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Melanie Blackwell and Angelos Pagoulatos

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Measuring the Specification Bias of the Cobb Douglas Functional Form for Alfalfa Production

by

Melanie Blackwell and Angelos Pagoulatos*

*The authors are Assistant Professor of Decision Science at Xavier University and Professor of Agricultural Economics at the University of Kentucky.

INTRODUCTION

The methodology of specifying a process-modeling (P-M) production function (often referred to as an engineering production function [4],[10],[15],[25]) was pioneered by Chenery. P-M production functions shift the basis of specifying production functions from "desirable" economic properties to the physical aspects of the dynamics of the crop production process being modeled [15]. Where standard economic production functions impose structures on the production functional form (such as homogeniety of degree one or perfectly competitive factor markets), process modeling results in summary representations of technology obtained from properties of reaction processes that transform a given set of factor inputs into desired output. In essence, the P-M approach results in a "summary" production function derived from rate equations describing the physical, biological and chemical processes that convert the inputs into final outputs. Since the parameters of these production functions have a technical basis and interpretation, it is they that are likely to constrain the economic relationships and parameters of interest rather than vice-versa.

Ample evidence of the advantages of employing process dynamics for threshold determination, analysis of chemical pesticide resistance and single-season pest-management decision models exists in the literature [6],[7],[8],[16],[17],[18],[19], [21],[26]. However, econometric investigations of input productivity, output supply and factor demands have ignored much of the contributions from the biological sciences by persisting to employ generalized production functional forms [1],[2],[3], [23]. Lichtenberg and Zilberman point out that the productivity of pesticides has been systematically overestimated due to omission of fundamental biological information in the production function specification [14]. In turn, they propose an ad hoc modification of the stastic Cobb-Douglas production function which reduces the bias that existed in the estimators of productivity.

Lichtenberg and Zilberman fail to eliminate the specification error that they uncover. By ignoring the effects of the length of the growing period upon output and pest prevalence, the true dynamics of the system are not incorporated into the crop production function. Since abundant knowledge of biological processes exists, we should be able to derive the appropriate form of the crop's dynamic production function as well as the manner in which damage abatement (both from chemical as well as natural control agents) should be specified. This involves using process models.

Due to the non-generality of P-M production functional forms, we will derive a production function for alfalfa Medicago sativa L infested with alfalfa weevil larvae (Hypera postica G.). We will begin the analysis by concentrating on a single-harvest production function. This will facilitate a comparison of the P-M summary production function with the Cobb-Douglas form suggested by Lichtenberg and Zilberman. Comparison of these models will center upon their respective measures of input factor productivity. In particular, we will use Kentucky data on the alfalfa ecosystem to measure input productivity and quantify the specification error associated in the Lichtenberg-Zilberman (L-Z) Cobb-Douglas form. It is argued that the P-M approach to production function specification leads to greater analytical precision and a broader range of input, output and productivity analysis.

DERIVATION OF A SUMMARY PRODUCTION FUNCTION

We argue that a summary crop production function can be derived directly from the interrelated growth rate equations characterizing the ecosystem. The following discussion attempts to do just that.

Temperature Dependency of Alfalfa and Alfalfa-Weevil Development

Biological processes progress over physiological time, not chronological time. Physiological time for the alfalfa plant progresses only when photosynthesis takes place--an activity which occurs when average air temperatures rise above 5°C. For the alfalfa weevil, physiological time for all phases is determined by respiration

activity, which occurs at air temperatures above 8°C.

A commonly used measure of physiological time is the cumulative degree-day [11], [20], [21]. Degree-days are a measure of the average temperature, over a 24 hour period, above the minimum temperature required for physiological progression. Each phase of alfalfa and weevil growth requires a specific number of cumulative degree-days to have transgressed before that particular phase is completed and the next growth phase can begin. Since the base temperatures for physiological growth of both the alfalfa plant and the weevil are very close, this study will measure physiological time in terms of cumulative degree-days base 5°C.

Alfalfa Growth

Alfalfa biomass exhibits a growth rate which is initially exponential in form and proportional to the level of biomass. As the biomass increases, environmental limitations force the proportional growth rate to decline, implying the proportional growth rate is a decreasing function of biomass [11], [20]. Such a growth process is termed a "compensatory process" [5] and it defines an ecosystem where overcrowding results in excessive and competitive demands for factors necessary to the growth process, leading to an inhibitory effect on biomass levels. The family of logistic growth models characterizes the compensatory nature of alfalfa growth and, thus, we chose the Gompertz growth model as the appropriate form to model alfalfa growth. We note that the Gompertz growth model has properties consistent with a "dynamic" Cobb-Douglas production functional form [5], [11].

A larval population can be thought of as a negative growth factor which results in a decline of the growth rate of alfalfa through an average feeding rate. As such, it would necessarily be proportional to the level of alfalfa biomass and, hence, growth of alfalfa remains a compensatory process. Subsequently, the Gompertz growth rate equation can be modified to incorporate the effects of larval feeding in the following manner:

$$(1) \qquad \frac{1}{Q(t)}\frac{dQ(t)}{dt} = \Sigma\alpha_i \ln(Z_i^{\circ}+Z_i^{*}) - \beta \ln Q(t) - \gamma \ln P(t) \quad t_o < t < \min\{t_1,t_p\}$$

where $Q(t)$ = biomass of alfalfa at physiological time t,

$P(t)$ = population of weevil larvae at time t,

$(Z_i^{\circ}+Z_i^{*})$ = level of growth factor i available in the ecosystem from natural sources (Z_i°) and supplements made by the decisionmaker (Z_i^{*}), for i = 1,..., n,

t = cumulative degree days base 5°C, where t_o is the cumulative degree days at the time of initial plant growth, t_1 is the cumulative degree days at the time of first harvest, and t_p is the cumulative degree days at the time of larval disappearance,

α_i = average intrinsic growth rate of alfalfa induced by the presence of growth factor i, $\alpha_i>0$,

β = compensation parameter controlling the growth of alfalfa within the confines of the environment, $\beta>0$, and

γ = average intrinsic larval feeding rate from the presences of a larval population P(t), $\gamma>0$.

We note that Eq.(1) is relevant only when a pest population exists. Because the larval stage of the weevil lasts for only a limited period of time during the spring, we need to examine the dynamics of the larval population.

Larval Population Growth

Since the majority of damage from larval feeding occurs from populations emerging in the spring [20], this study will ignore the larval population which arises from fall-laid eggs. An additional assumption to be employed is that there is no distinction between feeding rates of the different stages of the weevil larval population. These simplifying assumptions are made to avoid complications that would render subsequent analysis beyond any economic benefit.

Weevil larval population growth exhibits a net proportional growth rate [24]. That is, both the average rate of emergence from eggs as well as the average rate of maturation into the adult stage are proportional to the population size. However, the rate of emergence from eggs decreases as physiological time progresses. This implies the average rate of emergence from eggs is inversly proportional to physiological time

[19]. Accordingly, the weevil larval population growth rate is

$$(2)\quad \frac{1}{P(t)}\,\frac{dP(t)}{dt} = (k-1)\,t^{-1} - \mu k \qquad \text{for } 200 \leq t \leq \min\ \{t_1, t_p\}$$

where $P(t)$ = population of larvae at physiological time t,

$(k-1)$ = average rate at which larvae emerge from eggs, and

μk = average rate at which larvae mature into adults.

Since we are interested in a first-harvest production function for alfalfa which is weevil-infested, we restrict our time interval to one where the lower limit of the physiological time range is 200 degree days base 5°C (the earliest expected emergence of larvae from spring laid eggs [11]) and the upper limit is either the cumulative degree-days at first harvest (t_1) or the cumulative degree days when the larval population has effectively completed its maturation into an adult population (t_p), whichever occurs first.

Effects of Chemical Pesticides

Chemical pesticides do not alter the growth rate of the pest population; they change actual population levels [20]. When a chemical pesticide of level X is applied at some time $t = \tau$, the population at τ is reduced by an average "kill" percentage not related to the level or type of pesticide applied. Rather, the kill percentage is determined by the probability of larval exposure to the chemical. The greater the level of pesticide applied per unit of land, the greater the residual length of toxicity and the greater the total percentage of larvae killed.

Let $t_x(\tau,X)$ be the length of residual toxicity of a pesticide applied at time τ. Let $U(t_x,\tau,t,X)$ be the percentage of the potential pest population remaining at time t when a pesticide of level X is applied at time τ. It follows that $0 \leq U \leq 1$ and we will assume U decreases over the interval $[\tau,\tau+t_x]$. Furthermore, U becomes a constant (denoted $\overline{U}(X)$) at $t = \tau+t_x$ [20]. The sprayed larval population $P^s(t,X,t_x,\tau)$ can then be defined as a percentage (U) of the potential larval population (P(t)):

$$(3)\ P^{s}(t,X,t_x,\tau) = \begin{cases} P(t)\ U(t,X,t_x,\tau) & \text{for } \tau < t < (\tau+t_x) \\ P(t) - [1-\overline{U}(X)]P(\tau+t_x) & \text{for } (\tau+t_x) < t < t_p^* \end{cases}$$

where all parameters and variables are as previously defined except the effective disappearance of larvae, t_p^*, which defines the cumulative degree-days base 5°C such that $P^{s}(t_x,\tau,t=t_p^*,X)=0$.

It is assumed that the pesticide application will occur so that the population is not completely eradicated prior to the end of residual toxicity. Such an assumption is based on the fact that there would be wasted pesticide applied if any other action were undertaken.

Using Brown and Ruesink's estimate of a 40 percent probability of larval exposure to a chemical pesticide at time τ, the percentage of the potential pest population remaining at time t (given a pesticide application of level X at time τ) is

$$\frac{(t-\tau)}{t_x} \qquad (\tau+t_x-t)/t_x$$

$$(4)\ U(t,X,t_x,\tau) = \begin{cases} A(X) & [0.6] \quad \text{for } \tau < t < (\tau+t_x) \\ A(X) & \quad \text{for } (\tau+t_x) < t < t_p^* \end{cases}$$

where A(X) = percentage of potential larval population remaining at the end of residual toxicity (a pesticide specific constant), and

all other variables and parameters are as previously defined. That is, at the time of the spray application $t=\tau$, the percentage of the larval population remaining in the field is U-0.6. As time progresses such that residual toxicity is lessened (as $t \rightarrow (\tau+t_x)$), the percentage of the pest population remaining in the field approaches its lower limit (A(X)) in a negative exponential manner. Once residual toxicity has ceased to exist, or $t \geq (\tau+t_x)$, the percentage of the potential pest population remaining in the field becomes a constant at $\overline{U} = A(X)$.

A Summary Production Function for Weevil-Infested Alfalfa

A summary production function for weevil-infested alfalfa can be derived by solving differential equations Eq. (1) and Eq. (2) (i.e. the growth rate equations) simultaneously. Furthermore, the effects of chemical pesticides can be included by

substituting the sprayed larval population functional form $P^s(t,X,t_x,\tau)$ for the untreated potential larval population P(t) when appropriate. The result is a mixed discrete/continuous summary production (presented in logarithmic form):

$$(5)\quad \ln F(t,X,\tau,t_x,Z^*) = \Sigma\alpha_i'\ln(Z_i^\circ+Z_i^*) + [\ln Q(t_o)-\Sigma\alpha_i'\ln(Z_i^\circ+Z_i^*)]e^{-\beta(t-t_o)}$$
$$-\gamma'[\mu k/\beta + \ln P^*(t)]$$
$$+\gamma'[\ln P(200) + \mu k/\beta + (k-1)\int s^{-1}e^{\beta(s-200)}\,ds]e^{-\beta(t-200)}$$
$$\text{for } 200 \leq t \leq \min\{t_1,t_p^*\}$$

where $F(t,X,\tau,t_x,Z^*)$ = biomass of alfalfa at physiological time t,

$\ln Q(t_o)$ = initial level of alfalfa biomass,

t_o = initial physiological time in cumulative degree-days base 5°C,

$$P^*(t) = \begin{cases} P(t) & \text{for } x=o \text{ and } 200\leq t\leq\tau \\ P^s(t,X,t_x,\tau) & \text{for } X>O \text{ and } \tau<t\leq t_p^* \end{cases}$$

$\ln P(t)$ = $[\ln P(200) + (k-1)\ln 200 + \mu k200] + (k-1)\ln t - \mu kt$,

= unconstrained pest population at physiological time t (expressed in logarithmic form),

$P(200)$ = initial larval population,

α_1' = α_1/β, the environmentally adjusted intrinsic growth rate of alfalfa arising from the presence of growth enhancing factor $(Z_1^\circ+Z_1^*)$,

β = compensating parameter controlling the growth of alfalfa to within the confines of the environment, and

γ' = γ/β, the environmentally adjusted average feeding rate of weevil larvae.

The derived mixed discrete/continuous summary production function in Eq. (5) reflects the decisionmaker's option to spray or not, as well as his determination of the amount and timing of the chemical pesticide application.

More specifically, it can be shown that the first two terms on the right hand side of Eq. (5) constitute the functional form for uninfested alfalfa growth (i.e. the solution to differential Eq. (1) when $\gamma = o$). For uninfested alfalfa, we find that as the growing season progresses (or as $t \rightarrow 00$) alfalfa yields attain a maximum of

(6) $Q_{max} = \exp \{\Sigma \alpha_i' \ln(Z_i^\circ + Z_i^*)\}$.

Prior to this maximum, the term

(7) $\exp \{ [\ln Q(t_o) - \Sigma \alpha_i \ln(Z_i^\circ + Z_i^*)] e^{-\beta(t-to)}\}$

limits the growth of alfalfa as the stand's biomass increases over time (a common property of a compensatory process). Given our previously stated definitions of α_i and β, it follows that $\alpha' = \alpha_i/\beta$ parameterizes the maximum contribution to the growth of alfalfa that can be realized from input factor i as it operates within the confines of the environment.

The last two terms on the right-hand side of Eq. (5) comprise the effects of larval population feeding. We find the pest population results in only a proportion of the potential (uninfested) alfalfa yields being realized when larval feeding occurs. In particular, a compensatory process

(8) $\exp\{-[\ln P(200) + (k-1) \int^{-1} e^{\beta(s-200)}\, ds] e^{-\beta(t-200)}\}$

limits the growth of the potential pest population P(t) to within the confines of the environment. Given our prior definitions of γ and β, it follows that $\gamma' = \gamma/\beta$ is the environmentally-adjusted larval feeding rate. Finally we note that if the larval population exists for an extended period of time (as $t_p^* \rightarrow oo$), the effect of larval feeding on alfalfa yields reduces to

(9) $\exp\{-\gamma'\mu k/\beta\}$.

In other words, Exp (9) defines the residual effects of larval feeding upon alfalfa yields.

Damage Abatement Functions

Following Lichtenberg and Zilberman, damage abatement from any means of pest control can be defined as the proportion of potential yields realized when a control measure is implemented. Thus damage abatement at time t, DA (t), is defined to be the ratio of actual alfalfa yields (when both chemically-and naturally-controlled larval populations exists) to potential (uninfested) alfalfa yields. In logarithmic form, the damage abatement function for alfalfa is

$$(10)\quad \ln DA(t) = -\gamma'[\mu k/\beta + \ln P^{*}(t)] + \gamma'[\ln P(200) + \mu k/\beta + (k-1)\int s^{-1} e^{\beta(s-200)} ds] e^{-\beta(t-200)}$$

$$\text{for } 200 \leq t \leq \min\{t_1, t_p^{*}\}.$$

This damage abatement function represents the proportion of alfalfa yields remaining after larval feeding has occurred and is identical to the last two terms in Eq. (5).

Defining the natural damage abatement function DA"(t) to be that reflecting environmental restrictions upon larval population growth, then DA"(t) is identical to the damage abatement function in Eq. (10) when no chemical pesticides are applied (i.e. when $P^{*}(t)=P(t)$, or, in logarithmic form,

$$(11)\quad \ln DA^{\eta}(t) = -\gamma'[\mu k/\beta + \ln P(t)] + \gamma'[\ln P(200) + \mu k/\beta + (k-1)\int s^{-1} e^{\beta(s-200)} ds] e^{-\beta(t-200)}$$

as defined in Eqs. (2) and (5). Once we have accounted for natural damage abatement, Eq. (10) contains terms which reflect the additional damage abatement provided by chemical control of the larval population. That is, when chemical pesticides are applied, $P^{*}(t)$ in Eq. (10) is replaced with $P^{s}(t,X,tx,\gamma)$ and the resulting chemical damage abatement function (in logarithmic form) is

$$(12)\ \ln DA^{c}(t) = \begin{cases} -\gamma \ln U(t,X,tx,\tau) & \text{for } \tau \leq t \leq (\tau+tx) \\ -\gamma \ln\{1-[1-A(X)]\frac{P(\tau+tx)}{P(t)}\} & \text{for } (\tau+tx) \leq t \leq t_p^* \end{cases}$$

as defined in Eqs. (3), (4) and (5). Thus we delineate the natural damage abatement function from the chemical damage abatement function which together comprise Eq. (10).

Measures of Input Factor Productivity

Lichtenberg and Zilberman claim that the productivity of pesticides has been systematically overestimated because too little attention has been paid to the biological, physical and chemical effects of pesticides upon agricultural crops. In particular, they argue that pesticides should enter a production function not in the form of growth-enhancing elements, such as fertilizers, but as elements which reduce damage caused by pest feeding. Specifically, they recommend the modification of a standard myopic production function (such as a Cobb-Douglas form) where a chemical pesticide, X_i, is modeled to act on output levels through a damage abatement function, G(X). In the Cobb-Douglas production functional form, the proposed Lichtenberg-Zilberman (L-Z) model is

$$(13)\quad \ln F = \alpha_o + \Sigma\alpha_i \ln Z_i + \gamma \ln G(X),$$

where F = crop biomass,

Z_i = amount of growth-enhancing factor i which the decisionmaker controls, such as fertilizers,

G(X) = damage abatement function arising from the use of damage control agent X,

α_o = constant intercept term capturing the effects of "natural and omitted factors", taken to be a proxy for exogenous factors that affect production and are not under the control of the decisionmaker,

α_i = constant partial production elasticity of growth-enhancing factor i, and

γ = constant partial production elasticity of the damage abatement function.

G(X) is characteristic of a cumulative distribution function (cdf) defined on (0,1), with G(X)=1 denoting complete eradication of destructive capacity and G(X)=0 denoting zero elimination. The marginal productivity of the damage control agent is defined as $G_x(X)=g(X)$. Lichtenberg and Zilberman, after experimenting with alternative specifications of the fitted cdf to damage abatement data (Pareto, logistic, exponential, and Weilbull) concluded that estimation of standard production functions, where damage control agents (pesticides) are misspecified as growth enhancing factors, Z_i, results in overestimation of the marginal productivity of damage control agents and underestimation of the productivity of "natural factors and omitted variables".

Comparison of the L-Z and P-M Production Models

The P-M and L-Z models result in a type of Cobb-Douglas function. The main difference is that the P-M form is dynamic and directly incorporates the effects of "natural factors", which cannot be captured in the static L-Z form. Both the L-Z and P-M models provide for the influence of growth enhancing factors (Z_i in the L-Z model and $Z_i^{\circ}+Z_i^{*}$ in the P-M model) upon alfalfa yields where, by definition, we have that $Z_i=Z_i^{\circ}+Z_i^{*}$. Both studies assume that the influence of the growth enhancing factors can be combined into a single effect resulting from a composite growth enhancing factor Z. Thus, the partial production elasticity of the composite growth enhancing factor Z is denoted by α' in the L-Z model. The P-M model allows us to further define $\alpha_{\cdot}$ to be the environmentally-adjusted average growth rate of alfalfa resulting from the presence of composite growth factor Z.

Similarly, both production models claim that pesticides operate through a chemical damage abatement function to affect overall alfalfa yields. In order to compare the chemical damage abatement functions we will assume that the L-Z specification measures the maximum effect of chemical pesticides (i.e. the final damage abated at the end of residual toxicity). A consistent assumption for the P-M specification requires us to analyze the model at time $t=\tau+t_x$. It follows that the partial production elasticity of damage abatement γ in the L-Z form (Eq. 13) is, by

definition, identical to γ' in the P-M model Eq.(12). Because γ' is further defined to be the environmentally controlled average larval feeding, we will henceforth use γ' to denote the partial production elasticity of the chemical damage abatement function in both models.

It is here that the similarities between the two specifications end and their differences begin to emerge. The L-Z production function includes only one additional term in their model: an intercept parameter α_o which is defined as the effect of natural factors and omitted variables. Derivation of the P-M model reveals that this term is not a constant, rather, it is a dynamic specification of a natural damage abatement function and a compensatory process which limits the potential contribution from the composite growth-enhancing factor to within the confines of the environment. In addition, chemical damage abatement G(X) is defined in the L-Z specification as the percentage of the pest population killed (as measured at the end of residual toxicity) whereas in the P-M specification, chemical damage abatement A(X) is defined as the percentage of the pests remaining at time $t=\tau+t_x$ (thus A(X)=[1-G(X)]).

Incorporating the adjustments needed for comparative purposes, the estimating form of the P-M specification Eq.(12), is:

$$(14)\quad \hat{\ln F} = \hat{\alpha}_q e^{-\hat{\beta}(\tau+tx-to)} - \hat{\alpha}'[\ln Z]\,[e^{-\hat{\beta}(\tau+tx-to)}] - \hat{\gamma}' \ln h(\tau+tx) + \hat{\alpha}'\ln Z - \hat{\gamma}' \ln[1-G(X)] + \hat{\Sigma}$$

where the ^ denotes the estimated value of the associated parameter and $-\gamma' \ln h(\tau+tx) = \ln DA^{\eta}(\tau+tx)$ as defined in Eq. (11). In addition $\hat{\alpha}_q$ is the sample mean of $\ln Q(t_o)$. Similarly, the estimating form of the L-Z specification in Eq. 13, is:

$$(15)\quad \hat{\ln F} = \hat{\alpha}_o + \hat{\alpha}' \ln Z + \gamma' \ln (G(X) + \hat{\Sigma}^*$$

Assuming the P-M production function in Eq. (14) is the correct specification we can estimate the L-Z form in Eq. (15) using ordinary least squares (OLS), and

then examine the consequences of the resulting specification error. As we show formally in Appendix I, the expected value of the OLS estimator of α' from the L-Z model is

$$(16)\quad E(\hat{\alpha}')=\alpha'[1-E(e^{-\beta(\tau+tx-to)})] \leq \alpha'$$

The implication of Eq. (16) is that the L-Z specification results in an underestimation of the maximum partial production elasticity.

Similarly, the expected value of the OLS estimator of γ' resulting from estimation of the L-Z model is

$$(17)\quad E(\hat{\gamma}') = \gamma' E\left[\frac{G(X)}{1-G(X)}\right] - \gamma' E[\eta_h \eta_{\tau+tx}/\eta_g]$$
$$- \beta E\,[\eta_{\tau+tx}/\eta_g\ (\tau+tx)\ (\ln Q(t_o)-\alpha' \ln Z)e^{-\beta(\tau+tx-to)} \quad > \gamma'$$

where $\eta_h=[\partial h(\tau+tx)/\partial(\tau+tx)]\ [(\tau+tx)/h(\tau tx)] <0$ is the elasticity of natural damage abatement, $\eta_{\tau+tx}=[\partial(\tau+tx))/\partial X][X/(\tau+tx)]>0$ is the elasticity of the timing of maximum chemical effects and $\eta_g=[\partial G/\partial X]\ [X/G]$ is the elasticity of the kill function. That is, Eq. (17) implies the L-Z specification results in an over-estimation of the partial production elasticity of chemical damage abatement. We find the L-Z estimator of γ' is inflated due to the model's inability to distinguish between chemical damage abatement and the added effects of natural damage abatement (second term on the right-hand side of Eq. (17)) and increased productivity of the composite growth-enhancing factor at time τ+tx (the third term on the right-hand side of Eq. (17)).

Finally, we find that $E(\hat{\alpha}_0) = 0$, which follows from the fact that all the effects of the variables excluded in the L-Z model are embodied in the estimator $\hat{\gamma}'$ and the P-M model has, no constant intercept term. Of course, no bias can be determined since the L-Z model specifies no a priori: expected value or sign.

Empiral Analysis

The validity of our claim that the L-Z estimators are biased hinges upon whether the data supports the theory that the P-M summary production function is

the superior explanatory/predictive model. The traditional testing of this claim calls for parameter estimation and comparison through collection of sample field data covering as many conditions of alfalfa production as possible--a proposition which entails considerable expertise and many years. Although traditional testing is preferable, immediate assessment of the models is possible through use of an alfalfa ecosystem simulator developed independently of this study by an intercollegiate team of entomologists and agronomists [9], [19]. This system simulator generates "psuedo-data" which has been found to avoid many of the traditional problems of estimation: severe multicollinearity, limited sample ranges and inadequate technical and environmental detail [12], [13].

The simulator assumes that the growth of a pure stand of alfalfa at the peak of its productive capability takes place without severe soil fertility limitations. This implies that the influence of the composite growth-enhancing factor (α' ln Z) in both the P-M and L-Z specifications is a constant, henceforth to be denoted α_z. The simulator is initiated on September 1 for each year of simulation by specifying 1) the biomass of leaves, stems, basal buds and total non-structural carbohydrate reserves per square meter, 2) the latitude of the alfalfa field, 3) the daily weather patterns of high/low temperatures and precipitation, 4) the biotype of the weevil (eastern or western), 5) the number of adult weevils per square meter, 6) the type of pesticide applied (short-or long-residual) and 7) the timing of the pesticide application. The dynamics of the alfalfa ecosystem are then simulated by a system of first-difference equations which have been statistically tested for reliability and tractability with respect to actual yields realized at University of Kentucky's Spindletop Experimental Farm [19]. The model predicts values for several state variables of interest measured at intervals of 50 cumulative degree-days base 5°C: 1) the yield of alfalfa hay in tons/hectare, 2) the cumulative degree-days to date and 3) the total number of weevil larvae per square meter.

To estimate the parameters of the L-Z and P-M model, several intermediate models must first be estimated. Estimates of β (the compensatory parameter) can be

obtained and used to fit the environmentally adjusted growth curve for a population of alfalfa weevil larvae. Of course, this latter model forms the functional argument of our natural damage abatement function Eq. (11).

Four simulations of uninfested alfalfa yields were run using historical weather data collected at Spindletop (1979-1981 and 1983). This permitted us to estimate the parameters of the P-M summary production function when no larvae population exists and when Z is a fixed constant (Eqs. (6) and (7)). Gauss-Newton techniques of non-linear least squares (NLS) regression were employed to obtain a parameter estimate of β=.0055, as presented in Table 1. Next, pseudo-data consisting of weevil-infested alfalfa growth for three levels of infestation (high, medium and low) was simulated using the historical Spindletop weather data. This data was used to obtain the OLS parameter estimates of the environmentally-restricted alfalfa weevil larvae growth curve ((ln h (t)) from Eq. (14)) as presented in Table 1. It should be noted that the simulator does not distinguish between larvae arising from fall and spring laid eggs. Thus the initial measurements of the larval population at time t=200 consists of only a portion p_1 of which are spring-laid larvae. Similarly, only a portion p_1 of total environmental restrictions is attributed to the spring-laid larvae. These parameter estimates now allow use to fit the functional argument for natural damage abatement (Eq. (14)).

A similar approach to fitting the chemical damage abatement functions (Eq. (12) in the P-M model and G(X) in the L-Z model) was undertaken. The fitted form of two intermediate models were required: the length of residual toxicity in terms of degree-days and the pesticide kill function. Choosing long-and short-residual applications occuring on arbitrarily chosen dates of 90,270 and 450 degree-days, pseudo-data was generated from the four historical Spindletop weather patterns. Upon examination of the data, it was found that the length of residual toxicity, t_x, was approximately linear with respect to the timing of the pesticide application, τ. The level of pesticide, X, acts a both an intercept and slope shifter. The OLS parameter estimates of these relationships are presented in Table 2.

Table 1. Summary Statistics for the Intermediate Model Specifications of Uninfested Alfalfa Growth and Environmentally-Restricted Alfalfa Weevil Larval Population Growth.

Non-linear Least Squares Statistics for the Gompertz Model of Uninfested Alfalfa Growth

Equation Parameter	Parameter Estimate	Asymptotic Standard Error
d_z	.0107	.0006
β	.0055	.0003
α_q	-3.6569	.2513

Regression Sum of Squares	739.1395
Total Sum of Squares	770.6722
Uncorrected Degrees of Freedom	231

Ordinary Least Squares Statistics for the Logarithmic Form of the Environmentally-Constrained Weevil Larval Population

Variable	Parameter	Parameter Estimate	Standard Error
Intercept	P_o	-49.3347	4.5246
$[\ln P(200)]\,[1-e^{-\beta/(t-200)}]$	p_1	.5509	.0759
ln t	(k-1)	11.2734	.9155
t_t	$-\mu k$	-.0258	.0015
$\int_{200} s^{-1}e^{-\beta(t-s)}\,ds$	$-p_1(k-1)$	-2.9889	.3334

Regression Sum of Squares	466.9067
Total Sum of Squares	536.9069
Uncorrected Degrees of Freedom	228

For any given level of chemical control agent X, there exists a maximum number of larvae that can be eliminated, based upon the timing of the spray. This study will assume a chemical spray date τ which results in minimizing the total larval population over the time interval [200, t_p^*], where t_p^* is a function of both τ and X. According to Ruesink and Brown [19], approximately ten percent and one percent of the potential pest population will remain at the end of residual toxicity if a short-or long-residual pesticide is applied, respectively. For both a short-and long-residual application of the chemical Funadan 4F (X=.6178 lbs./hectare and X=1.2355 lbs/hectare, respectively), the fitted growth function for the environmentally-restricted larval population can be used, together with the equation determining the length of residual toxicity t_x and the kill percentages listed above, to find the spray date τ that will minimize the total larval population over the time interval [200, t_p^*]. The solution procedure involves use of the Newton-Ralphson algorithm and results in a short-residual pesticide minimizing the total larval population if applied at 405 degree-days and a long-residual pesticide achieves the same goal if applied at 370 degree-days.

From Regev, Shalit and Gutierrez [17], the functional form of the pesticide kill function is exponential with parameter $\tilde{a}$. Estimation of the equation parameter entails NLS techniques and is presented in Table 2. With this rate parameter estimate, we can now fit the functional arguments for the L-Z and P-M chemical damage abatement functions.

Dummy variable techniques provide an excellent means of combining the three pseudo-data sets used in the intermediate model estimation procedures to estimate the parameters of the P-M specification in a single linear regression equation. Defining 0-1 dummy variables as

Table 2. Summary Statistics for the Intermediate Model Specifications of Residual Toxicity and the Pesticide Kill Function.

Ordinary Least Squares Statistics for Residual Toxicity in Terms of Degree Days Base 5°C

Variable	Parameter Estimate	Standard Error
Intercept	-24.4386	38.0861
τ	.0520	.1239
X	97.8906	38.9921
τX	.0851	.1268

Regression Sum of Squares	53,335.918
Total Sum of Squares	25,856.240
Uncorrected Degrees of Freedom	29

Non-linear Least Squares Statisitcs for the Exponemtial Pesticide Kill Function

Parameter	Parameter Estimate	Asymptotic Standard Error
$\tilde{a}$	1.4082	.2132

Regression Sum of Squares	12.1220
Total Sum of Squares	13.5523
Uncorrected Degrees of Freedom	24

$$D_1 = \begin{cases} 0 \text{ if uninfested alfalfa data is used} \\ 1 \text{ otherwise,} \end{cases}$$

$$D_2 = \begin{cases} 0 \text{ if untreated, weevil-infested alfalfa data is used,} \\ 1 \text{ otherwise,} \end{cases}$$

$$D_3 = \begin{cases} 0 \text{ if chemically-treated, weevil-infested alfalfa data is used} \\ \quad \text{and } \tau \leq t \leq \tau + t_x, \\ 1 \text{ otherwise, and} \end{cases}$$

$$D_4 = \begin{cases} 0 \text{ if chemically-treated, weevil-infested alfalfa data is used} \\ \quad \text{and } t > \tau + t_x, \\ 1 \text{ otherwise,} \end{cases}$$

then, in the P-M specification, we multiply the logarithmic functional arguments of natural damage abatement (ln h(t) by D_1, chemical damage abatement (ln g(t, τ tx, X)) over [τ τ+tx] by $D_1{}^*D_2{}^*D_4$ and chemical damage abatement over [τ+tx, t_p^*] by $D_1{}^*D_2{}^*D_3$. We then estimate the parameters of the P-M production function using OLS techniques, the results of which are presented in Table 3. Using the same data set as above, only with observations limited to the end of residual toxicity τ+tx (483.391 and 524.647 degree-days for short- and long-residuals, respectively, and at 500 degree-days for uninfested and untreated data) the L-Z production function from Eq. (13) is estimated via OLS techniques. The results are presented in Table 3.

Using a confidence level of five percent, only the intercept term of the L-Z specification (i.e. the effects of "natural factors and omitted variables") can be deemed statistically significant. Our claim is that if the P-M model is correct, the intercept term of the L-Z model measures only the mean logarithm of alfalfa yields at time $\tau+t_x$ (which is 1.5185 from the pseudo-data). The OLS estimate of the L-Z intercept is 1.4380 with a 95 percent confidence interval of [1.2845, 1.5915]. The pseudo-data is thus consistent with the claim that the value for the L-Z intercept term (α_o) is zero and the L-Z estimate of α_z' is biased downward. Of course, the statistically significant measure of $\alpha_z' = 1.9775$ obtained from the P-M specification reinforces this claim. The data is also consistent with the P-M specification of a relevant natural damage

Table 3. Summary Statistics for the Process-Modeled (P-M) and Lichtenberg-Zilberman (L-Z) Production Functions.

Ordinary Least Squares Statistics for the P-M Summary Production Function

Variable	Parameter	Parameter Estimate	Standard Error
Intercept	α_z	1.9775	.0149
$e^{-s055(t-to)}$	$[\alpha_o-\alpha_z']$	-5.9613	.1176
ln h	$-\gamma'$	-.0105	.0025
ln g	$-\gamma'$	.0524	.0577

Regression Sum of Squares	400.4531
Total Sum of Squares	708.2243
Uncorrected Degrees of Freedom	2063

Ordinary Least Squares Statistics for the L-Z Production Function

Variable	Parameter	Parameter Estimate	Standard Error
Intercept	$[\alpha_o + \alpha_z']$	1.4380	.0783
ln G(X)	γ'	-.0616	.0697

Regression Sum of Squares	.0689
Total Sum of Squares	3.0707
Uncorrected Degrees of Freedom	35

abatement function (ln h(t)) which is omitted in the L-Z formulation.

Finally, we find the chemical damage abatement function (g(X) in the P-M model and G(X) in the L-Z model) is not a statistically significant explanatory term in either model. This insignificance could arise from several factors, only one of which is that the data generated by the simulator did not evidence significant reduction in larval feeding damage when a chemical pesticide was applied. A quick perusal of the pseudo-data indicates that a long-residual pesticide results in an average kill percentage of 81.48 percent by the end of residual toxicity but an average reduction in damage of only 1.61 percent. Similarly, a short residual pesticide has a mean kill percentage of 59.01 with a .9 percent average reduction in damage. Whatever the reason, analysis of any purported bias cannot be undertaken due to the insignificance of the chemical damage abatement function.

Conclusion

The dynamic summary production function for alfalfa arising from a predator-prey relationship existing in crop production was derived from the simultaneous solution of two differential equations describing the growth rates of the crop and the pest population. The resulting P-M model is dynamic and directly incorporates the effects of "natural factors" which cannot be captured in the static L-Z form. Lichtenberg and Zilberman thus fail to eliminate the problem they sought to cure.

Their model was proposed as a correction to previous misspecifications of crop production that continued to find pesticides to be severely underutilized. Yet, the P-M specification of alfalfa production shows that the proposed L-Z model continues to overestimate the productivity of chemical damage control agents X which will, in turn, continue to imply pesticides are underutilized. In addition, the P-M production function identifies a previously undiscovered bias of static economic production functions: the underestimation of the potential contribution of growth-enhancing factors.

Combining pseudo-data sets both the P-M and L-Z production functions were

estimated and compared. Due to specification error, we showed that the L-Z model under-estimates the partial elasticity of production with respect to decision-maker controls such as level of fertilization, etc. It was also found to over-estimate the partial elasticity of production in respect to chemical damage abatement. The effect attributed to chemical damage abatement includes the affects of natural damage abatement. The estimate of the two functional forms via OLS reinforces the conclusion that the P-M approach to production function specification leads to greater analytical precision and a broader range of imput, output and productivity analysis.

APPENDIX

According to the L-Z specification, $\hat{\alpha}'$ is the OLS estimator of the partial production elasticity of the maximum contribution from composite input factor Z, or

$$(A1)\quad E(\hat{\alpha}') = E\left\{\frac{\partial F}{\partial Z}\cdot\frac{Z}{F}\right\}$$

If the P-M specification is the true model, then the right-hand-side of Eq. (A1) is derived from Eq. (14):

$$(A2)\quad E\left\{\frac{\partial F}{\partial Z}\cdot\frac{Z}{F}\right\} = \alpha'\,[1 - E\{\,e^{-\beta(\tau+tx-to)}\,\}] < \alpha'$$

Thus we find the L-Z model results in a downwardly-biased estimator of α'.

Similarly, the L-Z specification defines γ' to be the OLS estimator of the partial production elasticity of chemical damage abatement, or

$$(A3)\quad E(\hat{\gamma}') = E\left\{\frac{\partial F}{\partial X}\cdot\frac{X}{F}\cdot\frac{1}{\eta_G}\right\}$$

From the assumed true P-M specification, we have that the right-hand-side of Eq. (A3) is derived from Eq. (14):

$$(A4)\quad E\left\{\frac{\partial F}{\partial X}\cdot\frac{X}{F}\cdot\frac{1}{\eta_G}\right\} = \gamma' E\left[\frac{G(X)}{1-G(X)}\right] - \gamma' E\left\{\frac{\eta_h \eta_{\tau+tx}}{\eta_G}\right\}$$
$$- \beta E\left\{(\tau+t_x)\,\frac{\eta_{\tau+tx}}{\eta_G}\,[\ln Q(t_o) - \alpha'\ln Z]e^{-\beta(\tau+tx-to)}\right\} > \alpha'$$
$$\text{for}\quad \frac{G(X)}{1-G(X)} > \tfrac{1}{2}\,.$$

where $\eta_h = [\partial h(\tau+tx)/\partial(\tau+tx)]\,[(\tau+tx)/h(\tau+tx)] < 0$ is the elasticity of natural damage abatement. $\eta_{\tau+tx} = [\partial(\tau+tx)/\partial X]\,[X/(\tau+tx] > 0$ is the elasticity of the timing of maximum chemical effects, and $\eta_G = [\partial G/\partial X]\,[X/G] > 0$ is the elasticity of the kill function. This time we find the L-Z model results in an upwardly-biased estimator γ'.

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