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Investment, disinvestment and policy impact analysis in the dairy sector: a real options approach

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Investment, disinvestment and policy impact analysis in the dairy sector: a real options approach

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Abstract

Due to strong changes in general economic conditions and increasing market volatility, a higher level of investments and disinvestments in the dairy sector can be observed. Therefore, the simultaneous analysis of firms' investment and disinvestment decisions under consideration of uncertainty and market interventions is of high relevance. In this paper, a flexible real options market model is applied to the German dairy sector. The model considers disinvestment besides investment options and, moreover, allows the integration and assessment of different market interventions that are relevant to the dairy sector. According to the results, production ceilings and investment subsidies are preferable to price floors because the welfare is less reduced for a given stimulation of the willingness to invest. Moreover, it is shown that not considering disinvestment options, which in reality often exist, can lead to incorrect valuations of investment strategies at firm level and incorrect policy impact analyses at macroeconomic level.

Keywords: Investment and disinvestment, real options, competition, policy impact analysis, dairy sector.

Zusammenfassung

Aufgrund von starken Veränderungen in den politischen Rahmenbedingungen und zunehmend volatiler Märkte nehmen sowohl Investitionen als auch Desinvestitionen im Milchsektor in den letzten Jahren deutlich zu. Aus diesem Grund ist die simultane Analyse von Investitions- und Desinvestitionsentscheidungen landwirtschaftlicher Unternehmen unter Berücksichtigung von Unsicherheit und Marktinterventionen von besonderer Relevanz. In dem vorliegenden Beitrag wird ein flexibles Realoptionsmodell auf den deutschen Milchsektor angewandt. Das Modell berücksichtigt neben Investitionsoptionen auch Desinvestitionsoptionen und erlaubt des Weiteren die Integration und Bewertung unterschiedlicher Marktinterventionen, die für den Milchsektor relevant sind. Entsprechend der Ergebnisse sind sowohl Produktionsobergrenzen als auch Investitionszulagen gleichsam Preisuntergrenzen vorzuziehen, da die sektorale Wohlfahrt für eine gegebene Stimulierung der Investitionsbereitschaft weniger stark eingeschränkt wird. Außerdem kann gezeigt werden, dass die Nicht-Berücksichtigung von Desinvestitionsoptionen, die in der Realität häufig vorliegen, zu fehlerhaften Investitionsentscheidungen auf einzelbetrieblicher Ebene und fehlerhaften Politikfolgenabschätzungen auf sektoraler Ebene führen kann.

Schlüsselwörter: Investition und Desinvestition, Realoptionen, Wettbewerb, Politikfolgenabschätzung, Milchsektor.

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1 Introduction

The dairy sector is currently globally exposed to strong changes in general economic conditions. Examples for this are the abolishment of the milk production quota in the EU by 2015, the allocation of additional production quota among the Canadian provinces in the years 2010 and 2011 as well as governmental investment support programmes for structually weak areas, like in the newly-formed German states. In addition, and, at least partially, as a result of these changes, milk prices have become more volatile in recent years (Informa Economics, 2010). In consequence of both the changes in general economic conditions and the increasing price volatility, adjustments in the dairy sector in the form of investments and disinvestments can be observed. In the German federal state of Mecklenburg-Western Pomerana, for instance, the dairy livestock grew by 4,000 heads in the past five years along with a massive national investment support programme; the number of farm firms, however, decreased (agrarheute, 2013). Hence, the simultaneous analysis of investment and disinvestment decisions under consideration of uncertainty and different political market interventions in agriculture in general, and in the dairy sector in specific, is of high practical relevance.

In the past two decades, agricultural economists showed that the Real Options Approach (ROA) is more advantageous for analysing investments in agriculture than traditional investment models based on the net present value (NPV) rule (e.g. Carey and Zilberman, 2002; Odening et al., 2005; Richards and Patterson, 1998). The reason is that investments in agriculture are mostly afflicted by (partially) irreversible costs, uncertainty of the future cash flows and temporal flexibility in making the investment. This particularly applies to investments in the dairy sector (e.g. Engel and Hyde, 2003; Purvis et al., 1995; Tauer, 2006). The ROA takes into account explicitly these characteristics through analysing investment decisions under dynamic-stochastic conditions and extending the NPV by the value of entrepreneurial flexibility.

The applicability of the ROA to investment problems in the dairy sector, especially under consideration of the aforementioned economic environment, however, is problematic (Feil and Musshoff, 2013). First, the simultaneous analysis of firms' investment and disinvestment decisions in the real options context is complex. For simplification purposes, the vast majority of real options models in agriculture implicitly assume perfect irreversibility of the investment costs, that is, disinvestment options do not need to be considered (e.g. McDonald and Siegel, 1986). In contrast to this, most investments in agriculture in general, and dairy farm investments in particular, can be abandoned and their costs partially reserved (e.g. Breustedt and Glauben, 2007). Moreover, the availability of at least one essential production factor of European dairy farming is usually limited, namely the milk quota. This causes strong interdependencies of firms' investment and disinvestment decisions within a country ore a region, because firms cannot grow unless other firms shrink or exit (e.g. Chavas, 2001). However, there are only a few real options models that analyse investment and disinvestment decisions simultaneously. These models depend on numerical solution procedures because a solution in closed form does not exist for the determination of both values (e.g. Hill, 2010; Isik et al., 2003).

Second, most real options models, in particular the aforementioned ones analysing investment and disinvestment decisions simultaneously, do not consider competition. With this, they implicitly exploit the finding of Leahy (1993), who shows that a competitive investor finds the same optimal investment strategy as a myopic planner that ignores other firms' investment and disinvestment decisions. The implication of this finding is that the analysis of optimal investment and disinvestment decisions is simplified considerably due to the fact that competition does not need to be taken into account: The firms' optimal investment and disinvestment thresholds can be determined without the burdensome and iterative derivation of the endogenous equilibrium price process. However, the preconditions for applying this optimality property of myopic planning to competitive markets are very restrictive and, at least partially, unrealistic. Furthermore, not considering competition is especially problematic for the analysis of investments in industries characterised by market interventions, like the dairy sector. For a myopic planner who takes prices as exogenously given, merely the effects of such measures can be investigated which can be transformed directly into the price process, for example, price ceilings (e.g. Dixit, 1991). For many other relevant interventions, however, this is not possible or at the very least complex; for example, production quotas. Finally, through limiting the perspective to only one myopic firm and not considering competition, the welfare effects of market interventions cannot be analysed and therefore, comparative policy impact analyses are not possible.

Third, although a few real options studies explicitly consider competition, they again do not allow for disinvestment options (e.g. Feil et al., 2013). The model of Feil et al., for example, is capable of directly determining firms' investment thresholds in competitive markets and thus, does not rely on the restrictions associated with exploiting the optimality property of myopic planning. For the intention of simplification, their model, however, assumes that the investment costs are sunk in total, whereby abandonment options do not need to be considered.

With these research gaps in mind, this paper has three objectives with regard to the dairy sector: First, firms' investment and disinvestment decisions shall be analysed simultaneously within the real options context under consideration of different market interventions. Second, a detailed assessment of different market interventions shall be enabled. Third, the relevance of disinvestment options, which exist in the dairy sector but which are not considered in many real options models, shall be investigated both for the valuation of investment projects at firm level and for the assessment of market interventions at the macroeconomic level. To achieve these objectives, the real options market model developed by Feil and Musshoff (2013) is applied to the German dairy sector. The model is solved numerically by linking genetic algorithms (GAs) with stochastic simulation. Through this, equilibria in competitive markets can be directly determined and a vast modelling flexibility is gained. In particular, different market interventions, which are relevant to agriculture in general and to the dairy sector in specific, can be integrated and their effects on the firms' investment and disinvestment decisions as well as on the sectoral welfare quantified. For demonstration, a comparative analysis of the effects of price floors maintained by governmental purchases of excess supply, production ceilings and subsidies on investments is carried out. Price floors constitute the reference to many analytical real options models in the literature; for example, Dixit and Pindyck (1994: 296ff.) who also consider investment and disinvestment options simultaneously. Moreover, this instrument is frequently used in agriculture, for example in the EU for many commodities including milk until the Fischler reform in 2003. Production ceilings can be seen as simplified representative for milk quota systems, like in the EU or Canada. And investment subsidies are paid in many agricultural sectors, for instance in the newly-formed German states for investments into livestock farming. Consequently, the analysis shows that the model, first, provides a conceptual decision support with regard to firms' optimal investment strategies and, second, enables detailed policy impact analyses in the dairy sector.

The remainder of the paper is structured as follows: Section 2 firstly explains the fundamentals of the ROA. In section 3, the structure and the functioning of the real options market model is illustrated. After the assumptions for the application of the model to the German dairy sector are presented in section 4, the results are discussed in section 5. The paper ends with a summary of the main findings and conclusions concerning the model's application potential (section 6).

2 Theoretical background

Real options models exploit the analogy between a financial option and an investment or disinvestment project (e.g. Abel and Eberly, 1994; Dixit and Pindyck, 1994; Trigeorgis, 1996). With an opportunity to invest (disinvest), a firm is holding an "option" analogues to a financial call (put) option; it has the right but not the obligation to buy (sell) an asset at any time in the future. If the firm invests, it exercises the option by giving up the opportunity of waiting for new information to arrive with a potential positive effect on the profitability of the investment. This lost continuation value of the option is an opportunity cost that should be included as part of the investment costs. Furthermore, it is highly sensitive to the uncertainty of the future cash flows. In conclusion, an irreversible investment under uncertainty should only be made if the present value of its expected returns exceeds the investment costs by an amount equal to the value of waiting for additional information. In comparison to the NPV rule, this means that the critical product price at which the firm should invest (in the following referred to as investment trigger price) is shifted upwards because the cash flows do not only have to compensate the investment costs but also the lost value from deferring the investment. Analogously, a firm will only abandon the investment if the present value of its expected returns falls below the liquidation value by a considerable amount. This means that the disinvestment trigger price is shifted downwards.

The direct transferability of the financial option pricing theory to real investment problems, however, is problematic. Financial options constitute exclusive rights for their owners, whereas real investment opportunities are also open to other market participants in competitive markets. Thus, exceeding (falling below) the (dis)investment trigger price will also lead to similar reactions of competitors, which, taken as a whole, will change sectoral supply and, thus, equilibrium prices. In consequence, the price process cannot be considered any longer as exogenous. As the price process determines again the value of the investment project and, with this, the investment and disinvestment trigger price, the direct determination of these values is considerably complicated. Leahy (1993), however, demonstrates that under perfect competition, an investor who decides myopically and ignores potential market entries of competitors finds the same trigger prices as a competitive investor. This is explained in the following.

Leahy assumes a perfectly competitive industry consisting of small homogeneous price-taking firms, which use the same constant-returns-to-scale technology for production. The production output of all firms at time t, which equals the aggregated market supply X_t, is subject to depreciation with rate λ as well as investments and disinvestments of the firms in additional production capacity. The product price P_t results from the reactions of all firms in the form of X_t on the exogenous stochastic demand parameter μ_t. It is defined by a time-invariant inverse demand function D, which is assumed to be isoelastic (e.g. Dixit, 1991):

$$P_t = D(X_t, \mu_t) = \left(\frac{\mu_t}{X_t}\right)^{\Pi} \tag{1}$$

with

$$\Pi = -\frac{1}{\eta}$$

where η is the price elasticity of demand. Following Leahy and many other real options applications, the demand shock is described by a geometric Brownian motion (GBM)[1]:

$$\mathrm{d}\mu_t = \alpha \cdot \mu_t \cdot \mathrm{d}t + \sigma \cdot \mu_t \cdot \mathrm{d}z \tag{2}$$

α denotes the drift rate and σ the volatility. Both parameters are assumed to be constant. $\mathrm{d}z$ is the increment of a Wiener process. The stochastic demand process according to equation (2) can be translated into a stochastic price process (Odening et al., 2007):

$$\mathrm{d}P_t = \hat{\delta}(P_t, X_t) \cdot \mathrm{d}X_t + \hat{\alpha} \cdot P_t \cdot \mathrm{d}t + \hat{\sigma} \cdot P_t \cdot \mathrm{d}z \tag{3}$$

with

$$\hat{\delta}(P_t, X_t) = -\Pi \cdot X_t^{-1} \cdot P_t$$

$$\hat{\alpha} = \Pi \cdot \alpha + \frac{1}{2} \cdot \sigma^2 \cdot (\Pi^2 - \Pi)$$

$$\hat{\sigma} = \Pi \cdot \sigma$$

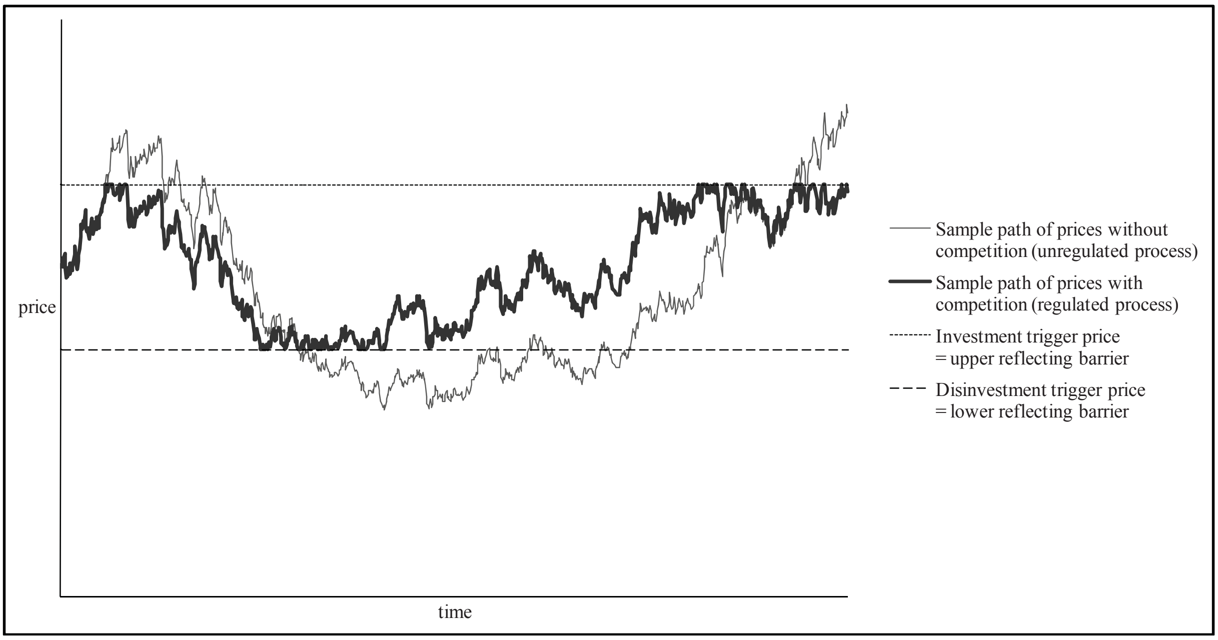
[1] By assuming a GBM, the exposition is simplified. However, the presence of a GBM is not essential for proving the validity of the optimality property of myopic planning. Baldursson and Karatzas (1997) deliver a generalisation.

Equation (3) specifies the regulated endogenous stochastic price process as anticipated by a competitive investor. The first term on the right-hand side of equation (3) captures price changes induced by investments and disinvestments of competitors. A myopic investor, however, ignores these effects and assumes an unregulated exogenous stochastic price process:

$$\mathrm{d}P_t = \hat{\alpha} \cdot P_t \cdot \mathrm{d}t + \hat{\sigma} \cdot P_t \cdot \mathrm{d}z \tag{4}$$

Figure 1 illustrates the respective differences between the regulated endogenous price process (cf. equation (3)) and the unregulated exogenous price process (cf. equation (4)) for the case of a GBM. Although both simulations utilise identical parameters with a drift rate of $\alpha = 0\%$ and a volatility rate of

Figure 1. Price dynamics with and without competition (Leahy, 1993)



Note: GBM with $\alpha = 0\%$ and $\sigma = 20\%$, η = -1.

$\sigma = 20\%$ and identical random numbers, the sample paths look different: As all firms behave in the same way, the endogenous price process anticipated by a competitive investor will be truncated upwards as soon as the product price climbs up to a specific investment trigger price level. Hence, the investment trigger price constitutes an upper reflecting barrier. By using analogous considerations, the disinvestment trigger price generates a lower reflecting barrier (Dixit and Pindyck, 1994: 261). The exogenous price process, as assumed by a myopic planner, does not show these boundaries.

Following Leahy, both the competitive investor and the myopic planner find identical optimal investment and disinvestment trigger prices. These trigger prices represent the competitive equilibrium meaning that the zero-profit-condition is fulfilled for the firms in the market in the long run. According to the model of Dixit and Pindyck (1994, 216ff.), the optimal investment trigger price $\overline{P}$ and the optimal disinvestment trigger price $\underline{P}$ of a myopic planner can be determined by the following system of equations:

$$-A \cdot \overline{P}^{\beta_1} + B \cdot \overline{P}^{\beta_2} + \frac{\overline{P}}{(r - \hat{\alpha})} = I \tag{5}$$

$$-\beta_1 \cdot A \cdot \overline{P}^{\beta_1 - 1} + \beta_2 \cdot B \cdot \overline{P}^{\beta_2 - 1} + \frac{1}{(r - \hat{\alpha})} = 0 \tag{6}$$

$$-A \cdot \underline{P}^{\beta_1} + B \cdot \underline{P}^{\beta_2} + \frac{\underline{P}}{(r - \hat{\alpha})} = -E \tag{7}$$

$$-\beta_1 \cdot A \cdot \underline{P}^{\beta_1 - 1} + \beta_2 \cdot B \cdot \underline{P}^{\beta_2 - 1} + \frac{1}{(r - \hat{\alpha})} = 0 \tag{8}$$

I represents the investment outlay that a firm must incur for one additional unit of output and E represents the cost to abandon it. It might be the case that the investment is partially reversible and a liquidation value can be generated, that is, E is negative with $I + E > 0$. r denotes the time-continuous discount rate. Variable costs are not explicitly considered. The constants β_1 and β_2 represent the two roots of the quadratic function (Dixit and Pindyck, 1994: 142f.):

$$\frac{1}{2} \cdot \hat{\sigma}^2 \cdot \beta \cdot (\beta - 1) + \hat{\alpha} \cdot \beta - r = 0$$

For β_1 and β_2 follows:

$$\beta_1 = \frac{1}{2} - \frac{\hat{\alpha}}{\hat{\sigma}^2} + \sqrt{\left(\frac{\hat{\alpha}}{\hat{\sigma}^2} - \frac{1}{2}\right)^2 + \frac{2 \cdot r}{\hat{\sigma}^2}}$$

$$\beta_2 = \frac{1}{2} - \frac{\hat{\alpha}}{\hat{\sigma}^2} - \sqrt{\left(\frac{\hat{\alpha}}{\hat{\sigma}^2} - \frac{1}{2}\right)^2 + \frac{2 \cdot r}{\hat{\sigma}^2}}$$

By using the four equations (5) to (8), the trigger prices $\overline{P}$ and $\underline{P}$ and the constants A and B are determined. The equation system cannot be solved in closed form; however, it can be proven that a unique solution for $\overline{P}$ and $\underline{P}$ exists (Dixit, 1989: Appendix A). This solution can be obtained by using iterative approximation procedures.

Nevertheless, applying the optimality property of myopic planning to competitive markets is problematic. For instance, it is not possible or at least very complex, to model any market interventions whose effects cannot be transformed directly into the price process, for instance, production ceilings. To solve this, a derivation of the endogenous equilibrium price process would be necessary, instead of just using the above system of equations. In the literature, this is commonly assessed as not practicable (e.g. Leahy, 1993). In the next section, a real options market model will be developed, allowing the derivation of exactly this equilibrium price process in competitive markets. Therefore, it does not rely on the preconditions of applying the optimality property of myopic planning and can be used more flexibly than other models, for example, by allowing the integration and assessment of different market interventions.

3 The real options market model

In the following section, a real options market model is presented which allows the simultaneous determination of firms' investment and disinvestment thresholds in the dairy sector under consideration of different market interventions. For this purpose, the model of Feil and Musshoff (2013) is extended by depreciation, as the production capacity of the firms in the dairy sector is not only subject to investments and disinvestments, but also to depreciation of most production factors, for example the milking equipment.

Within the model, a market consisting of $N = 50$ risk-neutral firms is considered. The firms are homogenous with regard to their production and investment capabilities. The firms plan in discrete time, which is a necessary assumption of numerical evaluation procedures.[2] Each firm has the option to repeatedly invest in production capacity within the period under consideration T, until an exogenously given maximum output capacity X_{cap} is reached. Investment outlay and production output are proportional, which means that there are no economies of scale for the firms. The investment project has an unlimited useful lifetime. However, in every period the production output declines corresponding to a geometric depreciation rate λ. After implementation, the investment can be abandoned and its costs partially reversed. Consequently, the production capacity of a firm n in t, resulting in a production output X_t^n, can be adjusted in two ways: Either via investments once in a period, resulting in an additional production output $Y_{t+\Delta t}^n$ in the following period, or via disinvestments once in a period, resulting in a reduction in production output $Z_{t+\Delta t}^n$ in the following period. Then production follows:

$$X_{t+\Delta t}^n = X_t^n \cdot (1-\lambda) + Y_{t+\Delta t}^n - Z_{t+\Delta t}^n. \quad (9)$$

The aggregated production output of all firms represents the market supply X_t. Prices result from the reactions of all firms in the form of X_t on the exogenous industry-wide demand shock and hence, need to be determined endogenously within the model. Without loss of generality, the relationship between X_t and P_t is defined by an isoelastic demand function

[2] However, the results according to the analytical model of Dixit and Pindyck (1994: 216ff.) in the absence of any market interventions, which are based on a continuous treatment of time, can be approximated by the real options market model (cf. subsection 4.1.).

according to equation (1). For modelling the stochastic demand parameter μ_t, any stochastic process can be applied as flexibly as needed.[3]

According to the model of homo economicus, all firms maximise their expected NPV. Furthermore, all firms have complete information regarding the stochastic demand process and the investment and disinvestment behavior of all competitors, whereby they build demand expectations for the respective next period. Consequently, all firms should have the same optimal investment and disinvestment trigger prices in equilibrium. To derive this Nash equilibrium within the model, the competing firms interact by gradually adjusting their (initially different) investment and disinvestment trigger prices ($\bar{P}^n$, $\underline{P}^n$), as explained in the next subsection. Furthermore, it is assumed that in a production period all firms first disinvest and then invest, based on their trigger prices and the expected market price. In this context, it is technically ensured that $\underline{P}^n < \bar{P}^n$ for all firms, that is a firm n will not invest if it has disinvested immediately before. Due to this system of chronological order, the disinvestments accumulated in a period impact the investment decisions of the same period, but not vice versa.

To derive the disinvestment volume of the firms in the first instance, it is assumed that firms with a higher disinvestment trigger price have a stronger tendency to abandon the investment. Accordingly, all firms are sorted according to their disinvestment trigger prices, starting with the highest, i.e. $\underline{P}^m \geq \underline{P}^{m+1}$. Consequently, firm $m+1$ does not disinvest if firm m has not already completely abandoned the investment. Likewise, it is obvious that if firm $m+1$ abandons the investment completely, firm m completely abandons the investment, too. Furthermore, in every period t, a marginal (or last) firm exists which disinvests to the extent that its disinvestment trigger price equals the expected product price of the next period. For the disinvestment volume of a firm $\tilde{m}$ in t, corresponding to its additional production output in $t + \Delta t$, follows:

$$Z_{t+\Delta t}^{\tilde{m}}\left(\underline{P}^{\tilde{m}}\right) = \max\left[0, \min\left(\begin{array}{c} X_t^{\tilde{m}} \cdot (1-\lambda), \\ \left(\sum_{m=1}^{N} X_t^m \cdot (1-\lambda) + \sum_{m=1}^{\tilde{m}-1} Z_{t+\Delta t}^m\left(\underline{P}^m\right)\right) - \frac{\hat{E}(\mu_{t+\Delta t})}{(\underline{P}^{\tilde{m}})^{-\eta}} \end{array}\right)\right] \tag{10}$$

The "max-query" of equation (10) ensures non-negativity of the disinvestment volume. Furthermore, the "min-query" makes sure that a firm $\tilde{m}$ cannot abandon more production capacity via disinvestments than it has built up in former periods. The "min-query" also guarantees that the total quantity of supply is just reduced as long as the disinvestment trigger price of the "last" firm equals the expected product price of the next period.

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[3] Besides industry-wide shocks, firm-specific shocks are not considered within the model for complexity reasons. For a combination of both industry-wide and firm-specific shocks, see the more general analytical model of Dixit and Pindyck (1994: 277ff.).

The investment volume of a firm is derived analogously, i.e. firms with lower investment trigger prices have a stronger tendency to invest. All firms are sorted according to their investment trigger prices, starting with the lowest, i.e. $\bar{P}^n \leq \bar{P}^{n+1}$. Thus, firm $n+1$ does not invest if firm n has not already invested in production capacity up to X_{cap}. In every period t, it is technically ensured that de facto a marginal (or last) firm exists which invests to the extent that its investment trigger price equals the expected product price of the next period. As a result of this and the relatively large number of firms ($N = 50$), the market within the model can be seen as an approximation of an atomistic market. For the investment volume of a firm $\tilde{n}$ in t, corresponding to its additional production output in $t + \Delta t$, follows:

$$Y_{t+\Delta t}^{\tilde{n}}\left(\bar{P}^{\tilde{n}}\right) = \max\left[0, \min\left(\begin{array}{c} X_{cap} - X_t^{\tilde{n}} \cdot (1-\lambda)\,, \\ \dfrac{\hat{\mathrm{E}}(\mu_{t+\Delta t})}{(\bar{P}^{\tilde{n}})^{-\eta}} - \left(\sum_{n=1}^{\mathrm{N}} X_t^n \cdot (1-\lambda) + \sum_{n=1}^{\tilde{n}-1} Y_{t+\Delta t}^n(\bar{P}^n) + \sum_{m=1}^{N} Z_{t+\Delta t}^m\left(\underline{P}^m\right)\right) \end{array}\right)\right] \tag{11}$$

Analogously to equation (10), the “max-query” of equation (11) ensures non-negativity of the investment volume. The “min-query” makes sure that a firm $\tilde{n}$ cannot build-up more production capacity via investments than it needs in order to produce its maximum production capacity X_{cap}. Additionally, the “min-query” ensures that the total quantity of supply is only expanded as far as the investment trigger price of the “last” invested firm equals the expected product price of the next period.

Finally, an objective function needs to be established which determines the optimal investment and disinvestment strategies of the firms. According to the above assumptions, each firm aims to maximize the expected NPV of the future cash flows F_0^n, in the real options terminology also referred to as an option value by choosing its firm-specific investment trigger price $\bar{P}^n$ and disinvestment trigger price $\underline{P}^n$:

$$\max_{\bar{P}^n,\underline{P}^n}\left\{F_0^n\left(\bar{P}^n, \underline{P}^n\right)\right\} = \max_{\bar{P}^n,\underline{P}^n}\left\{\sum_{t=0}^{\infty}\left((P_t - k)\cdot X_t^n\left(\bar{P}^n, \underline{P}^n\right) - i\cdot k\cdot \sum_{u=0}^{t} Z_u^n\left(\underline{P}^n\right)\right)\cdot e^{-r\cdot t}\right\} \tag{12}$$

The reversibility rate i determines what proportion of the investment costs can be recovered upon abandonment. The interest rate r is time-continuous. k represents the total costs of investment per output unit and period, consisting of the capital costs of the investment outlay I and all other costs to be paid c (e.g. material costs, labor costs):

$$k = I \cdot \left\{e^{r\cdot\Delta t} - (1-\lambda)\right\} + c \tag{13}$$

As there exists no closed-form solution for the described optimisation problem, the model is solved numerically by combining GAs with stochastic simulation following Feil and Musshoff (2013).

Finally, three market interventions are exemplarily implemented into the model. In the case of a price floor P_{min} maintained by governmental purchases of excess supply, the determination of the producer's price has to be modified. Considering the product price P_t according to equation (1), the following applies to the effective producer's price P_t':

$$P_t' = \max\{P_{min}, P_t\} = \max\left\{P_{min}, \left(\frac{\mu_t}{X_t}\right)^{\Pi}\right\} \tag{14}$$

Consequently, P_t in equation (12) is replaced by P_t'. As a reference point, P_{min} will be exogenously fixed as a proportion of the total costs of investment k. Following Dixit and Pindyck (1994) on the effects of price controls, it is assumed that governmental purchases are excluded from the market with no future impact on supply and demand.

A politically induced production ceiling X_{max} determines the maximum permissible overall production capacity in the market. As soon as X_{max} is exceeded, firms are not allowed to invest in additional capacity, even if their investment trigger prices are lower than the market price. Hence, the formula for the investment size of a firm $\tilde{n}$ according to equation (11) needs to be supplemented by a further "min-query":

$$Y_{t+\Delta t}^{\tilde{n}}\left(\bar{P}^{\tilde{n}}\right) = \max\left[0, \min\left(\begin{matrix} X_{cap} - X_t^{\tilde{n}} \cdot (1-\lambda)\,, \\ \dfrac{\hat{\mathrm{E}}(\mu_{t+\Delta t})}{(\bar{P}^{\tilde{n}})^{-\eta}} - \left(\sum_{n=1}^{\mathrm{N}} X_t^n \cdot (1-\lambda) + \sum_{n=1}^{\tilde{n}-1} Y_{t+\Delta t}^n(\bar{P}^n) + \sum_{m=1}^{N} Z_{t+\Delta t}^m\left(\underline{P}^m\right)\right), \\ X_{max} - \sum_{n=1}^{N} X_t^n \cdot (1-\lambda) - \sum_{n=1}^{\tilde{n}-1} \Delta X_{t+\Delta t}^n\left(\bar{P}^n\right) \end{matrix}\right)\right] \tag{15}$$

An investment subsidy s will be paid by the state to any firm undertaking investments in the respective industry. Accordingly, it reduces the initial investment outlay I by a fixed proportion. Thus, k in equation (12) is replaced by the effective producer's total costs of investment k':

$$k' = I \cdot (1-s) \cdot \left\{e^{r \cdot \Delta t} - (1-\lambda)\right\} + c \tag{16}$$

Finally, the welfare effects of the market interventions can be quantified by determining the economic efficiency following Feil and Musshoff (2013). This allows for assessing and comparing the different intervention measures on a macroeconomic level.

4 Model assumptions for the application to the German dairy sector

The developed real options market model is applied to the German dairy sector in this section. This sector is highly competitive, comprising 66,250 producers either classified as specialized dairy farms, or as dairying, rearing and fattening combined farms in 2010 (European Commission, 2013). This supports, or at least does not contradict, the applicability of the developed model framework. Mainly because of data availability problems, it is practically impossible to estimate the stochastic demand process μ_t and its parameters empirically. Instead, following many other real options applications (e.g. Engel and Hyde, 2003; Price *et al.*, 2005; Purvis *et al.*, 1995; Tauer, 2006), an unregulated GBM is assumed for the price process. For this, the stochastic demand process following equation (2) can be transformed into the stochastic price process (Odening et al., 2007). Its parameters, the drift rate $\hat{\alpha}$ and the volatility $\hat{\sigma}$, are estimated directly from empirical price data.[4] It is crucial to use historical prices that have not, or to a minor extent, been affected by any market interventions. Hence, historical milk prices of Germany do not seem to be appropriate because of the EU milk price intervention system until 2007 and the existing EU milk quota system. In contrast, the dairy sector in New Zealand is not characterized by any significant political interventions and, therefore, the inflation-adjusted average prices for milksolid in New Zealand from 1973 to 2012 are taken as a basis (LIC, 2012). Applying a variance ratio test to this time series, it is shown that the null hypothesis of non-stationarity cannot be rejected at a 5% significance level. Following common practice, this test result can be seen as a confirmation that an unregulated GBM represents an adequate model for the price process. Accordingly, the estimated drift rate $\hat{\alpha}$ is 1.31% and the volatility $\hat{\sigma}$ is 19.39%. Furthermore, in the literature a price elasticity for dairy products in Germany of $\eta =$ -0.99 is reported (Thiele, 2008). A typical investment to build up milk production capacity suggested by the German Association for Technology and Structures in Agriculture (KTBL, 2013) is considered: an average milk yield of 7,000 kg per place, an initial investment outlay of 4,371 € per place or $I = 0.62$ € per kg milk and a depreciation rate of $\lambda = 4.25\%$. With this information, the parameters of the stochastic price process $\hat{\alpha}$ and $\hat{\sigma}$ can be re-transformed into the parameters of the stochastic demand process α and σ, which yields $\alpha =$ -2.97% and $\sigma = 19.59\%$. Since the GBM as stochastic demand process according to eq. (2) assumes infinitesimal time length steps and hence is impractical for simulation purposes, it is transformed into a time-discrete version. This can be done by the use of Ito's Lemma (cf. Hull and White, 1987):

$$\mu_{t+\Delta t} = \mu_t \cdot e^{\left[\left(\alpha - \frac{\sigma^2}{2}\right)\cdot \Delta t + \sigma \cdot \varepsilon_t \cdot \sqrt{\Delta t}\right]} \tag{17}$$

[4] This 'standard procedure', however, ignores the fact that empirical price data generated from a competitive market are necessarily realizations of the regulated price process and not of the unregulated process. Therefore, estimates of the parameters of the GBM will be biased, as they implicitly capture the effect of competitive market entry by mistake. Odening et al. (2007) analyse and assess these estimation biases.

with a standard normally distributed random number ε_t and a time step length Δt. Eq. (7) represents an exact approximation of the time-continuous GBM for any Δt. For the risk-free discount rate, the arithmetic mean of the inflation-adjusted monthly average yields of listed federal securities with 15-30 years residual maturity for the period from 1989 to 2010 is calculated at 3.44% p.a. (Bundesbank, 2013). This corresponds to a time-continuous discount rate of $r = 3.38\%$. Including other relevant costs (e.g. heifer, fodder, labor and machine costs) and deducting the sales revenues for old cows and calves, the total costs of investment amount to $k = 0.35$ € per kg milk.

All parameters used for the calculations in the next two subsections are summarised in table 1.

Table 1. Model parameters for the application to the German dairy sector.

Number of firms N	50
Maximum output capacity X_{cap}	10 output units per firm
Period under consideration T	Infinite, approximated by 100 years
Milk yield	7000 kg per cow p.a.
Investment outlay I	0.62 € per kg milk
Geometric depreciation rate λ	4.25%
Useful lifetime of investment	Infinite
Risk-free time-continuous interest rate r	3.38% (corresponding to a time-discrete interest rate of 3.44%)
Total costs of investment k	0.35 € per kg milk (including 0.05 € capital costs of investment)
Reversibility rate of investment i	0%, 50%
Stochastic process of the demand parameter μ_t	Geometric Brownian motion (GBM)
Parameters of the stochastic process	
Drift rate α	-2.97%
Volatility σ	19.59%
Time step length Δt	1.00
Price elasticity of demand η	-0.99
Price floor P_{min}	80%, 90% of k

5 Results

The model results on firms' optimal investment and disinvestment trigger prices and on the economic efficiencies for different levels of market intervention in the German dairy market are presented in table 2. The price floors are exogenously given with $P_{min} = 0\%$, 80% and 95% of the total costs of investment $k = 1$. To ensure comparability, the production ceilings and the investment subsidies are fixed by iterative searching, so that the resulting investment trigger prices (nearly) equal the ones of the predefined price floors. Hence, the stimulation of the willingness to invest is the assumed main policy goal of the analysis in this subsection. To additionally analyse the effects of partial reversibility of the investment costs, the calculations

Table 2. Comparison of the effects of price floors, investment subsidies and production ceilings at different reversibility levels of the investment costs

	Price floor				Production ceiling				Investment subsidy			
Reversibility (%)	Level (% of k)	Investment trigger price (€)	Disinvest-ment trigger price (€)	Economic efficiency (%)	Level (units)	Investment trigger price (€)	Disinvest-ment trigger price (€)	Economic efficiency (%)	Level (% of k)	Investment trigger price (€)	Disinvest-ment trigger price (€)	Economic efficiency (%)
	0	0,4901	0,0705	100,00	n.a.	0,4901	0,0705	100,00	0,0	0,4901	0,0705	100,00
0	80	0,4320	0,0000	88,80	230	0,4323	0,0934	97,26	75,0	0,4377	0,0554	97,10
	95	0,3662	0,0000	75,81	170	0,3595	0,1245	93,27	185,0	0,3692	0,0337	94,41
	0	0,4780	0,1276	100,00	n.a.	0,4780	0,1276	100,00	0,0	0,4780	0,1276	100,00
50	80	0,4335	0,0000	88,84	240	0,4318	0,1446	97,74	65,0	0,4354	0,1233	97,59
	95	0,3659	0,0000	75,96	180	0,3647	0,1577	95,07	165,0	0,3665	0,1198	95,95

Note: GBM with α = -2.97% and σ = 19.59%, η = -0.99, λ = 4.25% r = 3.38%, N = 50, X_{cap} = 10, T = 100, k = 0.35 €.

in both subsections are each performed at $i = 0\%$, i.e. the investment costs are sunk in total, and at $i = 50\%$, i.e. only half of the investment costs are sunk and the other half can be recouped upon exit.

The results presented in table 2 can be summarised as follows:

1. In highly competitive markets like the dairy sector, in which it can be assumed that the zero-profit-condition is fulfilled for the firms in the long run, the implementation or the increased use of market interventions generally induces a decline in investment trigger prices leading to an increase in the willingness to invest. For the respective measure, this can be explained as follows: The price floor acts like a lower reflecting barrier for the firms, whereby the expected future price rises. Therefore, a lower investment trigger price can already ensure a compensation of the investment costs by the present value of the expected cash flows. The production ceiling causes a reduction of the quantity supplied, which likewise leads to a higher expected future price and hence, to a lower investment trigger price. The investment subsidy reduces the investment costs; the required trigger price to compensate for these is thus reduced as well.

2. The results show that market interventions have different effects on the disinvestment trigger price depending on the chosen measure. For the implementation of a price floor at both levels chosen ($P_{min} = 80\%$ and 95%), the disinvestment trigger price falls back to $\underline{P} = 0.000$ for both reversibility levels of the investment costs. The reason for this is that, at these guaranteed price floor levels, it is not worthwhile for the firms to abandon the investment at any expected product price level. In the case of implementation or the lowering of a production ceiling, the disinvestment trigger price increases. This can be explained by the lower investment trigger price (cf. 1.). Through this, potential upper price fluctuations, which could compensate for periods of low prices, are truncated and the firms abandon the investment earlier. Finally, the introduction and the increase of investment subsidies cause the disinvestment trigger price to decrease. This can be explained by two opposing effects: On one hand, the lowering of the investment trigger price induces a rise of the disinvestment trigger price, analogously to a production ceiling. On the other hand, the subsidy reduces the costs that can be recouped upon abandoning the investment (cf. equation (11)), whereby the firms abandon the investment later and the disinvestment trigger price decreases. As a result, this decreasing effect obviously overcompensates for the aforementioned increasing effect.

3. Not considering disinvestment options, which in reality exist in the dairy sector, can lead to biases in policy impact analyses. In the present example, the effectiveness of production ceilings and investment subsidies with regard to a stimulation of the willingness to invest is underestimated: The required intervention levels to achieve a certain reduction of the investment trigger prices are lower in the case of partially reversible investment costs ($i = 50\%$) compared to the case of investment costs, which are sunk in total ($i = 0\%$). The reason for this is that, for $i = 50\%$, the

disinvestment trigger prices are higher for both market interventions, compared to the case of $i = 0\%$ (cf. 2). Ceteris paribus, this already has a decreasing effect on the investment trigger price and hence, political intervention needs to be less intense to bring the investment trigger price down to the desired level. In the event of a price floor, the introduction of partial reversibility, by contrast, does not induce a lower intervention level. This directly follows the fact that the disinvestment trigger price is zero for both, totally sunk and partially reversible investment costs (cf. 2.). Therefore, the price floor levels to achieve a certain reduction of the investment trigger price ceteris paribus have to be the same for both scenarios ($i = 0\%$ and $i = 50\%$).

4. The welfare is reduced through the implementation or the increased use of market interventions leading to lower economic efficiencies. Depending on the applied measure, this has different reasons: By introducing or increasing a price floor, the price level increases over time and, through this, the consumer surplus is reduced. In addition, the state budget is burdened due to the expenditures for the purchases of excess supply. By implementing a production ceiling, the consumer surplus is reduced through limiting the market supply. By introducing an investment subsidy, the state budget is burdened through paying a financial compensation to the investing firms. Moreover, the introduction of partial reversibility of the investment costs has an increasing effect on the economic efficiencies of production ceilings and investment subsidies. This directly follows from the fact that the required intervention levels for these two measures are lower (cf. 3), thus, the welfare is reduced less. Although the economic efficiencies change in absolute values for two of the three market interventions, the overall ranking remains the same for both investigated reversibility levels: Under the given stimulation of the willingness to invest, production ceilings and investments are more advantageous than price floors.

6 Conclusive remarks

The simultaneous analysis of firms' investment and disinvestment decisions in the dairy sector under explicit consideration of different market interventions is of high practical relevance. The respective applicability of existing real options models, however, is very limited. Many of these models do not allow for the option to disinvest besides the option to invest. Furthermore, real options models typically do not consider competition and, in contrast to this, assume Leahy's optimality property of myopic planning for their application to competitive markets. This, however, is based on very restrictive preconditions and especially complicates the analysis of investment and disinvestment decisions in markets characterised by market interventions, like the dairy sector. Those few models, which directly consider competition, again do not allow for disinvestments and, in addition, are very restricted in their modelling flexibility, for instance with regards to different market intervention measures. Hence, the objective of this paper was to analyse firms' investment and disinvestment decisions in the dairy sector under simultaneous consideration of competition, uncertainty and market interventions. This was achieved by applying the model of Feil and Musshoff (2013) to the

German dairy sector. The model links GAs and stochastic simulation for its numerical solution. Through this, the direct determination of equilibria in competitive markets is enabled instead of determining the investment and disinvestment thresholds for a myopic planner according to Leahy's theorem. Consequently, vast modelling flexibility is gained, for instance with respect to the implementation of different market interventions, which are relevant to the dairy sector.

The results of the analysis underline the explanation potential of the model for both firms with regard to their optimal investment and disinvestment strategies in the dairy sector, as well as for agricultural politicians with regard to detailed policy impact analyses. Accordingly, it is shown that the implementation or the extension of market interventions, like the nation support programme for livestock farming in the newly-formed German states, generally increases the willingness to invest. At the same time, the effects on the willingness to abandon investments can be different depending on the measure used. While production ceilings increase the disinvestment speed, investment subsidies reduce it and price floors can even completely prevent the homogenous firms from disinvesting. Of course, these effects also apply in the opposite direction when market interventions are lowered or abolished completely, like the EU milk quota system by 2015. Furthermore, simulations of the model show that not considering partial reversibility of the investment costs can lead to an overestimation of the firms' investment thresholds. At the same time, the effectiveness of specific market interventions, i.e. production ceilings and investment subsidies, with respect to increasing the firms' willingness to invest can be underestimated. In result, the disregard of real existing disinvestment options can result in incorrect valuations of investment projects at firm level and in incorrect policy impact analyses at the macroeconomic level. To enable a direct comparison of competing market interventions, their welfare effects can be additionally quantified by means of the model. In the present case, the results suggest that for a given stimulation of the firms' willingness to invest, both production ceilings and investment subsidies are preferable to price floors.

However, it should be noted that the results of the present study are still based on some simplifying assumptions: In particular, the assumption of firms that are homogenous with respect to their production and investment possibilities represents a simplification from reality. Especially in the dairy sector, it is known that there can be considerable differences in the efficiency of firms in the same market, for example with some firms producing a milk yield of around 10,000 kg per cow and other firms producing of around 6,000 kg. In consequence, the investment trigger prices of the efficient firms will be considerable lower compared to those of the less efficient firms, rather than assuming just one equilibrium investment trigger price, which applies to all firms in the market. Moreover, the consideration of firm heterogeneity with regard to their efficiencies would also allow for the additional modelling of a quota market, where quota can be freely traded between the firms, like in the EU milk quota system.

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