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SESSION V: AGRICULTURAL AND ENVIRONMENTAL HAZARDS

PAPER 1: OPTIMUM NITROGEN USE UNDER GROUNDWATER POLLUTION CONSTRAINTS

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Optimum Nitrogen Use Under Groundwater Pollution Constraints

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by

K. William Easter and Satya N. Yadav*

Advances in agriculture since World War II substantially altered U.S. farming practices. High yielding varieties combined with greatly expanded use of fertilizers and pesticides allowed production to make unprecedented gains. Between 1960 and 1990, nitrogen use increased fourfold (Figure 1). Other primary nutrients and pesticides saw similar increases. This remarkable change in U.S. agriculture did not come without some environmental costs to society. The resulting water pollution, soil erosion, declining wildlife habitat, and the draining of wetlands have all raised serious concerns in the environmental community.

Minnesota is an important agricultural state in the U.S. that faces many of these same environmental concerns. One of the growing pollution concerns in Minnesota is groundwater contamination by nitrate-nitrogen from agriculture. Over 90% of the groundwater pollution in Minnesota is from nitrate-nitrogen. This is not surprising given that nitrates are water soluble and that in most years since 1975, Minnesota farmers annually use in excess of 500 thousand tons of nitrogen. The state produces 4-5 million acres of soybeans and approximately 2 million acres of alfalfa, both of which fix nitrogen and return a portion to the soil. In addition, Minnesota is among the top five states in dairy, swine, and turkey production, which creates sizeable quantities of nitrogen bearing wastes.

The heavy use of nitrogen combined with certain soil and water conditions can cause serious groundwater contamination. Several monitoring surveys have found 30-40% of the wells in the geologically sensitive areas of the state with nitrate-nitrogen concentrations

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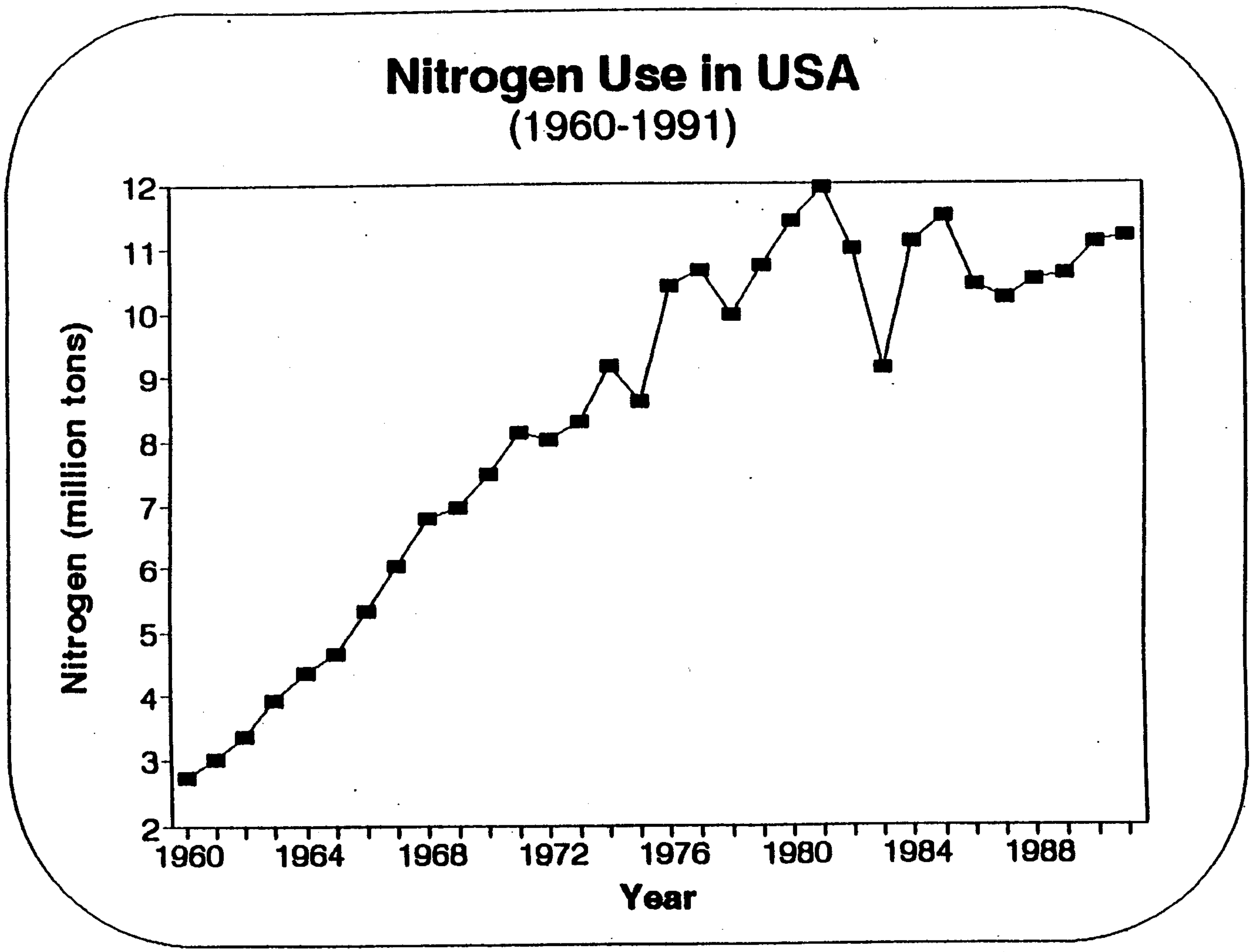

Figure 1. Nitrogen Application in the USA 1960-1991 (Vroomen, 1989)

that exceed EPA safe drinking water standard of 10 mg/l. The most sensitive areas in Minnesota for groundwater contamination are the sand plains of central Minnesota which may have shallow, superficial aquifers, and the shallow silt loam soils of southeastern Minnesota underlined by sandstone or fractured limestone (Karst topography). The Karst topography extends into northern Iowa and southwestern Wisconsin and is found in other parts of the U.S., such as Missouri.

The primary reason for the concern about nitrate contamination of groundwater is the threat it poses to human health. The consumption of nitrate-contaminated drinking water can cause methemoglobinemia in young children and animals. Methemoglobinemia, or the "blue baby" disease, can be fatal particularly for infants that consume infant formula mixed with nitrate-contaminated drinking water. Although the number of cases of blue baby disease are quite limited, it could increase in the future, particularly in rural areas where the major sources of drinking water are private wells. Seventy-five percent of the population in Minnesota depends on groundwater for its drinking water.

There are also some concerns that nitrates could be carcinogenic or cause central nervous system birth defects. However, so far, there is no significant evidence to show such effects. Thus currently, the major health problem appears to be methemoglobinemia for young children and livestock, particularly ruminant animals.

In this paper we will try to address two questions. First, are farmers using the level of nitrogen that would maximize profits, and second, does the profit maximizing level of nitrogen use exceed the social optimum? In the latter case, we determine the optimal level of nitrogen use for continuous corn when the farmer must try to maintain the groundwater quality level at 10 ppm (the EPA safe drinking water standard). We conclude with recommendations concerning how to achieve the social optimum level of nitrogen use.

Review of Nitrogen Use Rates

To determine the seriousness of the nitrate pollution problem in southeastern Minnesota, a University of Minnesota study was initiated during 1985 in the Ducshee Creek watershed in central Fillmore County, an area representative of much of southeastern Minnesota. The study found 63% of the wells tested had nitrate levels greater than 3 mg/l and 21% were in excess of 10 mg/l (Legg, et.al, 1989). The well testing was followed by nitrogen budget analysis of the six county southeastern Minnesota region. A comparison was made between crop requirements and the total amount of nitrogen available from commercial fertilizer, biological fixation by legume crops, and animal waste (manure). The study found that, on average, farmers in the region were applying 50 to 64 pounds more nitrogen than was needed for profit maximization. As part of the nitrogen budget analysis, four individual farms were surveyed in the region. The survey showed that there was a significant difference in nitrogen application rates among farm types. The grain farmer applied slightly less nitrogen than that required to maximize profits, while the dairy farmer applied over 133 pounds in excess of the profit maximizing level. The beef and hog farmers applied 60 and 27 pounds in excess of the optimum, respectively.

Based on this analysis, it was concluded that many southeastern Minnesota farmers were using too much nitrogen, even for profit maximization. However, it appeared that livestock farmers were the ones using excessive amounts of nitrogen. To confirm the results of the study, a more extensive survey of farmers was initiated that focused on livestock farmers.

A survey of 36 farmers in southeastern Minnesota was conducted in 1988. Thirty of the farmers raised livestock and grain, while 6 farmers raised only grain, primarily corn and soybeans. The results from the survey essentially confirmed the earlier findings. The grain farmers were using about the profit maximizing level of nitrogen, while the livestock farmers applied an average of 50 pounds more than was optimum. In addition, the average amount of nitrogen applied to unmanured corn on farms with livestock did not differ significantly from applications by farmers with no livestock. On average, nitrogen application on manured corn fields was over 100 pounds more than was optimum for profit maximization.

The overuse of nitrogen applied as manure can have several possible explanations. First, the farmers simply spread manure on corn fields as a disposal activity. Second, farmers may not be aware or lack information concerning the amount of nitrogen in the manure and thus, apply more than is needed just to be safe. A final explanation might be the uncertainty concerning crop response and rainfall. Under ideal growing conditions, the corn plant might need more nitrogen, or with high rainfall, some of the nitrogen may be lost through leaching. The latter explanation is not very likely, since it would also encourage farmers to apply more commercial fertilizer than was optimum, which they did not do.

Groundwater Contamination Constraints

The above analysis only addressed the question of whether or not farmers were using more than the private optimum level of nitrogen. It did not try to determine a social optimum level of nitrogen use when a groundwater pollution externality is present. If nitrates from fertilizer or manure application leach into the groundwater and cause damages, then the private optimum use of nitrogen is likely to be higher than the social optimum. This assumes that if less nitrogen is applied to the soil, less nitrogen will be leached into the groundwater. To determine the extent of this possible externality problem, a detailed study of nitrogen loss to the subsoil was conducted in southeastern Minnesota between 1987 and 1990.

Three sites on cooperating farms were selected where the movement of nitrates through the soil profile was measured by researchers from the University of Minnesota. Site 1 was near Rochester in Olmsted County, site 2 was in Winona County, and site 3 was in Goodhue County. Site 1 was a particularly good site since nitrogen fertilizer had not been applied when corn was grown on the fields. For site 2 and 3, a moderate amount of nitrogen (70 to 75 lbs of N/acre) had been applied on the corn grown in 1986.

Data

Samples of soil water were taken from these three sites during each of the four crop seasons (1987 through 1990). All the soil water sampling and testing were done by the Soil Science Department at the University of Minnesota. Altogether, there were 38 treatments, 10 at site 1, 12 and site 2, and 16 at site 3, each planted with continuous corn. Continuous corn was selected as a cropping pattern because it is a common cropping practice in Minnesota and the region. Moreover, corn utilizes a large share of nitrogenous fertilizers applied in the area.

A *randomized complete-block* design with four replications was established at each site, for every treatment, every year. Accordingly, the data set consisted of 608 (38x4x4) observations. Treatments among sites and within a site varied only with respect to fertilization (0 to 225 lbs/acre) and tillage management practices. All other aspects of cultivation practices were controlled except weather. Fertilizer and tillage management practices were varied in the experiment in order to see their impact on residual nitrogen build-up or, equivalently, on groundwater pollution. Variations in fertilization included time of application (fall vs spring; pre-plant vs four leaf stage), method of application (split vs single dose), with and without use of nitrogen inhibitor, and source of nitrogen (anhydrous ammonia vs manure). The variation in tillage practices included either deep tillage using chisel plow, or no tillage. The data set collected for the three sites consisted of information on corn yield (bu/acre), fertilizer use (lbs/acre), and nitrate concentration (lbs/acre) in the soil profile associated with each replication. Residual nitrogen in both ammonium and nitrate forms were recorded each year for every treatment in pounds per acre for each one foot incremental depth of the soil profile, down to an 8 ft. depth.

Nitrates in subsoil

The sampling at the three sites confirmed that nitrates were leaching into the subsoil below the root zone. Although the study did not measure the nitrates in the groundwater, it strongly suggests that eventually the nitrates will reach the groundwater. Given the Karst nature of the soil profile, the researchers from the Soil Science Department felt that the nitrate concentrations below the root zone is representative of nitrate concentrations in the underlying shallow aquifer. This is due to the rapid downward movement of the nitrates and the negligible denitrification in the Karst geology.

Since corn plant roots are mostly distributed in the upper 3 ft. depth of the soil profile, nitrate-nitrogen available beyond a three ft. depth cannot by utilized by plants.[1] For this study, therefore, the soil profile is divided into two zones, the rooting (0-3 ft. depth) zone and non-rooting (3-8 ft. depth) zone. The rooting zone is characterized as a *source* of nitrogen both for plant growth and groundwater pollution, while the non-rooting zone is considered the *sink* which is a source of groundwater pollution. It is assumed that nitrates reaching the sink will eventually contaminate the groundwater.

Yield response and nitrate pollution

As expected, mean corn yields for zero levels of nitrogen application were significantly lower (at the 1 percent level) than the non-zero levels for all three sites. What is somewhat surprising, until one considers the residual nitrogen, is that mean yields did not differ significantly among treatments, using non-zero levels of nitrogen (50 to 225 lbs/acre).

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[1] Rooting depth of plants depends on available soil moisture in root zone depth. During the moisture stress season, corn roots may go deeper than 3 feet depth in search of water and vice-versa.

In addition, corn yields among treatments involving the recommended applications of 150 lbs/acre of nitrogen were not statistically different from each other, irrespective of source, timing, nitrogen inhibitors, method of application, or tillage practices. This suggests that environmentally favorable cultural practices could be used without any reduction in yield. Furthermore, the potential loss to farmers from reducing their levels of nitrogen use is modified by the nitrogen that is already in the root zone (0-3 ft.). The yield response to nitrogen on the three sites clearly shows the importance of residual nitrogen.

If 10 mg/l of nitrates in the soil water below 3 feet (that is in the sink) is taken as a constraint, then there should be no application of nitrogen fertilizer at sites 2 and 3. Yet, to stop use of nitrogen fertilizer until the soil water levels of nitrogen drop below 10 mg/l is likely to mean a decline in corn yields and net returns. Therefore, there is a trade-off between the present value of revenue foregone from lower application rates and the present value of the future contamination prevented. Since there is an accumulation of the nitrates in the groundwater over time, the problem is dynamic rather than static. Any excess nitrogen applied today may not raise the level of nitrates in the groundwater above the 10 mg/l. Yet if excess nitrogen is applied over a number of years, the accumulated levels in the groundwater is likely to jump above this level.

Framework of Analysis

The data collected at the three sites in southeastern Minnesota allowed us to determine a social optimal level of nitrogen use over time. Since nitrogen accumulates in the groundwater over time, and only degrades very slowly, the problem is intertemporal in nature. Accordingly, as shown in the appendix, a net social benefit function is defined for nitrogen application on a representative farm with a contamination cost included for groundwater. The dynamic optimum of the present value of net social benefits is determined within the framework of a continuous time optimal control model.

Nitrogen recommendations

A comparison of nitrogen recommendations under a static profit maximization with one where a given level of nitrogen in the soil water is maintained shows a significant drop in the levels of nitrogen recommended (See Table 1). The profit maximizing level of nitrogen shown in column 2 of table 1 ranges from 131 to 158 lbs./acre. The amounts of nitrogen allowed in the sink (3 to 8 ft.) is varied from 45 to 55 lbs/acre to determine the impact on recommended nitrogen application (45, 50 and 55 lbs/acre are equivalent to 9, 10, and 11 mg/l in soil water).[2] To maintain current standards of nitrogen in drinking water (10 mg/l) the recommended application rates for nitrogen should drop by 36 to 65 lbs/acre. Lowering or raising the standard by 1 mg/l only changes the recommended application rate by 10 lbs/acre. Sensitivity analysis with prices suggest that the use rates would not be changed much with price movements. Changing the price of nitrogen by 10 percent only changes optimum fertilizer use by two lbs/acre while a change in the price of corn has a similar effect on fertilizer use. Thus, policy changes that alter the amount of nitrogen allowed in the groundwater would have a larger impact on the amount of fertilizer applied than would taxes or subsidies for corn or fertilizer.

When the amount of nitrogen already in the soil is considered in making nitrogen fertilizer recommendations, the recommended rates drop. The optimal policy rules for nitrogen applications are estimated from appendix equation 15 for the three sites (See Table 2). In the absence of additional information about residual nitrate-N, values recorded in 1987 (i.e., before

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[2] An interest rate of 5 percent, nitrogen price of $0.15 per pound, and corn price of $2.40 per bushel were used in the computation. Moreover, nitrate-N on a water basis (mg/l) in the sink was converted into soil basis (lbs/acre) in order to be comparable with the nitrate contamination function. The conversion of nitrate residual shows that 1 mg/l of nitrate-N on a water basis is equivalent to 5 lbs/acre of nitrate-N on a soil basis in the sink.

the start of the experiment) were used as the existing nitrate concentration built-up in the sink of the respective area. Using these residual nitrate concentrations of 15.2, 146.4, and 101.8 lbs/acre in the non-root zone (3-8 ft. depth) for sites 1, 2, and 3 respectively, the optimal nitrogen application rates are estimated for each site (158, 0, 0 lbs/acre). Since the levels of contamination exceed the 10 ppm standard in the sink at sites 2 and 3, the optimal policy rules require a reduction in nitrate-nitrogen in the groundwater to 10 ppm. This means that the recommended nitrogen application rates would drop to zero for sites 2 and 3. The recommendation is further reduced by 31 lbs/acre for site 1 to 121 lbs/acre when the root zones (0-3 ft. depth) concentration of residual nitrogen is counted (Table 3). Although the environmentally safe recommendations for sites 2 and 3 is zero nitrogen for the first year, in each successive year, nitrogen application rates should be dependent on the residual nitrogen found in subsoil tests. Even for site 1 subsoil tests should be run to determine the residual levels before recommending nitrogen application rates for successive years.

Policy Recommendations

Both a micro and macro look at southeastern Minnesota shows that, in many cases, public policy needs to encourage farmers to reduce their levels of nitrogen use. Some of the reduction can come from better use of manure and taking into account residual nitrogen already in the soil. The more difficult reductions will come for farmers that are using close to the private optimum level of nitrogen, but have nitrates accumulating in the non-root zone and/or in the groundwater. If society wants to reduce nitrogen in groundwater, then it must devise incentives that will encourage farmers to reduce nitrogen use below the private profit maximization level.

Table 1. Nitrogen Recommendations for Static Profit Maximization under Three Different Steady State Concentrations of NO_3-N in the 3-8 Ft. Depth of the Soil Profile

Sites	Profit Maximizing Levels of Nitrogen (N_π) use (lbs/acre)	9 ml/l of NO_3-N	10 ml/l of NO_3-N	11 ml/l of NO_3-N
		N*lbs/ac	N*lbs/ac	N*lbs/ac
Site 1	158	93	103	113
Site 2	131	93	96	113
Site 3	139	93	103	113

Table 2. Optimal Policy Rules for Nitrogen Application.

Sites	Optimal Policy Rules
Site 1	$N_t = 450.71 - 6.957\ C_t$, if $N_t < N_\pi$ $N_t = N_\pi$, if $N_t > N_\pi$
Site 2	$N_t = 442.35 - 6.790\ C_t$, if $N_t < N_\pi$ $N_t = N_\pi$, if $N_t > N_\pi$
Site 3	$N_t = 441.97 - 6.783\ C_t$, if $N_t < N_\pi$ $N_t = N_\pi$, if $N_t > N_\pi$

Note: N_t is the optimum nitrogen application rate under the consideration of externalities, while N_π is the profit maximizing level of nitrogen with no concern for externalities.

Table 3. Optimal Nitrogen Recommendation for Sites 1, 2, and 3 when Externalities from Nitrogen are Internalized.

Sites	Nitrogen Recommendations	
	No Residual Nitrogen in the Root Zone	Residual Nitrogen in the Root Zone
Site 1	158	127
Site 2	00	00
Site 3	00	00

A number of alternatives to reduce nitrogen use are available, however, several are likely to be the most acceptable. First, because of budget constraints, the efforts should be targeted at areas that have groundwater that is or may be used for human consumption, and that is susceptible to pollution from fertilizers applied in production practices (usually shallow aquifers). In areas where the groundwater is used primarily for irrigation, nitrate contamination should not be a major concern. Also, in areas that have good substitute supplies for the contaminated groundwater, doing something about nitrate pollution may be more expensive than using the alternative source of water. Consumers usually have the option of buying bottled water or a water filtration system (Yadav and Wall). They may also be able to use a deeper aquifer or a surface water source.

The major constraints to reducing nitrogen use are the loss in income to farmers, the long ingrained belief that a little more nitrogen fertilizer is better, the cost of enforcing nonpoint pollution standards, and the general lack of knowledge and information concerning the movement of nitrates through the soil into the groundwater. Research, education, and technical assistance can help overcome the knowledge and information problems which may be the most important for manure use. However, other strategies will be needed to deal with the income loss. Subsides could be targeted to make up the difference in income between net income with the private optimum use of nitrogen and the net income at the social optimum level of use. Table 4 shows the subsidies required to reduce nitrogen levels if only applied nitrogen is considered, e.g., no residual nitrogen in the soil or subsoil. These subsidies would have to increase substantially if existing levels of nitrogen in the soil water are considered. As shown above, no nitrogen would be applied in sites 2 and 3 if the residual nitrogen in the soil water is included. This would mean a drop in net income of greater than $4 to $5 per acre

Table 4. Optimal Subsidy for Regulating Nitrogen Reduction to Environmentally Safe Levels

Sites	Recommended Nitrogen Rates (lbs/acre) Profit Maximizing Level	 Environmentally Safe Level	Subsidy Amount ($/acre)
Site 1	158	103	9.29
Site 2	131	96	3.35
Site 3	139	103	4.58

The subsidy is likely to be considered too expensive for most states or the Federal government during this period of tight budgets. Direct regulation is also likely to be expensive because of high transaction costs. This means that public officials are going to have to be innovative if they plan to reduce nitrogen levels below the private optimum.

One possibility may be to give local communities more responsibility in setting and implementing nitrate control strategies. They will need assistance in terms of unbiased information concerning levels of pollution and potential problem areas. Once this is available, they should be given a certain amount of flexibility in deciding on standards and methods for meeting them. One key reason why the local community should be given greater flexibility in dealing with groundwater pollution, as compared to surface water, is the difference in mobility. Because of its mobility, polluted surface water has a much greater potential for damaging users outside the area where the pollution originates. In the case of groundwater, much of the damage is confined to the area where the pollution originates.

Unfortunately, this doesn't always hold true, particularly for southeastern Minnesota, where there are a number of underground streams. If pollution gets in these streams, it moves just as fast as surface water. This means that the flexibility of local communities may have to be limited by federal or state guidelines when the groundwater does move rapidly over a wide area or where there is a need for society to protect particular groundwater resources for future generations.

Conclusions

The current recommendation rate of nitrogen, 150 lbs/acre, exceeds the private profit maximizing level of nitrogen and should be revised downward in much of southeastern Minnesota. Our results indicate that the recommendation should be made site or area specific rather than one general figure for southeastern Minnesota. Moreover, the nitrogen application rates should be reduced from the profit maximizing level in the areas where nitrate levels in soil water exceed 10 ppm. The optimal policy rules developed in the study can be used to determine the extent of annual reductions. Such a policy will bring nitrate contamination back to a safe level for groundwater over time.

The current study shows a considerable residual nitrogen build-up in the soil profile. Since such nitrogen available in the *root zone* layers is also used by plants, appropriate credit must be given for this nitrogen (to avoid over application) in nitrogen recommendations. Credit for residual nitrogen will not only reduce production costs to for farmers, but will help prevent further water quality degradation.

A combined "policy package" consisting of *1) educational and technical assistance, and 2) regulation of nitrogen use* is needed for southeastern Minnesota if further pollution of the groundwater is to be avoided. The technical assistance in the form of tillage and nutrient management BMPs should be aimed at the management of nitrogen, applied externally as well as that already in the soil profile. Regulatory policy should be directed towards curtailing nitrogen application rates in critical areas to the level that would maintain nitrate concentrations at 10 ppm or lower in the aquifer.

Among the potential policies is a subsidy equal to the compliance cost. The reduction in net farm income associated with the reduced use of nitrogen is defined as the compliance cost. Our analysis shows that the required subsidy for the site 1 type of farms would be $9.29 per acre, followed by $3.35 per acre for the site 2 type of farms, and $4.58 per acre for site 3 type of farms. These amounts are much lower than the current subsidy rate of $15 per acre in one of the counties in southeastern Minnesota under Rural Clean Water Program (Yadav and Wall).

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Appendix

Dynamic Model

To determine economic levels of nitrogen application, given the groundwater contamination constraint, requires a dynamic model. Such a model can be formulated in terms of the representative farm that has a set constraint on its nitrogen use. For a typical representative farm, the net social benefit function (private net benefit minus social cost of contamination) from the use of nitrogen in crop production is $W(N_t,C_t)$, where the first two terms are

$$W(N_t,C_t) = P_y(a+bN_t-cN_t^2) - P_N N_t - \theta C_t^2$$

The first two terms on the right hand side of the objective function comprise the private net revenue per acre from the use of nitrogen, N. While the third term, θC_t^2, is the cost of contaminated groundwater to the society from nitrogen use, and is assumed to be proportional to the square of the nitrate concentration. Dynamic optimization of the present value of net social benefit in the framework of a continuous time optimal control model may be stated mathematically as:

$$\textit{maximize} \quad \int_{O}^{\infty} e^{-rt}[P_y(a+bN_t-cN_t^2) - P_N N_t - \theta C_t^2]\,dt$$

$$\textit{subject to } \dot{C}_t = \eta N_t - \delta C_t,$$

$$C_o = \bar{C}, \qquad \textit{given}$$

where, **r** is the discount rate and **t** is the time. Also, the equation of motion is a simplified version of an ideal equation of motion, discussed by others (Conrad and Olson; Kim, *et. al.*) Data limitations prompted us to resort to such a simplification. Nevertheless, it captures all the essentials of nitrate contamination function. The dynamics of nitrate-nitrogen concentration in the groundwater ($\dot{C}_t$) is explained by surface application of nitrogen (N_t) and denitrification of the existing nitrate concentration in the groundwater aquifer (C_t).

The relevant *current value* Hamiltonian is defined as,

$$H(.) = P_y(a + bN_t - cN_t^2) - P_N N_t - \theta C_t^2 + \lambda[\eta N_t - \delta C_t] \tag{2}$$

The three first order necessary (Pontryagin) conditions for maximum are:

$$\frac{\partial H(.)}{\partial N_t} = 0 \Leftrightarrow P_y(b - 2cN_t) - P_N + \lambda_t \eta = 0, \qquad \forall t \tag{3}$$

$$\dot{\lambda} = r\lambda_t - \frac{\partial H(.)}{\partial C_t} \Leftrightarrow -\dot{\lambda} = -2\theta C_t - \lambda_t(r+\delta), \qquad \forall\, t, \tag{4}$$

$$and \quad \dot{C}_t = \frac{\partial H(.)}{\partial \lambda_t} \Leftrightarrow \dot{C}_t = \eta N_t - \delta C_t, \qquad \forall\, t \tag{5}$$

Characterization of the Isoclines and Isosectors

The equations for $\dot{N}_t$ and $\dot{C}_t$, derived from solving the first two first order conditions, are:

$$\dot{N}_t = \frac{(r+\delta)[P_y(2cN_t - b) + P_N] + 2\theta\eta C_t}{2cP_y} \tag{6}$$

$$and \quad \dot{C}_t = \eta N_t - \delta C_t \tag{7}$$

These first order differential equations can further be solved for $\dot{N}_t = 0$ and $\dot{C} = 0$ isoclines.

The $\dot{N}_t = 0$ Isocline

The $\dot{N}_t = 0$ isocline implies,

$$N_t|_{\dot{N}_t=0} = \frac{P_y b - P_N}{2cP_y} - \frac{2\theta\eta}{2cP_y(r+\delta)}C_t \qquad (8)$$

The $\dot{N}_t = 0$ curve is a linear and a decreasing function of C_t.

The $\dot{C}_t = 0$ Isocline

The $\dot{C}_t = 0$ isocline implies,

$$N_t|_{\dot{C}_t=0} = \frac{\delta C_t}{\eta} = \frac{\delta}{\eta}C_t \qquad (9)$$

The $\dot{C}_t = 0$ curve is also a linear, but is an increasing function of C_t.

A graphical illustration of both isoclines with these properties is shown in Figure 1. The isoclines divide the positive orthant into four *isosectors*, labeled I, II, III, and IV. Each isosector has a *directional* indicating the movement of a point (N,C) over time. Isosectors I and III are *convergent*, while isosectors II and IV are *divergent*. The equilibrium (N^*,C^*) is classified as a *saddle point*. Note that the isosectors I and III each contain a trajectory which converges to $(N^{*}*,C^*)$ and is referred to as a *separatrix*. Taken together, the two *separatrices* define the optimal solution trajectories for our infinite horizon problem (since any other trajectory converges either to N = 0 or C = 0 as $t \to \infty$).[3] If these separatrix curves could actually be computed, we would have an explicit optimal nitrogen use policy specifying the optimal nitrogen use rate corresponding to any given stock level C. Such a rule is called a *feedback* or *closed-loop control policy*, since N_t is specified as a function of the current state C_t. Its computational procedure is discussed later.

Steady State Equilibrium

Graphically, it is the point (N^*,C^*) where both isoclines intersect each other in Figure

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[3] The convergent separatrices are said to form the *stable manifold*. Isosectors II and IV contain divergent separatrices which form the unstable manifold.

1, and is given algebraically as:

$$C^{*} = \frac{\eta(r+\delta)(P_y b - P_N)}{2[\delta c P_y(r+\delta)+\theta\eta^2]} \tag{10}$$

$$and \quad N^{*} = \frac{\delta(r+\delta)(P_y b - P_N)}{2[\delta c P_y(r+\delta) + \theta\eta^2]} \tag{11}$$

Equation 11 defines the optimal nitrogen application rate at steady state as a function of all parameters, including θ. Note that as $\theta \rightarrow 0$, $N^{*} \rightarrow N_{\pi}$ [= $(P_y b - P_N)/2cP_y$], the profit maximizing level of nitrogen use under static optimization with no concern for nitrate pollution of groundwater becomes optimal. Thus, if there is no cost to groundwater contamination, the static profit maximizing application rate would be long-run optimum.

A specific value for parameter θ in equations 10 and 11 is neither available nor possible to obtain. However, depending upon the level of nitrate concentration allowed in the groundwater, values for θ can be determined endogenously. Equation 12 defines such implied values for θ associated with the other parameters, including nitrate concentration, C, in the groundwater.

$$\theta = \frac{\eta(r+\delta)(P_y b - P_N) \; - \; 2\delta c P_y(r+\delta)C}{2\eta^2 C} \tag{12}$$

Optimal Policy Rule for Nitrogen Use

The optimal control problem discussed above is of the linear quadratic form. Hence, the optimal policy rule (defined earlier as a closed loop control policy) will be linear, and is given by,

$$C_{t+1} = \omega + \Omega C_t \tag{13}$$

where Ω, in the case of a saddle point equilibrium, is the smaller characteristic root, lying within the unit circle. For the steady state nitrate concentration (C^*) of 10 ppm in the groundwater, a numerical value for ω (such as $\bar{e}$) can be determined from equation 13, thereby giving the optimal policy rule as,

$$C_{t+1} = \bar{e} + \Omega C_t \tag{14}$$

Finally, an optimal nitrogen application rate can be obtained by substituting the nitrate contamination function (equation of motion) in equation 14, and then solving for N_t as a function of C_t ,

$$N_t = N(C_t) \tag{15}$$