway, raising its rating always decreases its market-clearing quantity.

[17] With these high ratings, ethanol use far surpasses blend wall constraints. This case therefore requires a large fraction of the fleet to be flex-fuel vehicles capable of using fuel with 85% ethanol.

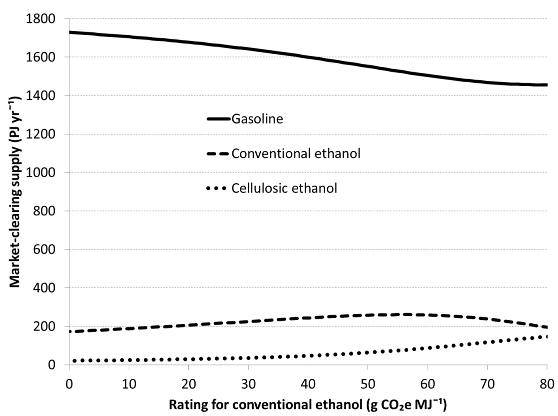
[18] Each contour plot uses 11^2 uniformly distributed nodes.



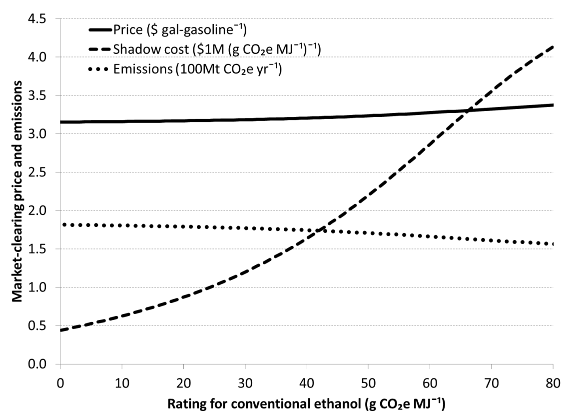
(a) Supply, without cellulosic



(b) Prices and emissions, without cellulosic



(c) Supply, with cellulosic



(d) Prices and emissions, with cellulosic

Figure 5: Market-clearing quantities and prices as a function of the rating for conventional ethanol, in cases with and without available cellulosic ethanol. Also the total emissions produced in equilibrium. Plots use the specifications with more elastic demand and less elastic ethanol supply. Cellulosic ethanol's rating and emission factor are 0 g CO_2e MJ^{-1}. Emission calculations assume that conventional ethanol's emission factor is 75 g CO_2e MJ^{-1}.

Conventional ethanol's optimal rating | **Cellulosic ethanol's optimal rating**

Low supply elasticities

High supply elasticities

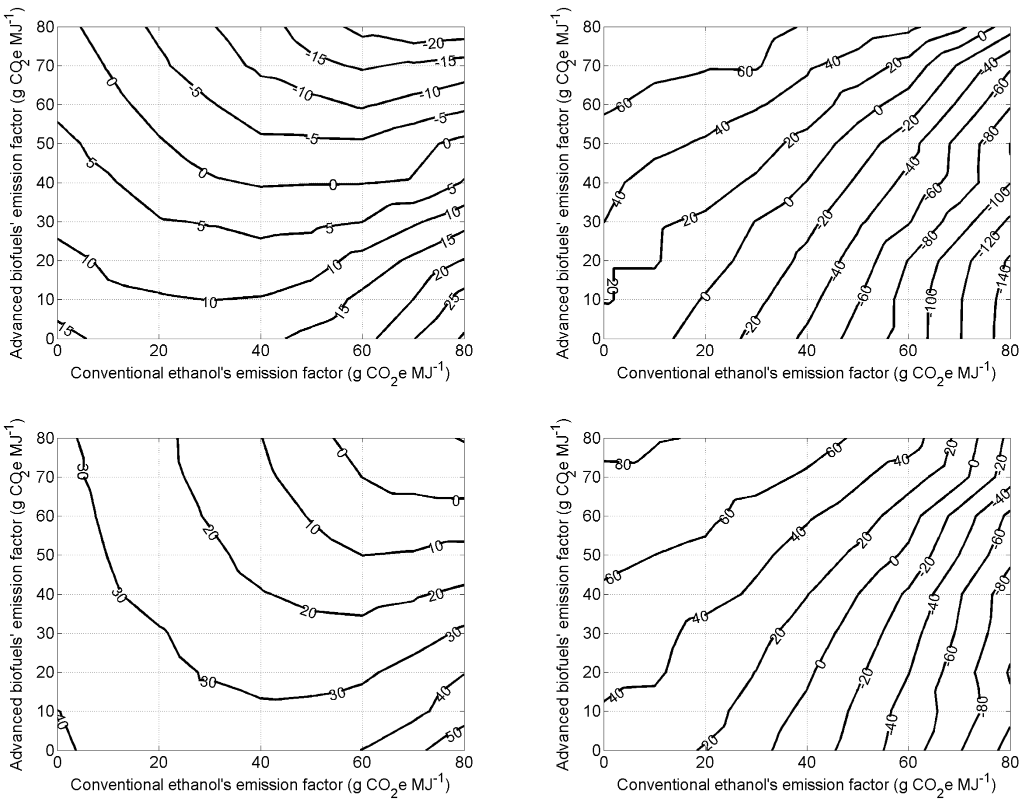
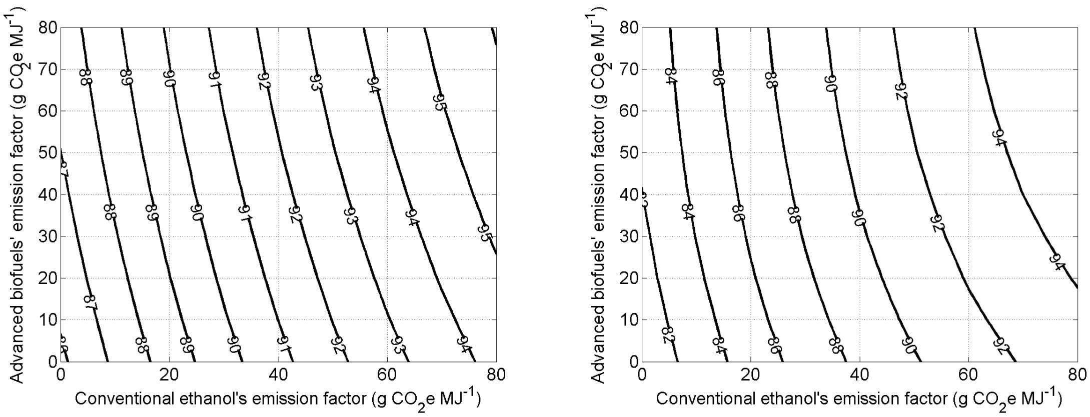


Figure 6: Level curves for the optimal ratings for conventional ethanol (left) and cellulosic ethanol (right) when both fuel types are available and the emission ratings are jointly optimized. All plots use the specification with more elastic demand and a social cost of carbon of \$50/tCO_2.

However, the results are different for conventional ethanol: as its own emission intensity increases, the optimal rating for conventional ethanol first decreases and then increases (moving towards the right in the left panels). This pattern arises because of the regulator's ability to optimize the rating for cellulosic ethanol. When conventional ethanol is not very emission-intensive relative to cellulosic ethanol, increasing the estimated emissions from conventional ethanol makes the regulator want to decrease its quantity without strongly increasing the quantity of cellulosic ethanol. In this case, the optimal rating for cellulosic ethanol is relatively unresponsive to conventional ethanol's emissions (the contours in the right panels become more widely spaced as we move diagonally up and to the left). The optimal rating for conventional ethanol then behaves as in the two-product world analyzed earlier. In contrast, when cellulosic ethanol generates relatively few emissions, increasing the emission intensity of conventional ethanol makes the regulator sharply reduce the rating for cellulosic ethanol. The complementary move in the rating for conventional ethanol is to raise its rating, shifting compliance towards cellulosic ethanol.



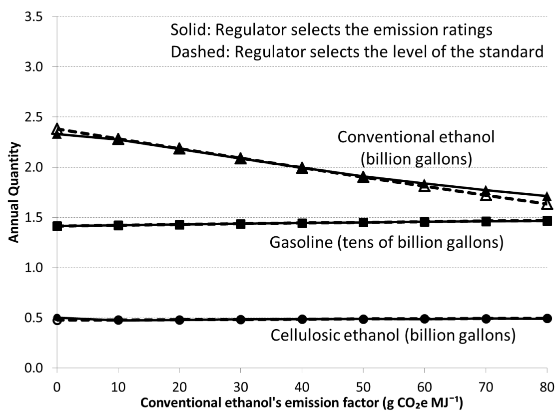
(a) Less elastic supply (b) More elastic supply

Figure 7: Level curves for the optimal level of the low-carbon fuel standard when both types of biofuels are available and their emission ratings are constrained to equal their expected emission factors. Plots use the specification with more elastic demand and a social cost of carbon of \$50/tCO_2.

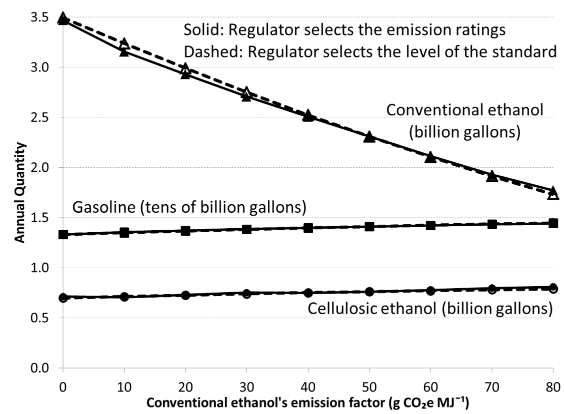
Finally, observe that the ratings for conventional ethanol are generally greater when cellulosic ethanol is available (compare Figures 4 and 6). When cellulosic ethanol is at its most emission-intensive, the ratings in the two charts are similar: cellulosic ethanol is rated at a sufficiently high level that it does not contribute much to compliance, so the regulator is effectively in a two-product world. However, as already discussed, lowering the emission intensity for cellulosic ethanol raises the rating for conventional ethanol in order to make cellulosic ethanol more competitive. While it is common for cellulosic ethanol to receive a strongly negative rating, conventional ethanol now generally receives a positive rating, though still below its estimated emission intensity.

4.3 Advantage of selecting both ratings

The theoretical analysis showed that a regulator who can select ratings can almost always do better—and should never do worse—than a regulator who can only select the level of the standard. Figure 7 plots the optimal level of the standard in the case where both conventional and cellulosic ethanol are available. It uses the specifications with more elastic demand and a social cost of carbon of \$50/tCO_2. As either fuel becomes more emission-intensive, the emission benefit of a given standard falls and the optimal level of the standard therefore rises. The optimal standard is relatively insensitive to the emission intensity of cellulosic ethanol (the contours are nearly vertical) because that fuel plays a small role in achieving compliance. Importantly, the optimal standard is above the actual standard of 87.65 g CO_2 per MJ unless conventional ethanol generates very few emissions. Indeed, for high emission intensities, the optimal standard comes close to the point at which it would no longer bind. The high optimal standard tells us the same thing as the low optimal ratings: the current LCFS is difficult to justify on the basis of its direct climate benefits.



(a) Less elastic supply



(b) More elastic supply

Figure 8: The annual market-clearing quantities of each fuel under the welfare-maximizing emission ratings. Also, the annual market-clearing quantities under the welfare-maximizing standard when the emission ratings are constrained to equal estimated emission factors. Cellulosic ethanol does not generate any emissions in these simulations. These specifications use the more elastic demand function and a social cost of carbon of \$50/tCO_2.

Figure 8 shows how the welfare-maximizing regulator designs the intensity policy to increase biofuel consumption at the expense of gasoline. Relative to the baseline (no-policy) quantities in Table 1, the optimal intensity policy decreases gasoline consumption by 0.5–1 billion gallons per year, increases conventional ethanol consumption by 0.1–2 billion gallons per year, and increases cellulosic ethanol consumption by 0.2–0.5 billion gallons per year. The optimal policy has a stronger effect with the greater supply elasticities because a given policy then obtains more biofuel and because, as seen previously, the optimal policy is more stringent. Whether the regulator selects the emission ratings (solid lines) or only selects the level of the policy (dashed lines), the optimal policy makes the market-clearing quantities of gasoline and cellulosic ethanol generally increase in conventional ethanol's estimated emission factor. Total fuel consumption also increases in the estimated emission factor for conventional ethanol. However, the regulator adjusts the policy so that the market-clearing quantity of conventional ethanol decreases in its estimated emission factor.

The key comparison is across policy environments. In Figure 8, the effect of the policy environment is visually apparent for conventional ethanol: its market-clearing quantity is more sensitive to its emission factor when the regulator can only select the level of the standard. Table 2 shows that the same is true of the other fuels' quantities. The two policy environments produce equivalent outcomes when conventional ethanol's estimated emission factor is just below 20 g CO_2 MJ^{-1} in the case with lower supply elasticities or around 50 g CO_2 MJ^{-1} in the case with higher supply elasticities. In all other cases, the extra degree of freedom afforded to a regulator who can adjust both ratings enables better smoothing of policy outcomes. When conventional ethanol has high estimated emissions and biofuel supply elasticities are low (high), the regulator who can only select the level of the standard reduces ethanol consumption by 76 (38) million gallons per year more than necessary while obtaining 53 (39) million excess gallons per year of gasoline. In the specification with low supply elasticities, that constrained regulator obtains 3 million excess gallons per year of cellulosic ethanol, but in the specification with high supply elasticities, that constrained regulator obtains 20 million fewer gallons per year of cellulosic ethanol than the best LCFS would have. These numbers suggest only small inefficiencies from constraining the regulator to manipulate only the level of the standard. Yet the inefficiency increases with the number of products, and the actual LCFS defines dozens of products.

Greater estimated emissions from conventional ethanol make the optimal intensity policy laxer, but a regulator who can only adjust the level of the standard makes it overly lax. Similarly, lower emission estimates for conventional ethanol make the optimal policy stricter by increasing the value of displacing gasoline with biofuel, but a regulator who can only adjust the level of the standard makes the policy overly strict. That regulator's policy adjustments are relatively brute in that they cannot help but affect each fuel's effective rating at once. In contrast, the regulator who can adjust the emission ratings can control how policy outcomes respond along multiple dimensions and so can more finely tune the policy response. The regulator who can control both ratings can optimize both the stringency of the policy and fuels' relative competitiveness, as opposed to a regulator who can only adjust the policy's stringency. When conventional ethanol has a high estimated emission intensity and cellulosic ethanol is responsive, the former regulator can loosen the policy's stringency while also shifting compliance towards cellulosic biofuels.

5 Discussion

Our theoretical model analyzed the factors determining the optimal choice of emission ratings. Actual policies are implemented in more complex environments. Nonetheless, our results do help understand how additional, complicating factors affect optimal ratings. In particular, this section discusses two important features of the LCFS missing from the analysis so far: technological change, and interactions with broader fuel markets and federal biofuel mandates. Table 3 summarizes how these additional features affect optimal ratings.

5.1 Technology forcing

A potentially important aspect of the LCFS is its role in spurring technological change. In fact, one of the main arguments for California's policy was that the state needed to spur low-carbon fuels in order to achieve its long-term carbon goals (Farrell and Sperling, 2007; Farrell et al., 2007). Because plausible near-term carbon taxes only slightly increase the gasoline price, complementary policies might be required to produce the desired change in transportation technologies. While a complete analysis would require a dynamic setting that traced out the channels of technological change, our static analysis does provide insight into how technology forcing objectives should affect the optimal rating for conventional ethanol.

First, consider technology goals that are advanced by production of conventional ethanol. Introducing such objectives is like lowering conventional ethanol's actual emission factor: conventional

Table 2: The difference in equilibrium fuel consumption induced by constraining the regulator to only select the level of the intensity standard rather than the emission ratings. Positive numbers indicate greater consumption under the constrained policy.

	Change in consumption (million gal per year)			
Emission factor ($g\ CO_2e\ MJ^{-1}$)	Gasoline	Conventional ethanol	Cellulosic ethanol	Total
Less elastic supply				
0	-19	51	-27	4.4
20	-3.3	2.8	1.5	1.1
40	2.0	-2.9	-0.027	-0.85
60	15	-27	6.5	-5.5
80	53	-76	3.0	-19
More elastic supply				
0	-9.0	26	-15	2.6
20	-42	61	-3.1	16
40	-13	19	1.2	6.6
60	8.5	-7.3	-5.3	-4.1
80	39	-38	-20	-19

Cellulosic ethanol does not generate any emissions in these simulations. These specifications use the more elastic demand function and a social cost of carbon of $\$50/tCO_2$.

Table 3: How the optimal rating for conventional ethanol responds to additional factors

Consideration	Effect on optimal rating
Technology forcing via production of conventional ethanol	$\uparrow^*$
Technology forcing via production of advanced biofuels	$\uparrow$
Rebound effect in the global oil markets	$\downarrow$
Rebound effect in international ethanol markets	$\downarrow^*$
Interaction with the federal Renewable Fuel Standard	$\uparrow^*$

[*] Analytically ambiguous; conclusion based on the numerical simulations

ethanol produces greater marginal social benefits and its optimal quantity increases. The theoretical analysis thus indicates that the effect of technological change objectives will depend on the fuel market, and the numerical results suggest that these objectives should in practice *increase* ("toughen") the rating assigned to conventional ethanol.

Next, consider technology goals that are achieved through production of next-generation biofuels like cellulosic ethanol. This case is like lowering the estimated emission factor for next-generation biofuels. The theoretical analysis and numerical results both suggest that the regulator should then *increase* the rating for conventional biofuels in order to more strongly favor advanced biofuels. Either type of channel for technological change therefore increases the optimal rating for conventional ethanol.

5.2 Rebound effects and the Renewable Fuel Standard

A second important aspect of the LCFS is its interaction with broader markets and policies. In particular, California is integrated into the U.S. fuel market and the world oil market. This has two implications. First, reductions in gasoline use in California will be at least partially offset by increased gasoline use elsewhere (the "rebound effect"). We could model this by reducing the marginal emissions attributed to gasoline consumption. By Proposition 2, this change *decreases* the optimal rating for conventional ethanol. Further, recent reductions in U.S. ethanol tariffs have stimulated an international ethanol market. Increasing use of cane ethanol in California might now be partially offset by Brazilian fuel consumption shifting away from domestic cane ethanol and towards gasoline and imported ethanol. This is a second type of rebound effect. We could model it by raising the emission factor for conventional ethanol, which *decreases* its optimal rating in our simulations.

Second, the U.S. fuel market must meet the federal Renewable Fuel Standard (RFS), which mandates minimum quantities of biofuels.[19] Biofuels used in the LCFS also count towards the RFS. If the biofuels called forth by the LCFS are more than sufficient to meet the national RFS, then the rating's marginal effects are unchanged from the central analysis. However, our simulations only come close to the mandated 15 billion gallons of conventional ethanol in cases with inelastic demand, elastic ethanol supply, and a very high emission rating. It therefore appears as if the California LCFS induces too little domestic biofuel consumption to meet the RFS. The federal RFS still binds. In this case, biofuels carry no emission penalty because they would (to a first approximation) have been produced anyway. This policy interaction decreases conventional ethanol's net emissions, which *increases* its optimal rating in our simulations. Both the RFS and the LCFS subsidize biofuels, but the LCFS couples California's biofuels to an implicit tax on gasoline.[20] A more stringent LCFS carries social benefits by enhancing some RFS biofuels' ability to reduce gasoline consumption.

[19] The RFS requires that 16.55 billion gallons of "renewable fuel" be used in 2013, with further requirements for particular types of biofuel. Renewable fuel must reduce greenhouse gas emissions by at least 20%, unless it is made by a grandfathered facility. Other fuel categories are defined similarly, making the RFS another ratings-based standard.

[20] In fact, the RFS actually acts like a set of intensity standards because the Environmental Protection Agency implements it by converting the total quantity mandate to a percentage requirement for each regulated party (Holland et al., 2011; Lapan and Moschini, 2012). With this implementation scheme, both the RFS and the LCFS combine gasoline taxes with biofuel subsidies.

6 Conclusions

Ratings-based intensity standards are increasingly common, second-best means of regulating emission externalities. I have shown that optimal emission ratings do not generally equal even certainly known rates of emission generation. Moreover, when other products are sufficiently unresponsive, the optimal rating for a low-emission product decreases as its estimated emission intensity increases. If regulators do not consider product market responses when translating scientific information into product ratings, then their chosen ratings might end up exacerbating the emission externality rather than mitigating it. Further analyses could fruitfully consider the political economy of this high-stakes regulatory setting by modeling the production of scientific information and the political constraints within which regulators operate.

Simulations of the California Low-Carbon Fuel Standard suggest that the welfare-maximizing emission rating for conventional ethanol does move opposite to its estimated emission intensity. Constraining the regulator to use expected emission factors can distort the state's fuel market outcomes by millions of gallons per year. Further, the welfare-maximizing rating for conventional ethanol is very small and even negative for standard values of the social cost of carbon. This result suggests that the implemented LCFS is overly stringent unless non-modeled factors are crucial. A desire to force technological change and interactions with the national Renewable Fuel Standard would both raise the optimal rating from its simulated level, but rebound effects in broader fuel markets would further lower it. Future simulations could model these technology objectives and broader interactions, use geographically-refined estimates of potential supply for the many compliance pathways recognized by the California LCFS, and explicitly account for constraints imposed by the blend wall and the vehicle fleet.

Finally, the results should introduce a note of pessimism towards the use of intensity standards. Policies like low-carbon fuel standards have been promoted as market-based: the market, not the regulator, picks winners and losers subject to the intensity constraint. However, the assignment of emission ratings determines how strongly products are taxed and subsidized by the policy, which determines market outcomes. In order to achieve the best possible intensity policy, or even to know in what direction to adjust the policy in response to new scientific information, the regulator must be able to forecast product market outcomes. What looks like a market-based policy that allows firms to choose their combination of compliance pathways begins to require an unrealistic level of information about each product's supply and demand. And the information problem grows more severe as the number of regulated products grows: the California LCFS defines several dozen product types in gasoline markets alone, and the arrival of scientific information about any one product type requires that optimal ratings be recalculated for all of these products. In addition to the standard efficiency advantage of an emission charge relative to the best possible intensity standard, economic evaluations should also account for the likelihood that even the best-intentioned regulator will be unable to achieve the optimal parameterization of such a complicated policy.

References

Andress, David, T. Dean Nguyen, and Sujit Das (2010) "Low-carbon fuel standard—Status and analytic issues," *Energy Policy*, Vol. 38, No. 1, pp. 580–591, DOI: 10.1016/j.enpol.2009.10.010.

Brons, Martijn, Peter Nijkamp, Eric Pels, and Piet Rietveld (2008) "A meta-analysis of the price elasticity of gasoline demand. A SUR approach," *Energy Economics*, Vol. 30, No. 5, pp. 2105–2122, DOI: 10.1016/j.eneco.2007.08.004.

Ebert, Udo (1998) "Relative standards: a positive and normative analysis," *Journal of Economics—Zeitschrift Fur Nationalokonomie*, Vol. 67, No. 1, pp. 17–38, DOI: 10.1007/BF01227761.

Farrell, A. E., D. Sperling, S.M. Arons, A.R. Brandt, M.A. Delucchi, A. Eggert, B.K. Haya, J. Hughes, B.M. Jenkins, A.D. Jones, D.M. Kammen, S.R. Kaffka, C.R. Knittel, D.M. Lemoine, E.W. Martin, M.W. Melaina, J.M. Ogden, R.J. Plevin, B.T. Turner, R.B. Williams, and C. Yang (2007) "A Low-Carbon Fuel Standard for California, Part 1: Technical Analysis,"Technical report, UC Berkeley and UC Davis.

Farrell, Alex and Daniel Sperling (2007) "Getting the carbon out," *San Francisco Chronicle*, p. B11.

Fischer, Carolyn and Richard G. Newell (2008) "Environmental and technology policies for climate mitigation," *Journal of Environmental Economics and Management*, Vol. 55, No. 2, pp. 142–162, DOI: 10.1016/j.jeem.2007.11.001.

Fischer, Carolyn and Michael Springborn (2011) "Emissions targets and the real business cycle: Intensity targets versus caps or taxes," *Journal of Environmental Economics and Management*, Vol. 62, No. 3, pp. 352–366, DOI: 10.1016/j.jeem.2011.04.005.

Fullerton, Don and Garth Heutel (2010) "The general equilibrium incidence of environmental mandates," *American Economic Journal: Economic Policy*, Vol. 2, No. 3, pp. 64–89, DOI: 10.1257/pol.2.3.64.

Gerlagh, Reyer and Bob van der Zwaan (2006) "Options and instruments for a deep cut in CO2 emissions: Carbon dioxide capture or renewables, taxes or subsidies?" *Energy Journal*, Vol. 27, No. 3, pp. 25–48.

Greenstone, Michael, Elizabeth Kopits, and Ann Wolverton (2013) "Developing a social cost of carbon for US regulatory analysis: A methodology and interpretation," *Review of Environmental Economics and Policy*, Vol. 7, No. 1, pp. 23–46, DOI: 10.1093/reep/res015.

Hatcher, Aaron (2007) "Firm behaviour under pollution ratio standards with non-compliance," *Environmental and Resource Economics*, Vol. 38, No. 1, pp. 89–98, DOI: 10.1007/s10640-006-9063-6.

Helfand, Gloria E. (1991) "Standards versus standards: The effects of different pollution restrictions," *American Economic Review*, Vol. 81, No. 3, pp. 622–634.

Holland, Stephen P. (2009) "Taxes and trading versus intensity standards: Second-best environmental policies with incomplete regulation (leakage) or market power," Working Paper 15262, NBER.

——— (2012) "Emissions taxes versus intensity standards: Second-best environmental policies with incomplete regulation," *Journal of Environmental Economics and Management*, Vol. 63, No. 3, pp. 375–387, DOI: 10.1016/j.jeem.2011.12.002.

Holland, Stephen P., Jonathan E. Hughes, and Christopher R. Knittel (2009) "Greenhouse gas reductions under Low Carbon Fuel Standards?" *American Economic Journal: Economic Policy*, Vol. 1, No. 1, pp. 106–146, DOI: 10.1257/pol.1.1.106.

Holland, Stephen P., Jonathan E. Hughes, Christopher R. Knittel, and Nathan C. Parker (2011) "Some inconvenient truths about climate change policy: The distributional impacts of transportation policies," NBER Working Paper 17386.

Hughes, Jonathan E., Christopher R. Knittel, and Daniel Sperling (2008) "Evidence of a shift in the short-run price elasticity of gasoline demand," *Energy Journal*, Vol. 29, No. 1, pp. 113–134.

Kim, Daesik and Anthony M. Santomero (1988) "Risk in banking and capital regulation," *The Journal of Finance*, Vol. 43, No. 5, pp. 1219–1233, DOI: 10.2307/2328216.

Koehn, Michael and Anthony M. Santomero (1980) "Regulation of bank capital and portfolio risk," *The Journal of Finance*, Vol. 35, No. 5, pp. 1235–1244, DOI: 10.2307/2327096.

Lapan, Harvey and GianCarlo Moschini (2012) "Second-best biofuel policies and the welfare effects of quantity mandates and subsidies," *Journal of Environmental Economics and Management*, Vol. 63, No. 2, pp. 224–241, DOI: 10.1016/j.jeem.2011.10.001.

Lee, Hyunok and Daniel A. Sumner (2010) "International trade patterns and policy for ethanol in the United States," in Madhu Khanna, Jurgen Scheffran, and David Zilberman eds. *Handbook of Bioenergy Economics and Policy*, Vol. 33, New York: Springer, pp. 327–345, doi:10.1007/978-1-4419-0369-3_19.

Liska, Adam J. and Richard K. Perrin (2009) "Indirect land use emissions in the life cycle of biofuels: regulations vs science," *Biofuels, Bioproducts and Biorefining*, Vol. 3, No. 3, pp. 318–328, DOI: 10.1002/bbb.153.

Luchansky, Matthew S. and James Monks (2009) "Supply and demand elasticities in the U.S. ethanol fuel market," *Energy Economics*, Vol. 31, No. 3, pp. 403–410, DOI: 10.1016/j.eneco.2008.12.005.

Park, Sung Y. and Guochang Zhao (2010) "An estimation of U.S. gasoline demand: A smooth time-varying cointegration approach," *Energy Economics*, Vol. 32, No. 1, pp. 110–120, DOI: 10.1016/j.eneco.2009.10.002.

Parry, Ian and Roberton C. Williams III (2012) "Moving US climate policy forward: Are carbon taxes the only good alternative?" in Robert W. Hahn and Alastair Ulph eds. *Climate Change and Common Sense: Essays in Honour of Tom Schelling*, Oxford, UK: Oxford University Press.

Rochet, Jean-Charles (1992) "Capital requirements and the behaviour of commercial banks," *European Economic Review*, Vol. 36, No. 5, pp. 1137–1170, DOI: 10.1016/0014-2921(92)90051-W.

Searchinger, Timothy, Ralph Heimlich, R. A. Houghton, Fengxia Dong, Amani Elobeid, Jacinto Fabiosa, Simla Tokgoz, Dermot Hayes, and Tun-Hsiang Yu (2008) "Use of U.S. croplands for biofuels increases greenhouse gases through emissions from land-use change," *Science*, Vol. 319, No. 5867, pp. 1238–1240, DOI: 10.1126/science.1151861.

Yeh, Sonia and Daniel Sperling (2010) "Low carbon fuel standards: Implementation scenarios and challenges," *Energy Policy*, Vol. 38, No. 11, pp. 6955–6965, DOI: 10.1016/j.enpol.2010.07.012.

Appendix

The first section develops graphical intuition for how the optimal rating of some product j responds to new scientific information about emissions from some other product i. The second section contains proofs.

A. Graphical intuition for comparative statics of optimal ratings with respect to other products' emissions

Figures A.1 and A.2 provide graphical intuition for parts ii and iii of Proposition 2. Each plot illustrates how the product on the y-axis responds to a higher rating for the product on the x-axis. First, the left-hand plot in Figure A.1 shows the tradeoffs between two low-emission products L and L', holding the quantities of all other products fixed at their initial equilibrium levels. The intensity constraint has a strictly positive intercept because the other products in the market generate compliance debits in excess of credits, requiring some production of these low-emission products. The intensity constraint slopes down because producing additional product L means that less of product L' is required to achieve compliance. Equilibrium production of these two products occurs where the constraint is tangent to an iso-private-surplus curve.

Raising the rating for product L rotates the intensity constraint upward. It also shifts the intercept in some direction via other products' responses. If the new constraint kept the firm on the original iso-private-surplus curve, then the constraint's upward rotation makes the firm increase the quantity of L' by substitution for L. Because a tougher constraint is more costly, the upward rotation also shifts the firm to a lower iso-private-surplus curve. For a given slope, shifting the intensity constraint outward to a curve representing less iso-private-surplus also increases the quantity of L'. Both effects combine to make the quantity of L' increase in the rating for L.

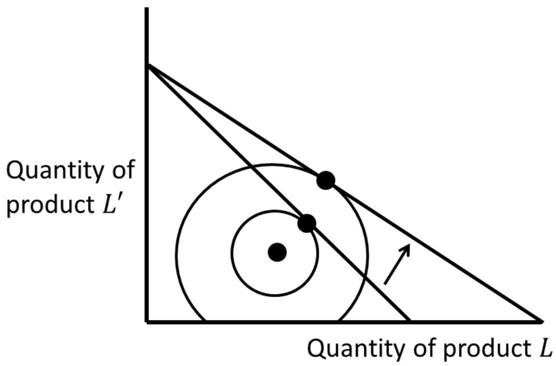
The right-hand plot in Figure A.1 is similar, except illustrating the response of some high-emission product H to an increase in the rating for L.[21] The higher rating now rotates the intensity constraint downward. Along a given iso-private-surplus curve, the downward rotation reduces the quantity of H, and for a given slope of the constraint, a shift towards a curve representing less private surplus also reduces the quantity of H. Both effects combine to make the quantity of H decrease in the rating for L.

Matters become more complicated when we consider the effect of raising the rating for some high-emission product H (result iii). The left-hand plot in Figure A.2 illustrates the response of some other high-emission product H'. The intensity constraint slopes downward because, holding other products' quantities fixed, producing additional product H requires the firm to produce less of product H' in order to maintain compliance. Increasing the rating for product H rotates the constraint down. If the firm maintained its position along a given iso-private-surplus curve, the steeper constraint would require it to produce more of product H' and less of H. However, the increased cost of the constraint pushes the firm to a curve representing less private surplus. For a given slope of the constraint, this shift reduces its production of both H' and H. The two effects combine to unambiguously reduce production of H, but the net effect on H' is ambiguous. If the firm's profits do not change much, then the quantity of H' can increase because it now competes

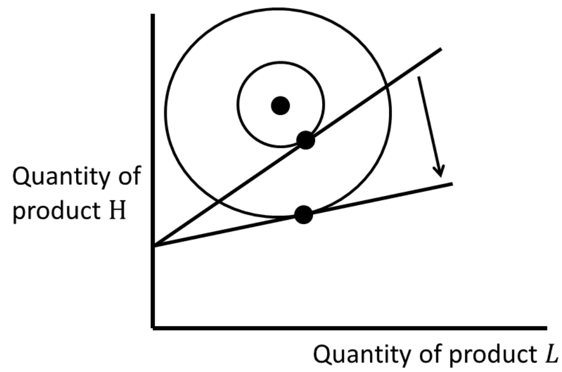
[21] The constraint's intercept could now be positive or negative, but it is not generally zero.

more effectively with H (its tax is lower relative to that on H). However, if H is hard to substitute away from, then the standard becomes much more costly and the tax on H' increases by enough in absolute terms to overcome the substitution induced by its decrease relative to the tax on H.

The right-hand plot in Figure A.2 shows the effect on some low-emission product L of raising the rating for H. The higher rating rotates the constraint upward. Along a given iso-private-surplus curve, a steeper constraint reduces production of L and of H, but shifting to a curve representing less private surplus increases production of L and decreases production of H. As before, the effect on product H of an increase in its own rating is to unambiguously decrease its quantity, but the effect on product L is ambiguous. The shift to a curve representing less private surplus occurs because the constraint has become more costly, and that increase in its shadow cost increases the subsidy to product L. However, if the firm were kept on the same private surplus curve but forced to reorganize its production of these two quantities in accord with their new emission ratios, then it would decrease production of both quantities. The more crucial is product H to the firm, the greater the effect on the shadow cost of the constraint. And the greater the effect on the shadow cost of the constraint, the more likely it is that a higher rating for product H increases the quantity of product L.

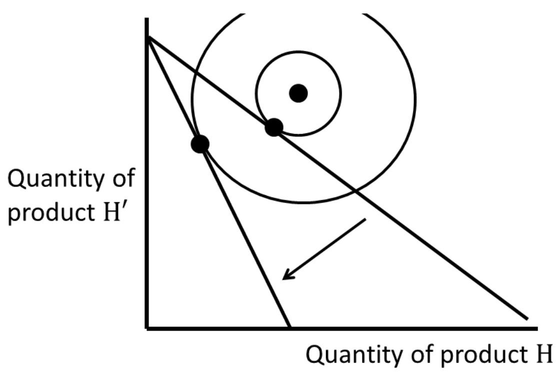


(a) Response of L'

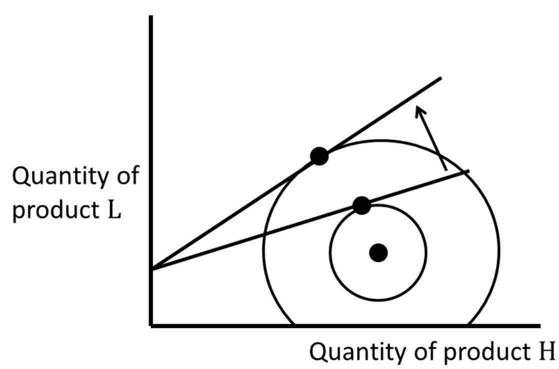


(b) Response of H

Figure A.1: Illustrative response of other products to a higher rating for low-emission product L, holding other equilibrium quantities constant.



(a) Response of H'



(b) Response of L

Figure A.2: Illustrative response of other products to a higher rating for high-emission product H, holding other equilibrium quantities constant.

B. Proofs

I introduce three lemmas before proving the propositions. The first describes how equilibrium quantities and emissions respond to the chosen ratings:

Lemma 4 *Assume that utility is separable in each type of product.*

(i) There exists $x > 0$ such that marginally raising the rating for product i decreases its quantity ($\partial q_i^e / \partial \alpha_i < 0$) if and only if $\alpha_i > \sigma - x$.

(ii) If $\alpha_j < \sigma$, then marginally raising the rating for product j raises the quantity of product i ($\partial q_i^e / \partial \alpha_j > 0$, $i \neq j$) if and only if $\alpha_i < \sigma$.

(iii) If $\alpha_j > \sigma$, then there exists $x > 0$ such that marginally raising the rating for product j raises the quantity of some other product i ($\partial q_i^e / \partial \alpha_j > 0$) if and only if either $\alpha_j < \sigma + x$ with $\alpha_i < \sigma$ or $\alpha_j > \sigma + x$ with $\alpha_i > \sigma$.

Proof The equilibrium outcome $\{q_1^e, ..., q_N^e, \lambda^e\}$ solves the following system of equations:

$$\begin{aligned}
F^1(\boldsymbol{q}, \lambda; \boldsymbol{\alpha}) &\equiv & -\textstyle\sum_{i=1}^N [\alpha_i - \sigma]\, q_i &= 0\\
F^2(\boldsymbol{q}, \lambda; \boldsymbol{\alpha}) &\equiv & \partial U(\boldsymbol{q})/\partial q_1 - C_1'(q_1) - \lambda[\alpha_1 - \sigma] &= 0\\
&\vdots & &\vdots\\
F^{N+1}(\boldsymbol{q}, \lambda; \boldsymbol{\alpha}) &\equiv & \partial U(\boldsymbol{q})/\partial q_N - C_N'(q_N) - \lambda[\alpha_N - \sigma] &= 0.
\end{aligned}$$

The Jacobian of this system is the bordered Hessian H of the original system. At a global maximum, the determinant of this bordered Hessian must have the same sign as $(-1)^N$. By the Implicit Function Theorem and Cramer's Rule, we have:

$$\frac{\partial q_i^e}{\partial \alpha_j} = -\frac{det(H_{i+1})}{det(H)},$$

where H_{i+1} is the matrix H with column $i+1$ replaced by the partial derivative of the column vector $\boldsymbol{F}$ with respect to α_j. This column vector of partial derivatives has $-q_j$ in its first row, $-\lambda$ in row $j+1$, and zeroes elsewhere. If we interchange column $i+1$ with each of the $N-i$ columns to its right and also interchange row $i+1$ with each of the $N-i$ rows beneath it, we obtain the following convenient block matrix form that has the same determinant as H_{i+1}:

$$det(H_{i+1}) = det\left(\begin{bmatrix} A & B \\ C & D \end{bmatrix}\right) = det(A)\, det\left(D - CA^{-1}B\right).$$

B is the $N \times 1$ vector with $-q_j$ in the first row, $-\lambda$ in row $j+1$ if $j < i$ or in row j if $j > i$, and zeroes elsewhere. C is the $1 \times N$ vector with α_i in its first column and zeroes elsewhere. D is a real number such that

$$D = \begin{cases} -\lambda & \text{if } i = j, \\ 0 & \text{if } i \neq j. \end{cases}$$

The $N \times N$ matrix A is the bordered Hessian for the system lacking product i. By the second-order condition for a constrained maximum, the determinant of A has the same sign as $(-1)^{N-1}$ and the determinant of H has the same sign as $(-1)^N$. We therefore have:

$$\frac{\partial q_i^e}{\partial \alpha_j} \propto det(D - CA^{-1}B) = D - CA^{-1}B.$$

Further, we only need to know the first row of A^{-1} because C has zeroes in all but its first column.

We invert A using Gauss-Jordan elimination. Entry $(1,1)$ of A^{-1} is

$$\left[\sum_{k=1,k\neq i}^{N} \frac{(\alpha_k - \sigma)^2}{C_k''(q_k) - \partial^2 U(\boldsymbol{q})/\partial q_k^2}\right]^{-1}.$$

For $m \in \{1, \ldots, N-1\}$, entry $(1, m+1)$ of A^{-1} is

$$\frac{-(\alpha_n - \sigma)}{\left[C_n''(q_n) - \frac{\partial^2 U(\boldsymbol{q})}{\partial q_n^2}\right] \sum_{k=1,k\neq i}^{N} \frac{(\alpha_k-\sigma)^2}{C_k''(q_k) - \partial^2 U(\boldsymbol{q})/\partial q_k^2}},$$

with $n = m$ for $m < i$ and $n = m+1$ for $m \geq i$. We have:

$$\frac{\partial q_i^e}{\partial \alpha_j} \propto D - CA^{-1}B = \begin{cases} -\lambda - q_i \frac{\alpha_i - \sigma}{\sum_{k=1,k\neq i}^{N} \frac{(\alpha_k-\sigma)^2}{C_k''(q_k) - \partial^2 U(\boldsymbol{q})/\partial q_k^2}} & \text{if } i = j, \\ \frac{\alpha_i - \sigma}{\sum_{k=1,k\neq i}^{N} \frac{(\alpha_k-\sigma)^2}{C_k''(q_k) - \partial^2 U(\boldsymbol{q})/\partial q_k^2}} \left(-q_j + \lambda \frac{\alpha_j - \sigma}{C_j''(q_j) - \partial^2 U(\boldsymbol{q})/\partial q_j^2}\right) & \text{if } i \neq j. \end{cases}$$

For $i = j$, the left-hand term is negative and the right-hand term has the opposite sign as $\alpha_i - \sigma$. The whole expression is negative if and only if $\alpha_i - \sigma > -\lambda\, q_i^{-1} \sum_{k=1,k\neq i}^{N} [\alpha_k - \sigma]^2 [C_k''(q_k) - \partial^2 U(\boldsymbol{q})/\partial q_k^2]^{-1}$, where the right-hand side is negative (part i). For $i \neq j$, the term outside parentheses has the same sign as $\alpha_i - \sigma$. The expression in parentheses is clearly negative if $\alpha_j < \sigma$. The whole expression is therefore unambiguously signed when $\alpha_j < \sigma$, with sign opposite to that of $\alpha_i - \sigma$ (part ii). If $\alpha_j > \sigma$, then the expression in parentheses is positive if and only if $\alpha_j - \sigma > q_j\, \lambda^{-1} [C_j''(q_j) - \partial^2 U(\boldsymbol{q})/\partial q_j^2]$. The right-hand term is positive. The sign of the whole expression is the same as the sign of the term in parentheses if and only if $\alpha_i - \sigma > 0$. Part iii follows. ∎

The second lemma describes how equilibrium quantities respond to changes in the level of the standard.

Lemma 5 *Marginally raising the standard has the same effect on the quantity of product i as marginally lowering every rating $\left(\partial q_i^e/\partial \sigma = -\sum_{k=1}^{N} \partial q_i^e/\partial \alpha_k\right)$. If $\alpha_i > \sigma$ and, $\forall j \neq i$, $\alpha_j < \sigma$, then marginally raising the standard increases the quantity of product i ($\partial q_i^e/\partial \sigma > 0$).*

Proof The proof follows that of Lemma 4, except with $D = \lambda$ and B a column vector with first element $\sum_{j=1}^{N} q_j$ and remaining elements λ. We have:

$$\frac{\partial q_i^e}{\partial \sigma} \propto D - CA^{-1}B = \lambda + \frac{\alpha_i - \sigma}{\sum_{n=1,n\neq i}^{N} \frac{(\alpha_n-\sigma)^2}{C_n''(q_n)-\partial^2 U(\boldsymbol{q})/\partial q_n^2}} \left(\sum_{j=1}^{N} q_j - \lambda \sum_{k=1,k\neq i}^{N} \frac{\alpha_k - \sigma}{C_k''(q_k) - \partial^2 U(\boldsymbol{q})/\partial q_k^2} \right),$$

$$\Rightarrow \frac{\partial q_i^e}{\partial \sigma} = -\sum_{k=1}^{N} \frac{\partial q_i^e}{\partial \alpha_k}. \tag{7}$$

The bottom expression uses the results for $\partial q_i^e / \partial \alpha_j$ in the proof of Lemma 4 to establish the first part. In the top expression, the left-hand term (λ) is positive. The term multiplying the parentheses is positive if and only if $\alpha_i > \sigma$. The first term in parentheses is positive. The second term in parentheses is positive if and only if $\alpha_k < \sigma \; \forall k \neq i$. The top expression is unambiguously positive if all of these terms are positive, which establishes the second part of the corollary. ∎

The third lemma extends Proposition 3(i) in Holland et al. (2009) to the case of N product categories and unconstrained ratings:

Lemma 6 *Around an optimum, marginally increasing any rating α_j must decrease aggregate emissions $\left(\sum_{i=1}^{N} \partial \left[\beta_i q_i^e\right] / \partial \alpha_j \leq 0\right)$.*

Proof Differentiate the intensity constraint with respect to some $\hat{\alpha}_j$ to obtain:

$$\sum_{i=1}^{N} \hat{\alpha}_i \frac{\partial q_i(\hat{\boldsymbol{\alpha}})}{\partial \hat{\alpha}_j} = -q_j. \tag{8}$$

This equality must hold at the equilibrium outcomes. Substitute into equation (6) to get

$$\sum_{i=1}^{N} \tau \frac{\partial \beta_i q_i^e(\hat{\boldsymbol{\alpha}})}{\partial \hat{\alpha}_j} \propto -q_j \leq 0 \tag{9}$$

around the optimum. ∎

Proof of Proposition 1

Recall that a regulator who can adjust only the ratings can achieve every combination of market-clearing quantities compatible with some intensity standard and with the sign of the difference between the numeraire rating and the level of the standard. If we let the highest-emitting product be the numeraire, then that sign is also positive when the regulator selects the level of the standard with ratings fixed at expected emission factors. Since the latter type of regulator can only achieve a subset of market-clearing quantities achievable by the regulator who can set the ratings, the first claim must be true: $V_N^{\alpha}(\sigma) \geq V_N^{\sigma}(\boldsymbol{\beta})$.

Now consider when the set of market outcomes achievable by a regulator who can set only the level of the standard includes the market outcome selected by a welfare-maximizing regulator who can set the emission ratings. First, differentiate the intensity standard with respect to σ to obtain

$$\sum_{i=1}^{N}[\alpha_i - \sigma]\frac{\partial q_i(\sigma;\boldsymbol{\alpha})}{\partial\sigma} = \sum_{i=1}^{N} q_i. \tag{10}$$

We now define the optimal intensity standard for fixed ratings. Let $W(\sigma,\boldsymbol{\alpha})$ be welfare under an intensity standard defined by σ and $\boldsymbol{\alpha}$. With fixed ratings $\boldsymbol{\alpha}$, the regulator selects σ to maximize welfare:

$$V_N^{\sigma}(\boldsymbol{\beta}) \equiv \max_{\sigma} W(\sigma,\boldsymbol{\beta}) = \max_{\sigma}\left\{U(\boldsymbol{q^e}(\sigma;\boldsymbol{\beta})) - \sum_{i=1}^{N} C_i(q_i^e(\sigma;\boldsymbol{\beta})) - \tau\,\boldsymbol{\beta}^T\boldsymbol{q^e}(\sigma;\boldsymbol{\beta})\right\}.$$

Equilibrium quantities are functions of σ for a given set of ratings. The optimal level of the standard solves the first-order condition:

$$0 = \sum_{i=1}^{N}\left(\frac{\partial U(\boldsymbol{q^e}(\sigma;\boldsymbol{\beta}))}{\partial q_i} - C_i'(q_i^e(\sigma;\boldsymbol{\beta})) - \tau\,\beta_i\right)\frac{\partial q_i^e(\sigma;\boldsymbol{\beta})}{\partial\sigma}. \tag{11}$$

Substituting in the equations governing equilibrium outcomes, the first-order condition becomes

$$\lambda^e(\sigma;\boldsymbol{\beta})\sum_{i=1}^{N}[\beta_i - \sigma]\frac{\partial q_i^e(\sigma;\boldsymbol{\beta})}{\partial\sigma} = \tau\sum_{i=1}^{N}\beta_i\frac{\partial q_i^e(\sigma;\boldsymbol{\beta})}{\partial\sigma}. \tag{12}$$

This equation implicitly defines the optimal level of the standard (σ^*) as a function of the estimated emission intensities, with the emission ratings fixed to equal the estimated emission intensities. Apply the Implicit Function Theorem:

$$\frac{\partial\sigma^*}{\partial\beta_i} = \frac{(\tau-\lambda^e)\frac{\partial q_i^e}{\partial\sigma} - \sum_{k=1}^{N}\left[\frac{\partial\lambda^e}{\partial\alpha_i}(\beta_k-\sigma)\frac{\partial q_k^e}{\partial\sigma} + \left(\lambda^e\left[\beta_k-\sigma\right] - \tau\,\beta_k\right)\frac{\partial^2 q_k^e}{\partial\sigma\partial\alpha_i}\right]}{\sum_{k=1}^{N}\left[\frac{\partial\lambda^e}{\partial\sigma}(\beta_k-\sigma)\frac{\partial q_k^e}{\partial\sigma} - \lambda^e\frac{\partial q_k^e}{\partial\sigma} + \left(\lambda^e\left[\beta_k-\sigma\right] - \tau\,\beta_k\right)\frac{\partial^2 q_k^e}{\partial\sigma^2}\right]}, \tag{13}$$

suppressing the arguments for the equilibrium quantities and shadow cost.

Assume the regulator can reach the best possible intensity standard by manipulating only σ. In that case, altering any rating could not further increase welfare. That is, σ^* and the fixed ratings $\boldsymbol{\alpha} = \boldsymbol{\beta}$ jointly achieve the optimal standard if and only if

$$0 = \left.\frac{\partial W(\sigma^*,\boldsymbol{\alpha})}{\partial\alpha_j}\right|_{\boldsymbol{\alpha}=\boldsymbol{\beta}} \equiv \chi_j(\boldsymbol{\beta}),\ \forall j\in\{1,..,N-1\},$$

where we recognize that one rating is redundant. This condition is identical to the first-order condition for the regulator's optimal choice of ratings, except evaluated at σ^* and $\boldsymbol{\beta}$. From equation (6), we have:

$$\chi_j(\boldsymbol{\beta}) = \lambda^e\sum_{k=1}^{N}\left(\beta_k - \sigma^*(\boldsymbol{\beta})\right)\frac{\partial q_k^e}{\partial\alpha_j} - \tau\sum_{k=1}^{N}\beta_k\frac{\partial q_k^e}{\partial\alpha_j},\ \forall j\in\{1,..,N-1\}, \tag{14}$$

where we suppress the arguments for the equilibrium quantities and shadow cost but make the argument of σ^* explicit. For it to be possible to attain the optimal standard for any combination of emission factors, the N equations in (14) must each equal zero for any choice of β_i. Therefore, they must hold for any perturbation of $\boldsymbol{\beta}$, implying:

$$0 = \frac{\partial \chi_j(\boldsymbol{\beta})}{\partial \beta_i} = (\lambda^e - \tau)\frac{\partial q_i^e}{\partial \alpha_j} + \left[\frac{\partial \lambda^e}{\partial \alpha_i} + \frac{\partial \lambda^e}{\partial \sigma}\frac{\partial \sigma^*}{\partial \beta_i}\right]\sum_{k=1}^{N}(\beta_k - \sigma^*)\frac{\partial q_k^e}{\partial \alpha_j} - \lambda^e \frac{\partial \sigma^*}{\partial \beta_i}\sum_{k=1}^{N}\frac{\partial q_k^e}{\partial \alpha_j}$$
$$+ \sum_{k=1}^{N}(\lambda^e[\beta_k - \sigma^*] - \tau\beta_k)\left\{\frac{\partial^2 q_k^e}{\partial \alpha_j \partial \alpha_i} + \frac{\partial^2 q_k^e}{\partial \alpha_j \partial \sigma}\frac{\partial \sigma^*}{\partial \beta_i}\right\},$$
$$\forall j \in \{1,..,N-1\},\ \forall i \in \{1,..,N\}.$$
$$\Rightarrow \frac{\partial \sigma^*}{\partial \beta_i} = \frac{(\lambda^e - \tau)\frac{\partial q_i^e}{\partial \alpha_j} + \sum_{k=1}^{N}\left[\frac{\partial \lambda^e}{\partial \alpha_i}(\beta_k - \sigma^*)\frac{\partial q_k^e}{\partial \alpha_j} + (\lambda^e[\beta_k - \sigma^*] - \tau\beta_k)\frac{\partial^2 q_k^e}{\partial \alpha_j \partial \alpha_i}\right]}{\sum_{k=1}^{N}\left[\lambda^e \frac{\partial q_k^e}{\partial \alpha_j} - \frac{\partial \lambda^e}{\partial \sigma}(\beta_k - \sigma^*)\frac{\partial q_k^e}{\partial \alpha_j} - (\lambda^e[\beta_k - \sigma^*] - \tau\beta_k)\frac{\partial^2 q_k^e}{\partial \alpha_j \partial \sigma}\right]},$$
$$\forall j \in \{1,..,N-1\},\ \forall i \in \{1,..,N\}. \tag{15}$$

Combining equation (13) (defining the choice of σ that maximizes welfare) and equation (15) (derived from the assumed overall optimality of the intensity standard with ratings fixed at the expected emission factors), we have:

$$\frac{(\tau - \lambda^e)\frac{\partial q_i^e}{\partial \sigma} - \sum_{k=1}^{N}\left[\frac{\partial \lambda^e}{\partial \alpha_i}(\beta_k - \sigma)\frac{\partial q_k^e}{\partial \sigma} + (\lambda^e[\beta_k - \sigma] - \tau\,\beta_k)\frac{\partial^2 q_k^e}{\partial \sigma \partial \alpha_i}\right]}{\sum_{k=1}^{N}\left[\frac{\partial \lambda^e}{\partial \sigma}(\beta_k - \sigma)\frac{\partial q_k^e}{\partial \sigma} - \lambda^e \frac{\partial q_k^e}{\partial \sigma} + (\lambda^e[\beta_k - \sigma] - \tau\,\beta_k)\frac{\partial^2 q_k^e}{\partial \sigma^2}\right]}$$
$$= \frac{(\lambda^e - \tau)\frac{\partial q_i^e}{\partial \alpha_j} + \sum_{k=1}^{N}\left[\frac{\partial \lambda^e}{\partial \alpha_i}(\beta_k - \sigma^*)\frac{\partial q_k^e}{\partial \alpha_j} + (\lambda^e[\beta_k - \sigma^*] - \tau\beta_k)\frac{\partial^2 q_k^e}{\partial \alpha_j \partial \alpha_i}\right]}{\sum_{k=1}^{N}\left[\lambda^e \frac{\partial q_k^e}{\partial \alpha_j} - \frac{\partial \lambda^e}{\partial \sigma}(\beta_k - \sigma^*)\frac{\partial q_k^e}{\partial \alpha_j} - (\lambda^e[\beta_k - \sigma^*] - \tau\beta_k)\frac{\partial^2 q_k^e}{\partial \alpha_j \partial \sigma}\right]},$$
$$\forall j \in \{1,..,N-1\},\ \forall i \in \{1,..,N\}. \tag{16}$$

Use equation (7) to rewrite equation (16) as

$$\frac{(\lambda^e - \tau)\sum_{k=1}^{N}\frac{\partial q_i^e}{\partial \alpha_k} + \sum_{k=1}^{N}\left[\frac{\partial \lambda^e}{\partial \alpha_i}(\beta_k - \sigma)\sum_{m=1}^{N}\frac{\partial q_k^e}{\partial \alpha_m} + (\lambda^e[\beta_k - \sigma] - \tau\,\beta_k)\sum_{m=1}^{N}\frac{\partial^2 q_k^e}{\partial \alpha_i \partial \alpha_m}\right]}{\sum_{k=1}^{N}\left[\lambda^e \sum_{m=1}^{N}\frac{\partial q_k^e}{\partial \alpha_m} - \frac{\partial \lambda^e}{\partial \sigma}(\beta_k - \sigma)\sum_{m=1}^{N}\frac{\partial q_k^e}{\partial \alpha_m} - (\lambda^e[\beta_k - \sigma] - \tau\,\beta_k)\sum_{m=1}^{N}\frac{\partial^2 q_k^e}{\partial \alpha_m \partial \sigma}\right]}$$
$$= \frac{(\lambda^e - \tau)\frac{\partial q_i^e}{\partial \alpha_j} + \sum_{k=1}^{N}\left[\frac{\partial \lambda^e}{\partial \alpha_i}(\beta_k - \sigma^*)\frac{\partial q_k^e}{\partial \alpha_j} + (\lambda^e[\beta_k - \sigma^*] - \tau\beta_k)\frac{\partial^2 q_k^e}{\partial \alpha_j \partial \alpha_i}\right]}{\sum_{k=1}^{N}\left[\lambda^e \frac{\partial q_k^e}{\partial \alpha_j} - \frac{\partial \lambda^e}{\partial \sigma}(\beta_k - \sigma^*)\frac{\partial q_k^e}{\partial \alpha_j} - (\lambda^e[\beta_k - \sigma^*] - \tau\beta_k)\frac{\partial^2 q_k^e}{\partial \alpha_j \partial \sigma}\right]},$$
$$\forall j \in \{1,..,N-1\},\ \forall i \in \{1,..,N\}. \tag{17}$$

This condition holds only if

$$\sum_{m=1}^{N}\frac{\partial q_k^e}{\partial \alpha_m} = \frac{\partial q_k^e}{\partial \alpha_j},\ \forall j \in \{1,..,N-1\},\ \forall k \in \{1,..,N\}, \tag{18}$$

which is true if and only if

$$\sum_{m=1,m\neq j}^{N} \frac{\partial q_k^e}{\partial \alpha_m} = 0,\ \forall j \in \{1,..,N-1\},\ \forall k \in \{1,..,N\}. \tag{19}$$

If $N = 2$, this necessary condition reduces to

$$\frac{\partial q_k^e}{\partial \alpha_N} = 0,\ \forall k \in \{1,..,N\},$$

which is in fact true by the definition of product N as the numeraire. Further, in a two-product setting, incrementally raising the standard is identical to incrementally lowering both ratings. Therefore when $N = 2$, equation (16) always holds and the best possible intensity standard can be achieved by manipulating only the level of the standard.

If $N > 2$, the necessary condition implies

$$\sum_{m=1,m\neq j}^{N} \frac{\partial q_k^e}{\partial \alpha_m} = \sum_{m=1,m\neq s}^{N} \frac{\partial q_k^e}{\partial \alpha_m} = 0,\ \forall s,j \in \{1,..,N-1\},\ \forall k \in \{1,..,N\},$$

which implies

$$\frac{\partial q_k^e}{\partial \alpha_j} = \frac{\partial q_k^e}{\partial \alpha_s},\ \forall s,j \in \{1,..,N-1\},\ \forall k \in \{1,..,N\}$$

and so

$$\frac{\partial q_k^e}{\partial \alpha_N} = -(N-1)\frac{\partial q_k^e}{\partial \alpha_j},\ \forall j \in \{1,..,N-1\},\ \forall k \in \{1,..,N\}.$$

Therefore, by the definition of α_N as the numeraire,

$$\frac{\partial q_k^e}{\partial \alpha_j} = 0,\ \forall j \in \{1,..,N-1\},\ \forall k \in \{1,..,N\}.$$

However, all partial derivatives equaling zero contradicts the assumptions of nonzero production and a binding intensity constraint. This contradiction shows that if the regulator can attain the best possible intensity standard by adjusting only the level of the standard for some set of expected emission factors, then perturbing that set by even an infinitesimally small amount would prevent the regulator from attaining the best possible intensity standard without also selecting at least one rating. When ratings are constrained to equal expected emission factors, the set of emission factors that can achieve the best possible intensity standard is a set of measure zero. We have proved that $V_N^{\alpha}(\sigma) > V_N^{\sigma}(\boldsymbol{\beta})$ if $N > 2$ and $\boldsymbol{\beta} \notin A$, where A is a set of measure zero. ∎

Proof of Proposition 2

In a two-product system, the regulator only selects a single rating. In that case or in the N-product case in which the regulator is only free to adjust a single rating, the optimal rating is defined by the form of equation (6) corresponding to that one rating. Apply the Implicit Function Theorem:

$$\frac{\partial \hat{\alpha}_j^*}{\partial \beta_i} = \frac{\tau \frac{\partial q_i^e(\hat{\boldsymbol{\alpha}})}{\partial \hat{\alpha}_j}}{\frac{\partial \lambda^e(\hat{\boldsymbol{\alpha}})}{\partial \hat{\alpha}_j} \sum_{k=1}^N \hat{\alpha}_k \frac{\partial q_k(\hat{\boldsymbol{\alpha}})}{\partial \hat{\alpha}_j} + \lambda^e(\hat{\boldsymbol{\alpha}}) \frac{\partial q_j^e(\hat{\boldsymbol{\alpha}})}{\partial \hat{\alpha}_j} + \sum_{k=1}^N \left[\lambda^e(\hat{\boldsymbol{\alpha}})\, \hat{\alpha}_k - \tau\, \beta_k\right] \frac{\partial^2 q_k(\hat{\boldsymbol{\alpha}})}{\partial \hat{\alpha}_j^2}}.$$

All partial derivatives in this proof are evaluated at $\hat{\alpha}_j = \hat{\alpha}_j^*$. The denominator is negative by the second-order condition for a global maximum. The numerator is negative if and only if $\partial q_i^e(\hat{\boldsymbol{\alpha}})/\partial \hat{\alpha}_j < 0$. Parts (i)-(iii) follow by Lemma 4.

Now consider how the optimal rating for product j changes in the marginal damage from emissions:

$$\frac{\partial \hat{\alpha}_j^*}{\partial \tau} = \frac{\sum_{k=1}^N \frac{\partial \beta_k q_k^e(\hat{\boldsymbol{\alpha}})}{\partial \hat{\alpha}_j}}{\frac{\partial \lambda^e(\hat{\boldsymbol{\alpha}})}{\partial \hat{\alpha}_j} \sum_{k=1}^N \hat{\alpha}_k \frac{\partial q_k(\hat{\boldsymbol{\alpha}})}{\partial \hat{\alpha}_j} + \lambda^e(\hat{\boldsymbol{\alpha}}) \frac{\partial q_j^e(\hat{\boldsymbol{\alpha}})}{\partial \hat{\alpha}_j} + \sum_{k=1}^N \left[\lambda^e(\hat{\boldsymbol{\alpha}})\, \hat{\alpha}_k - \tau\, \beta_k\right] \frac{\partial^2 q_k(\hat{\boldsymbol{\alpha}})}{\partial \hat{\alpha}_j^2}}.$$

The denominator is as above. The numerator is the change in total emissions from an increase in rating j. The claim in (iv) follows by Lemma 6. ∎

Proof of Corollary 3

When the regulator can select the standard's level σ but not the emission ratings $\boldsymbol{\alpha}$, the optimal level of the standard is defined by equation (12) but without the ratings set to $\boldsymbol{\beta}$:

$$\lambda^e(\sigma;\boldsymbol{\alpha}) \sum_{i=1}^N [\alpha_i - \sigma] \frac{d\, q_i^e(\sigma;\boldsymbol{\alpha})}{d\,\sigma} = \tau \sum_{i=1}^N \beta_i \frac{d\, q_i^e(\sigma;\boldsymbol{\alpha})}{d\,\sigma}.$$

Apply the Implicit Function Theorem:

$$\frac{\partial \sigma^*}{\partial \beta_i} = \frac{\tau \frac{d\, q_i^e(\sigma;\boldsymbol{\alpha})}{d\,\sigma}}{\sum_{k=1}^N \left[\frac{d\,\lambda^e(\sigma;\boldsymbol{\alpha})}{d\,\sigma}[\alpha_k - \sigma] \frac{d\, q_k^e(\sigma;\boldsymbol{\alpha})}{d\,\sigma} - \lambda^e(\sigma;\boldsymbol{\alpha}) \frac{d\, q_k^e(\sigma;\boldsymbol{\alpha})}{d\,\sigma} + \left(\lambda^e(\sigma;\boldsymbol{\alpha})[\alpha_k - \sigma] - \tau \beta_k\right) \frac{d^2 q_k^e(\sigma;\boldsymbol{\alpha})}{d\sigma^2}\right]}.$$

In this proof, all derivatives are evaluated at σ^*. The denominator is negative by the second-order condition for a global maximum. The numerator is positive if and only if $dq_i^e(\sigma;\boldsymbol{\alpha})/d\sigma$ is positive. Part (i) follows by Lemma 5.

Now consider the effect of marginal damage on the optimal standard, assuming the convexity of the damage function does not change:

$$\frac{\partial \sigma^*}{\partial \tau} = \frac{\sum_{k=1}^N \frac{d\,\beta_k q_k^e(\sigma;\boldsymbol{\alpha})}{d\,\sigma}}{\sum_{k=1}^N \left[\frac{d\,\lambda^e(\sigma;\boldsymbol{\alpha})}{d\,\sigma}[\alpha_k - \sigma] \frac{d\, q_k^e(\sigma;\boldsymbol{\alpha})}{d\,\sigma} - \lambda^e(\sigma;\boldsymbol{\alpha}) \frac{d\, q_k^e(\sigma;\boldsymbol{\alpha})}{d\,\sigma} + \left(\lambda^e(\sigma;\boldsymbol{\alpha})[\alpha_k - \sigma] - \tau \beta_k\right) \frac{d^2 q_k^e(\sigma;\boldsymbol{\alpha})}{d\sigma^2}\right]}.$$

The denominator is as above. The numerator is the change in estimated emissions from raising the standard. Substituting equation (10) into the first-order condition, it is easy to show that the numerator is positive (total expected emissions increase when the standard is loosened from its optimal level). The claim in (ii) follows.

Finally, consider the effect of changing some exogenously given rating α_i:

$$\frac{\partial \sigma^*}{\partial \alpha_i} = \frac{-\lambda^e(\sigma;\boldsymbol{\alpha})\frac{d\, q_i^e(\sigma;\boldsymbol{\alpha})}{d\,\sigma} - \frac{\partial \lambda^e(\sigma;\boldsymbol{\alpha})}{\partial \alpha_i}\sum_{k=1}^N [\alpha_k - \sigma]\frac{d\, q_k^e(\sigma;\boldsymbol{\alpha})}{d\,\sigma} - \sum_{k=1}^N \left(\lambda^e(\sigma;\boldsymbol{\alpha})[\alpha_k - \sigma] - \tau\beta_k\right)\frac{\partial^2 q_k^e(\sigma;\boldsymbol{\alpha})}{\partial\sigma\partial\alpha_i}}{\sum_{k=1}^N \left[\frac{d\,\lambda^e(\sigma;\boldsymbol{\alpha})}{d\,\sigma}[\alpha_k - \sigma]\frac{d\, q_k^e(\sigma;\boldsymbol{\alpha})}{d\,\sigma} - \lambda^e(\sigma;\boldsymbol{\alpha})\frac{d\, q_k^e(\sigma;\boldsymbol{\alpha})}{d\,\sigma} + \left(\lambda^e(\sigma;\boldsymbol{\alpha})[\alpha_k - \sigma] - \tau\beta_k\right)\frac{d^2 q_k^e(\sigma;\boldsymbol{\alpha})}{d\sigma^2}\right]}.$$

The denominator is as above. Substitute from equation (10):

$$\frac{\partial \sigma^*}{\partial \alpha_i} \propto \lambda^e(\sigma;\boldsymbol{\alpha})\frac{d\, q_i^e(\sigma;\boldsymbol{\alpha})}{d\,\sigma} + \frac{\partial \lambda^e(\sigma;\boldsymbol{\alpha})}{\partial \alpha_i}\sum_{k=1}^N q_k^e(\sigma;\boldsymbol{\alpha}) + \sum_{k=1}^N \left(\lambda^e(\sigma;\boldsymbol{\alpha})[\alpha_k - \sigma] - \tau\beta_k\right)\frac{\partial^2 q_k^e(\sigma;\boldsymbol{\alpha})}{\partial\sigma\partial\alpha_i}.$$

This expression is positive if and only if

$$\frac{d\, q_i^e(\sigma;\boldsymbol{\alpha})}{d\,\sigma} > -\frac{1}{\lambda^e(\sigma;\boldsymbol{\alpha})}\left[\frac{\partial \lambda^e(\sigma;\boldsymbol{\alpha})}{\partial \alpha_i}\sum_{k=1}^N q_k^e(\sigma;\boldsymbol{\alpha}) + \sum_{k=1}^N \left(\lambda^e(\sigma;\boldsymbol{\alpha})[\alpha_k - \sigma] - \tau\beta_k\right)\frac{\partial^2 q_k^e(\sigma;\boldsymbol{\alpha})}{\partial\sigma\partial\alpha_i}\right].$$

Ignoring second-order responses of equilibrium quantities, we have

$$\frac{d\, q_i^e(\sigma;\boldsymbol{\alpha})}{d\,\sigma} > -\frac{1}{\lambda^e(\sigma;\boldsymbol{\alpha})}\frac{\partial \lambda^e(\sigma;\boldsymbol{\alpha})}{\partial \alpha_i}\sum_{k=1}^N q_k^e(\sigma;\boldsymbol{\alpha}).$$

The first claim in (iii) follows from recognizing that the right-hand side is negative. The second claim follows from applying Lemma 5. ∎