Dynamic Interactions between Pasture Production, Milk Yields and Economic Viability of NSW Dairy Farms

Peter R. Tozer and Ray G. Huffaker *

Abstract

Previous bioeconomic studies have mainly concentrated on beef operations, principally stocker activities, and rangeland conditions. These studies have assumed that the rancher determines some desired weight gain per head over a period, usually one year, and this weight gain is achieved by utilising a resource such as pasture or grazing rangeland. This study differs to the previous research as we are interested in the interactions between pasture productivity and milk yield in an intensive grazing situation, rather than extensive grazing, and incorporate more than one type of pasture or forage type into a model of a dairy system.

We develop a discrete optimal control model based on the energy demand of a herd of dairy cows and the supply of energy available from the various forages produced on a model dairy farm. The objective of the model is to maximise the net present value of the flow of profits generated by the dairy. Incorporated into the model is a herd dynamics sub-model, a transferable quota trading equation, and a milk revenue function.

Introduction

The dairy industry in Australia is in general pasture based, with concentrates or grains being fed to supplement the dietary requirements of the cows and to improve milk productivity on most dairy farms (ABARE 1995, ADC). There are two distinct production patterns in the Australian dairy industry, in the states of New South Wales, Queensland, South Australia and Western Australia milk production is based on a supply to the market milk sector, in the remaining states, Tasmania and Victoria, milk is supplied principally to the manufacturing sector. The milk supply in the market milk states is relatively constant over the year with some seasonal fluctuations, however, in Victoria and Tasmania there is a pronounced seasonal pattern of milk production with production peaking in the spring and relatively little milk produced in the autumn and winter months (ADC). Victorian and Tasmanian pasture production is winter dominant due to the prevailing weather patterns, and milk production is not as strictly controlled as in the other states. Institutional arrangements in the other states force producers to supply a constant level of market milk throughout the year, hence there is less seasonality in production.

Given the pasture based dairy farming systems in this country, and the locations of some of the dairy regions, i.e. the Murray River Valley or the dryland areas of coastal NSW and Queensland, and the costs associated with pasture renovation, the Dairy Research and Development Corporation has recognised the need to develop land practices that could reduce these costs (Bartsch and Mason). Economic incentives are also driving dairy farmers to reduce costs of production, particularly competition from New Zealand in the world market, and pasture based systems are still the most cost effective method of producing milk. The Dairy Research and Development Corporation has found that pasture costs are stable at around 13.5 per cent of milk income over the last 11 years, whereas the costs of grains and/or concentrates have increased from near zero to about 7 per cent in the same period. Hence the need to sustain a pasture based production system is seen as a desirable objective for the Australian dairy industry, rather than turn towards the more costly feedlot type systems of North America and Europe, particularly with the prices of feed grains expected to rise over the short to medium term due to shortages world-wide (Bartsch and Mason).

The aim in this study is to incorporate the dynamic interactions of a dairy farm's systems, such as changes in pasture composition or prices of output, into a representation of the farm system that would yield output that can be readily understood, and is applicable to the current situation of the farm, by farm managers or extension staff. This simple representation will capture the complex dynamic nature of

the dairy system yet still provide measures of viability of the farm business that can be used as a basis for long-term management decision-making. A discrete optimal control model is built to capture the dynamic interactions of the systems of equations within the dairy system. This model is based on the work of Standiford and Howitt (1992), and Howitt (1996) who show that a discrete optimal control model can be solved as a non-linear dynamic optimisation problem using readily available mathematical programming packages such as GAMS, providing the equations within the model satisfy certain criteria. The restrictions are that the functions be continuous, differentiable, and satisfy the first and second order conditions to ensure the existence of a solution, Howitt (1996). We are currently immersed in the solution phase of the research and we are solving the model via a programming approach found in Standiford and Howitt (1992) and Howitt (1996).

Previous Research

Much of the previous bioeconomic research concerning grazing enterprises has usually concentrated on extensive beef operations. Karp and Pope (1984), and Pope and McBryde (1984), examined the profitability of a cow-calf operation in the rangelands of southern Texas employing stochastic dynamic or quadratic programming, respectively, to model rancher behaviour. Torrell, Lyon and Godfrey (1991) extended this analysis to incorporate an equation of motion of forage production, the previous researchers assumed an average level of forage availability over the grazing period. Torrell *et al.* (1991) argue that the average forage production function does not capture the variation in forage production throughout the grazing season, thus the resultant stocking rate may not be realistic as there would be insufficient fodder to feed the stock at some times in the grazing period.

Huffaker and Wilen (1989) use an optimal control framework to present a model of pasture-grazing interaction in terms of a forage density function and an animal search and harvest of forage function. They then introduce a weight gain function for the grazing animals so that an economic interpretation can be applied to the research, in this article it was assumed the animals were only gaining weight not reproducing. Given the three functions mentioned it was possible for them to derive a dynamic profit function that was then used to derive a stocking isocline that was analysed for stability with respect to stocking rate and forage production. This method of analysis is a departure from that of the previous work in that both the stocking rate and forage production functions are continuous, and the effects of changes in economic and physical parameters can be readily ascertained by examination of the phase diagrams.

Work in dairy systems analysis can be divided into two distinct categories, simulation analysis and mathematical programming. Economic, bioeconomic or biological simulation models of dairy farms or parts of dairy systems have been undertaken in previous research, i.e. Congleton (1984), Gao, Spreen and DeLorenzo (1992), and Parker, Muller and Buckmaster (1992). In most cases it was assumed that dairy farmers had access to an infinite supply of feed for their dairy and this feed was purchased, in reality this is not the case. In Australia, dairy farms offer pastures with finite forage yields and dairy farmers do not have access to either unlimited feed sources or face capital constraints limiting the amount of feed that can be purchased.

Other researchers have modelled dairy systems using linear programming, see Olney and Kirk (1989), Olney and Falconer (1985), Gunn and Silvey (1967), Conway and Killen (1987), or Tozer (1993). This technique linearises non-linear equations or systems of equations within a model, such as pasture growth or milk yield functions. Imposing these types of restrictions yield solutions that may seem ideal given the linearity of the equations, but which may not be economically or biologically consistent.

The Farmer's Problem and Optimal Control

The dairy farmer's problem is to maximise the economic benefit from all the resources, pastures, land and cows, of the farm subject to the physical and financial constraints of the farmer. Hence, the decision for the farmer is to choose the stocking rate, S_t, that will maximise the present value of current and future profits of the dairy farm, that is;

$$(1) \; \underset{S_t}{\text{Max}} \; NPV = \sum_{t=1}^{T} (\frac{1}{(1+r)^t} \{\pi_t^M + \pi_t^L - C_t^S - C_t^P\})$$

where π_t^M is the profit function of the dairy enterprise, and π_t^L is the profit to the farmer of trading in livestock, female and male calves, and cull cows. C_t^S is the cost of feeding supplements to the producing herd due to a shortage of energy from pasture production and C_t^P are the costs of producing pastures for the primary energy supply to the milk producing herd.

In the management of a dairy system the dairyman can usually control few variables, and in general may control only the stocking rate, the level of supplements fed, and the pasture rotations of the farm. If we assume that pasture rotations are fixed, meaning that the area of the farm under particular pasture types remains constant, which is not unusual, then the dairy farmer really has control only over the stocking rate and

supplements fed. As the farmer can control the stocking rate, then this control will effect the state of variables the farmer has no control over, such as pasture growth or the herd size. Thus in the context of this problem the control variable is the stocking rate, and the state variables are the herd size and pasture availability. The dairyman also controls the level of supplements fed by determining the stocking rate of pasture and the level of milk production, thus these three variables are dependent control variables.

In the following discussion we will develop a model to explain the behaviour of the dairyman, and show how each sub-system of the dairy system can be incorporated into an optimal control framework which can be solved to maximise equation 1.

Herd Dynamics.

The herd dynamics within a dairy herd are fairly complex and have been studied in great deal in previous work such as van Arendonk (1985, 1986, and 1988), and Stewart, Burnside, Wilton and Pfeiffer (1976). In this study we will abstract from the complex dynamics and provide a simplified model of herd dynamics. We will assume all deaths, transfers from age class to age class, and culling from the different age classes occurs on the first day of month t. Another assumption is that the average monthly total calving rate (α_{TC})[1] of the herd is constant across age categories. The culling rate δ_c^i and death rate δ_d^i will be constants but will vary across the age classes. Also, it will be assumed that a constant proportion of cows calve in each period. This is not an uncommon assumption and has been applied in previous research, (see for example Standiford and Howitt 1992, Karp and Pope 1984, Pope and McBryde 1984).

The structure of the whole herd is determined by the calving rate, death and culling rates within age groups and the number of breeding females, and can be denoted by the following equations.

The number of retained heifer calves less than one year old in period t is:

$$(2) \quad H_t^0 = (1 - \alpha_2)(1 - \delta_d^0)(0.5\alpha_1 \sum_{k=0}^{11} H_{t-k})$$

where k represents the lag caused by animals being born over a twelve monthly period and remaining in the same age class as those at time t. The number of replacement heifers less than two years old, but older than one in period t is:

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[1] α_{TC} is employed rather than α_1, the average monthly live calving rate, as some calves could die between birth and sale, thus even though the calf has died the cow will continue to lactate as normal.

$$(3) \quad H_t^1 = \sum_{k=12}^{23} H_{t-k}^0 \, (1 - \delta_d^1)$$

Number of breeding cows in age class j in period t is:

$$(4) \quad H_t^J = \sum_{k=12}^{23} H_{t-k}^{J-1} \, (1 - \delta_c^1 - \delta_d^1)$$

Summing the number of cattle in each of the age classes in the breeding herd yields the total herd size in month t:

$$(5) \quad H_t = \sum_{j=2}^{n} H_t^j \qquad j = 2,\dots,n, \text{ is the number of age classes in the breeding herd.}$$

The total herd of the farm, including retained heifers under one year old and replacement heifers older than one, but less than two years old, is given as;

$$(6) \quad H_t^T = (1 - \alpha_2)(1 - \delta_d^0)(0.5\alpha_1 \sum_{k=0}^{11} H_{t-k}) + (1 - \delta_d^1) \sum_{k=12}^{23} H_{t-k}^0$$

$$+ (1 - \delta_c^1 - \delta_d^1) \sum_{j=2}^{n} (\sum_{k=12}^{23} (H_{t-k}^{J-1})$$

This equation shows us that the structure of the herd depends only on the state variable H_t and not on the stocking rate control variable, S_t.

Energy Supply

There are numerous possible pasture types a dairy farmer can produce for consumption by the dairy herd, and the energy supplied by these pastures at any one time will be dependent on the growth of that pasture over time. We will assume there are four types of pastures grown on the farm pastures w, x, y and z, and the pasture production in period t from these different pastures can be written as;

$$(7) \quad F_t^{(w,x,y,z)} = F_{t-1}^{(w,x,y,z)} + G_{t-1}^{(w,x,y,z)}(F_{t-1}^{(w,x,y,z)}) - Cn\,(F_{t-1}^{(w,x,y,z)})\,S_{t-1}$$

where $F_t^{(w,x,y,z)}$ is the amount of pasture available at the beginning of period t measured in kg DM/ha, $F_{t-1}^{(w,x,y,z)}$ is the pasture dry matter at the beginning of the previous period, $G^{(w,x,y,z)}(F_{t-1}^{(w,x,y,z)})$ is the growth function of the pasture in the previous period, and $Cn(F_{t-1}^{(w,x,y,z)})$ is the consumption of pastures in the previous which is multiplied by the stocking rate in the previous, S_{t-1}. S_{t-1} is the stocking rate of the pastures of the dairy farm at time t-1 or the number of cows per hectare of pasture, and differs from the total stocking rate, H_t, by the amount of cows being fed supplements. Using the logistic growth function and the Michaelis-Menten consumption function as discussed in Huffaker and Wilen (1989 p555) we can define for pasture x the growth and consumption functions, which are respectively;

(8) $G^{(x)}(F_{t-1}^{(x)}) = a_x F_{t-1}^x - b_x (F_{t-1}^x)^2$

and

(9) $Cn(F_{t-1}^{(x)}) = \dfrac{q F_{t-1}^x}{F_{t-1}^x + K}$

where a_x and b_x are coefficients of the growth function. The parameters of the consumption function are q, which is the average satiation rate per animal over all age classes in the breeding herd, and K is the Michaelis-Menten constant in kg DM/ha. The Michaelis-Menten constant is a parameter of the model that determines the steepness of the pasture consumption function. Adapted from the original definition of the Michaelis-Menten function, K is defined as the value of forage production for half maximal intake, i.e. $(K = 1/2q_{max})$, Thornley (1976, pp11-14). This constant is inversely related to the search efficiency of the grazing animal, a search efficiency decline would occur in a pasture of high forage production as the animal does not have to search very hard to find sufficient palatable feed to satisfy it's needs, therefore as pasture production increases K would decline, Huffaker and Wilen (1989).

From equations 8 and 9 we can derive the pasture available at the beginning of period t,

(10) $F_t^x = F_{t-1}^x + a_x F_{t-1}^x - b_x (F_{t-1}^x)^2 - \dfrac{q F_{t-1}^x}{F_{t-1}^x + K} S_{t-1}$

From previous research we know that the energy content of a well-managed pasture sward is relatively constant, Newberry and Bowen (1969), and Lazenby (1988), so we

can derive the energy production of this pasture sward by the following method. If we assume the pasture x has a constant energy level, γ^x, measured in megajoules/kg DM, then we have the total energy production of the pasture is E_t^x MJ/ha, hence;

(11) $E_\gamma^x = \gamma^x F_t^x$

this implies that:

(12) $F_t^x = \dfrac{E_t^x}{\gamma^x}$

By substituting equation 12 into equation 10 we get:

(13) $\dfrac{1}{\gamma^x} E_t^x = \dfrac{1}{\gamma^x} E_{t-1}^x + \dfrac{a_x}{\gamma^x} E_{t-1}^x - \dfrac{b_x}{(\gamma^x)^2}(E_{t-1}^x)^2 - \dfrac{\dfrac{q}{\gamma^x}E_{t-1}^x}{\dfrac{1}{\gamma^x}E_{t-1}^x + K} S_t$

and multiplying through by $\dfrac{1}{\gamma^x}$ gives us the following equation for the energy available from pasture x in period t:

(14) $E_\gamma^x = E_{t-1}^x + a_x E_{t-1}^x + \dfrac{b_x}{\gamma^x}(E_{t-1}^x)^2 - \dfrac{qE_{t-1}^x}{\dfrac{1}{\gamma^x}E_{t-1}^x + K} S_t$

Energy for milk production or weight gain can come from sources other than the pasture resource. Producers may choose to feed animals prepared concentrates or grains of various types. In the context of the problem being studied the dairy farmer has a choice of energy sources and can choose to place cows on pasture or feed supplements, and the number of cows on supplements will depend on the herd size, the stocking rate of pasture and the relative profitability of this option. Hence we have;

(15) $E_t^e = (H_t - S_t)X^e$,

where E_t^e is the energy gained from feeding supplements in period t, $(H_t - S_t)$ is the stocking rate receiving supplemental feeding, and X^e is the energy available from the supplements fed in period t to the stock. The energy supplied from supplements is restricted by a constraint on feeding dairy cows too much grain or concentrates as this

could lead to the problem of *acidosis*, or grain poisoning. The recommended maximum grain intake is approximately 60 per cent of a cow's total energy intake, McDonald *et al.*.

From the above discussion it can be seen that the total energy available in any one period, E_t, is the sum of the energy from pastures and the energy from supplements. Hence,

$$(16) \; E_t^s = E_t^{(w,x,y,z)} + E_t^e$$

which can be written in general functional form as;

$$(17) \; E_t^s = E_t^{(w,x,y,z)} (E_{t-1}^{(w,x,y,z)}, \; S_{t-1}) + E_t^e \, (H_t, \; S_t)$$

This equation of motion of energy tells us that the total energy supply is a function of the state variables $E_{t-1}^{(w,x,y,z)}$ and H_t, and the control variables S_{t-1} and S_t.

Energy Demand

The demand for energy by an individual cow is contingent upon the physiological condition and the physical size of the cow. The energy requirements of a cow can be shown on a time line depicting the time from when the cow calves (t=0) until she calves again (t=12) (This is an ideal situation, the actual time between calving for individual cows will vary across a herd);
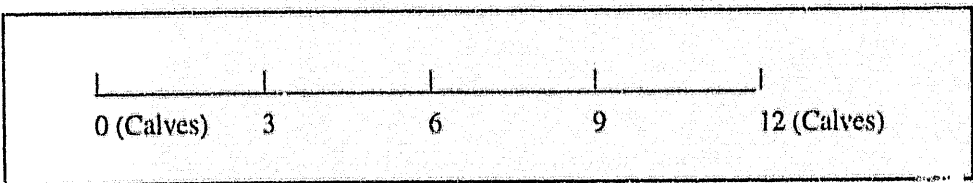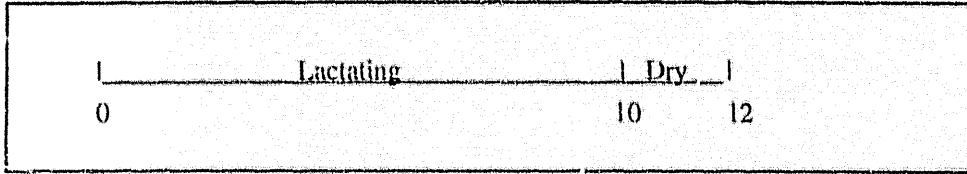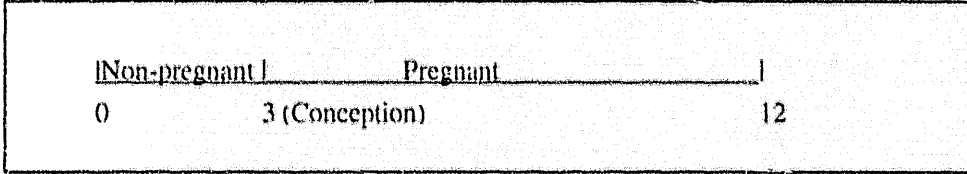
Figure 1: Calving Time Line

0 (Calves)	3	6	9	12 (Calves)

Figure 2: Milk Production Time Line

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|_____Lactating_____|__Dry___|
0                                          10   12
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Figure 3: Pregnancy Status Time Line

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|Non-pregnant |_____Pregnant_____|
0             3 (Conception)                          12
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From t=0 until t=2 the cow's energy requirements are separated into three physiological actions. First, energy is required to maintain bodyweight (PE^m), the next requirement is for energy to produce milk (PE^l), and the last demand for energy is for the weight loss (PE^g) that occurs in the first 3 periods of lactation, (Goodall and McMurray). The demand for energy in the period t=3 until the cow is dried off at t=9 is made up of the energy for maintenance, weight change and lactation as in the period t=0-2, plus the extra energy required for foetal growth (PE^f). In the periods t=10 and 11 the demands for energy are in the form of foetal growth, maintenance and weight change requirements. The demand for energy of a cow in these various physiological stages is as follows:

$$(18) \quad PE_{t-i}^{NP} = PE_{t-i}^{m} + PE_{t-i}^{l} + PE_{t-i}^{g} \qquad i = 0, 1, 2.$$

$$(19) \quad PE_{t-i}^{PL} = PE_{t-i}^{m} + PE_{t-i}^{l} + PE_{t-i}^{g} + PE_{t-i}^{f} \qquad i = 3, 4, ..., 9.$$

$$(20) \quad PE_{t-i}^{PD} = PE_{t-i}^{m} + PE_{t-i}^{g} + PE_{t-i}^{f} \qquad i = 10, 11$$

where PE_{t-i}^{NP} is the per capita energy demand of a non-pregnant lactating animal, PE_{t-i}^{PL} is the per capita demand of a pregnant lactating cow, and PE_{t-i}^{PD} is the per capita demand of a pregnant dry, or non-lactating, cow, (i indicates the number of months after the cow last calved).

There are various relationships between the weight of the cow, the period of time from conception, the amount of weight change, the quality of milk, and the stage of lactation and energy demands. These relationships are specified below;

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$$PE_{t-i}^m = 0.58 W^{0.73}$$

$$PE_{t-i}^f = 1.08e^{0.318t}$$

$$PE_{t-i}^g = \text{energy of weight change} = +34\text{MJ/kg gain}$$
$$= -28\text{MJ/kg loss}$$

$$PE_{t-i}^l = 1.694\,EV_1 * L_{t-i} = \tau\,L_{t-i}$$

where W = average cow's weight in kg, EV_1 = energy value of milk (MJ/litre) = (0.0386BF + 0.0205SNF - 0.236), BF = butterfat content of milk in g/kg, SNF = solids non-fat content of milk also in g/kg, L_{t-i} is the yield per head t-i months after calving (i=0,1...9), and $\tau = 1.694\,EV_1$.

The number of cattle in each physiological category is a function of the breeding herd size at t, which in turn is a function of the number of cows that calved t-i periods previously. In the following equations we are assuming that cows are not pregnant, but lactating for three months after calving, pregnant and producing milk for the next six months, and are pregnant and dry for the remaining three months of the "cow" year. We use α_{TC} here again as the number of cows that lactate in any one period depends on the total number of calves born, dead and alive, rather than just the live calving percentage, as discussed in footnote one. The number of cows in each category can be shown to be as follows:

$$(21)\ H_t^{NP} = \alpha_{TC}\sum_{i=0}^{2}H_{t-i} \qquad \text{The number of non-pregnant lactating cows,}$$

$$(22)\ H_t^{PL} = \alpha_{TC}\sum_{i=3}^{9}H_{t-i} \qquad \text{The number of pregnant lactating cows,}$$

$$(23)\ H_t^{PD} = \alpha_{TC}\sum_{i=10}^{11}H_{t-i} \qquad \text{The number of pregnant dry cows.}$$

Therefore we can determine the total energy requirements of the herd from the numbers in each physiological class and the energy requirements of the animals in these classes. Total energy demanded by each energy category is given by the following equations. Beginning with the lactating non-pregnant category;

$$(24)\ E_t^{NP} = \alpha_{TC} \sum_{i=0}^{2} (PE_{t-i}^{m} + PE_{t-i}^{l} + PE_{t-i}^{g})\ H_{t-i}$$

as PE_{t-i}^{m} and PE_{t-i}^{g} are constants we have,

$$= (PE_t^{m} + PE_t^{g})\ H_t^{NP} + \alpha_{TC} \sum_{i=0}^{2} PE_{t-i}^{l}\ H_{t-i}$$

and by substitution from the definition of PE_{t-i}^{l}, we have the following relationship for the demand for energy by the non-pregnant lactating portion of the herd.

$$(24a)\ E_t^{NP} = (PE_t^{m} + PE_t^{g})\ H_t^{NP} + \alpha_{TC} \sum_{i=1}^{2} \tau\ L_{t-i}\ H_{t-i}$$

Similar methods yield the energy demands for the lactating pregnant portion of the herd and the dry cow herd. These equations are shown below;

$$(25)\ E_t^{PL} = \alpha_{TC} \sum_{i=3}^{9} (PE_{t-i}^{m} + PE_{t-i}^{l} + PE_{t-i}^{g} + PE_{t-i}^{f})\ H_{t-i}$$

$$= (PE_t^{m} + PE_t^{g} + PE_t^{f})H_t^{PL} + \alpha_{TC} \sum_{i=3}^{9} PE_{t-i}^{l}\ H_{t-i}$$

once again as PE_{t-i}^{m}, PE_{t-i}^{g} and PE_t^{f} do not depend on the interval after calving we have,

$$(25a)\ E_t^{PL} = (PE_t^{m} + PE_t^{g} + PE_t^{f})H_t^{PL} + \alpha_{TC} \sum_{i=3}^{9} \tau\ L_{t-i}\ H_{t-i}$$

and

$$(26)\ E_t^{PD} = \alpha_{TC} \sum_{i=10}^{11} (PE_{t-i}^{m} + PE_{t-i}^{g} + PE_{t-i}^{f})\ H_{t-i}$$

as PE_{t-i}^{m}, PE_{t-i}^{g} and PE_t^{f} do not depend on the interval after calving and by rearranging equation 26 and substituting for $\alpha_{TC} \sum_{i=10}^{11} H_{t-i}$ from equation 23 we have;

$$= (PE_t^m + PE_t^g + PE_t^f) \, H_t^{PD}$$

From the above representations for the energy required by the different energy categories within the herd we can derive an equation for the total energy required by the dairy herd in one month.

(27) $E_t^d \quad = E_t^{NP} + E_t^{PL} + E_t^{PD}$

$$= (PE_t^m + PE_t^g)^{NP} H_t^{NP} + \alpha_{TC} \sum_{i=0}^{2} \tau L_{t-i} \, H_{t-i}$$

$$+ (PE_t^m + PE_t^g + PE_t^i)^{PL} H_t^{PL} + \alpha_{TC} \sum_{i=3}^{9} \tau L_{t-i} \, H_{t-i}$$

$$+ (PE_{t-i}^m + PE_{t-i}^g + PE_{t-i}^i)^{PD} H_t^{PD}$$

$$= \eta_t^{NP} H_t^{NP} + \alpha_{TC} \sum_{i=0}^{2} \tau L_{t-i} \, H_{t-i} + \eta_t^{PL} H_t^{PL} + \alpha_{TC} \sum_{i=3}^{9} \tau L_{t-i} \, H_{t-i} + \eta_t^{PD} H_t^{PD}$$

(27a) $\quad = \eta_t^{NP} H_t^{NP} + \eta_t^{PL} H_t^{PL} + \eta_t^{PD} H_t^{PD} + \alpha_{TC} \sum_{i=0}^{9} \tau L_{t-i} \, H_{t-i}$

where $(PE_t^m + PE_t^g)^{NP} = (\eta_t^{NP})$, $(PE_t^m + PE_t^g + PE_t^f)^{PL} = (\eta_t^{PL})$, and $(PE_t^m + PE_t^g + PE_t^f)^{PD} = (\eta_t^{PD})$.

As the dairy herd is made up of groups of cows in various stages of lactation, the total milk production is the sum of the number of cows that calved i periods previously multiplied by the production of these cows. Thus, the total milk production in any one month, Y_t, is the average monthly total calving rate (α_{TC}) multiplied by the number of cows and their lactation yield.

(28) $Y_t = \alpha_{TC} \sum_{i=0}^{9} L_{t-i} \, H_{t-i}$

By substituting Y_t from equation 28 into equation 27a we can define the total demand for energy E_t^d as,

(29) $E_t^d = \eta_t^{NP} H_t^{NP} + \eta_t^{PL} H_t^{PL} + \eta_t^{PD} H_t^{PD} + \tau Y_t$

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and by rearranging equation 29 we can derive the milk production function in terms of energy.

$$(30)\ Y_t = \frac{E_t^u - (\eta_t^{NP}H_t^{NP} + \eta_t^{PL}H_t^{PL} + \eta_t^{PD}H_t^{PD})}{\tau}$$

This last equation provides us with a production function for milk in terms of the energy requirements and the herd size, and tells us that milk will only be produced if there is a surplus of energy, above the requirements for maintenance, weight changes, and foetal growth, available to the dairy herd. An alternative interpretation of equation 30 is that milk production is the difference between energy supply and energy demand weighted by the energy value of milk, τ.

If we assume that all energy supplied in period t is consumed over all demands, that is energy supplied equals energy demanded then we can substitute equation 17 into equation 30 to yield a maximum milk production function such as follows;

$$(31)\ Y_t = \frac{E_t^s - \eta_t^{NP}H_t^{NP} + \eta_t^{PL}H_t^{PL} + \eta_t^{PD}H_t^{PD}}{\tau}$$

This milk production function tells us the maximum amount of milk that could be produced if all available feed is consumed, however this is usually not the case, thus the total yield of milk of the dairy farm will be less than Y_t. From the definition of E_t^s we can show that total milk production is a function of various lags of the state variable H_t, the lag of the pasture energy state variable, $E_{t-1}^{(w,x,y,z)}$, and the control variables S_t and S_{t-1}.

$$(32)\ Y_t = Y_t\ (E_{t-1}^{(w,x,y,z)},\ H_t,\ H_{t-1},\ H_{t-2},\ H_{t-3},\ H_{t-4},\ H_{t-5},\ H_{t-6},\ H_{t-7},\ H_{t-8},\ H_{t-9},\ H_{t-10},\ H_{t-11};\ S_t,\ S_{t-1})$$

or alternatively

$$(32a)\ Y_t = Y_t\ (E_{t-1}^{(w,x,y,z)},\ H_{t-i},\ i=0...,11;\ S_t,\ S_{t-1})$$

Revenue and Cost Functions

Dairy revenue is determined by the amount of quota milk, Y_t^q, and manufacturing milk, Y_t^m, sold, the net prices per litre of these two types of milk, P_t^q and P_t^m, and the costs of

acquiring, or returns from disposing of, milk quota, C_q. P_t^q and P_t^m are prices net of the variable costs of producing a litre of milk. Thus we have;

$$(33) \quad \pi_t^M = P_t^q Y_t^q + P_t^m Y_t^m - C_q (Y_t^q - Y_{t-12}^q)$$

Y_t^q is the amount of quota the farmer holds in period t, and Y_{t-12}^q is the allocation of quota the farmer has in the same period in the previous year, we are assuming that the farmer can purchase quota, for the supply of quota milk in period t, any time between t and t-12. This function, $C_q (Y_t^q - Y_{t-12}^q)$, is the costs or revenue generated from trading in quota. Now we also know that $Y_t = Y_t^m + Y_t^q$, or alternatively $Y_t^m = Y_t - Y_t^q$, hence we can substitute this into equation 33 to get,

$$= P_t^q Y_t^q + P_t^m (Y_t - Y_t^q) - C_q (Y_t^q - Y_{t-12}^q)$$

$$= P_t^q Y_t^q + P_t^m Y_t - P_r^m Y_t^q - C_q (Y_t^q - Y_{t-12}^q)$$

$$(34) \qquad = (P_t^q - P_t^m) Y_t^q + P_t^m Y_t - C_q (Y_t^q - Y_{t-12}^q)$$

and from equation 31a we know that Y_t is a function of the control variables S_t and S_{t-1}. Therefore we can write the milk profit function in terms of the control variables of quota levels and the stocking rate, and the herd state variable H_{t-i}.

$$(35) \quad \pi_t^M = \pi_t^M (Y_t^q, Y_t, S_t, S_{t-1}; H_{t-i}, i=0,...11)$$

The profit generated by the livestock operation in each period depends on the prices received for the types of livestock sold, the number available for sale, which is a function of the herd size, and the weight of the cull cows. We are assuming that the prices in the livestock profit function are net of marketing costs, thus there are no costs explicitly defined in the profit equation.

$$(36) \quad \pi_t^L = [0.5 \, \alpha_1 \, H_t(P_{bc} + \alpha_2 \, P_{hc}) + \pi_t^B]A$$

The first term in this function, $(0.5 \, \alpha_1 \, H_t(P_{bc} + \alpha_2 \, P_{hc}))$, defines the total number of calves sold each month, and is determined by the number of live calves born, α_1, and the number of heifers needed for replacements in the herd, α_2. Not all heifer calves born are kept as replacements for several reasons. First, the number born may exceed the number required to keep the herd in a desired state. Second, some female calves

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may have undesirable physical or genetic characteristics, which require their removal from the herd. P_{bc} and P_{hc} are the prices received, in dollars per head, for bull calves and heifer calves, respectively.

The profits from culling cows, π_t^B, the second term in equation 36, depends on the weight of the cull cows, which will differ across age classes and the number of culls from these classes, which depends on the culling parameter δ_c^j. It will be assumed that the weight of cows within each age group is constant, but differs across age groups. Therefore, the profit from cull cow sales will be:

$$(37) \quad \pi_t^B = P_b \sum_{j=2}^{n} \delta_c^j W^j H_t^j$$

The costs of producing pasture will be a function of the area of the pasture, the type of pasture sown and the annual maintenance costs of the pasture. Hence, the pasture costs will be independent of the stocking rate, and could be treated as a fixed cost in an optimisation problem with stocking rate as the decision variable. As we are assuming a constant area of each type of pasture, A^x, we can define the total pasture production costs, C_t^P, as:

$$(38) \quad C_t^P = \sum_{x=1}^{n} C^x A^x$$

where C^x is the annual cost of pasture production of pasture x, including maintenance, and A^x is the area of each pasture. Hence,

$$(39) \quad A = \sum_{x=1}^{n} A^x$$

and n is the number of different pasture types.

The costs of feeding supplementary energy to the dairy cows will depend on how much extra energy each cow needs, the number of cows on this feeding regime, and the cost per unit of energy. This relationship can be represented as follows;

$$(40) \quad C_t^e = C^e F^e (H_t - S_t) X^e A$$

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where C^e is the cost per kilogram of the supplementary ration, F^e is the weight of the ration that yields one megajoule of energy, (i.e. kg/MJ), the third term $(H_t - S_t)$ is the stocking rate of the herd on supplemental feeding, X^e is the total energy required per head of the stocking rate. A different interpretation of the equation can be that the first terms C^e F^e represent the cost per megajoule of energy and $(H_t - S_t)$ X^e is the total amount of energy required to supplement the stocking rate on feed.

Equation 40 is also a function of the control variables S_t and X^e, and the state variable H_t, which means that the amount and cost of supplementary feeding will be dependent on the solution to the profit maximisation problem.

The Objective Function Revisited

Now that we have explicitly defined each of the components of the objective function, we can now redefine the objective function in a more complete form. By substitution from equations 34, 36, 39, and 40, the objective function of equation 32 can now be rewritten in terms of the state variables H_t and H_{-t} and the control variables Y_t^q, X^e, S_t and S_{-t}:

$$(41) \; \underset{S_t}{\text{MaxNPV}} = \sum_{t=1}^{T} (\frac{1}{(1+r)^t}) \{\pi_t^M (Y_t^q, S_t, S_{-t}, X^e; H_{-t}) + \pi_t^L(H_t)$$
$$- C_t^S(S_t, X^e; H_t) - C_t^P \})$$

This form is now suitable for use in a profit maximisation problem with the objective to maximise the net present value of current and future profits based on the stocking rate decision variable.

Discrete Optimal Control and the Farmer's Problem

From the functions presented in the previous and the current chapters we can see that the farmer's problem is now a function of the state and control variables which is the same functional form as that required for solution of a discrete optimal control problem as discussed in the second section of this chapter. The only requirements now to fully define the problem, as shown on the second section, is to specify the initial and terminal values of the state variables energy and total herd and to specify the equations of motion for the state variables, energy and herd size. The energy equation of motion was defined in chapter three but we will repeat it here for convenience:

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(42) $E_t^s = E_t^{(x,y,z)} (E_{t-1}^{(x,y,z)}, S_{t-1}) + E_t^e (H_t, S_t)$

The equation of motion for the total herd size can be derived from the definition of the herd structure given in equation 5. We know that the equation of motion is the difference between the current and previous period's herd structure, or in other words;

$$(43) \; H_t^T - H_{t-1}^T = [(1 - \alpha_2)(1 - \delta_d^0)(0.5\alpha_1 \sum_{k=0}^{11} H_{t-k}) + (1 - \delta_d^1) \sum_{k=12}^{23} H_{t-k}^0$$

$$+ (1 - \delta_c^1 - \delta_d^1) \sum_{j=2}^{n} (\sum_{k=12}^{23} (H_{t-k}^{j-1})] - [(1 - \alpha_2)(1 - \delta_d^0)(0.5\alpha_1 \sum_{k=0}^{11} H_{(t-1)-k})$$

$$+ (1 - \delta_d^1) \sum_{k=12}^{23} H_{(t-1)-k}^0 + (1 - \delta_c^1 - \delta_d^1) \sum_{j=2}^{n} (\sum_{k=12}^{23} (H_{(t-1)-k}^{j-1})]$$

$$= (1 - \alpha_2)(1 - \delta_d^0)(0.5\alpha_1)(H_t - H_{t-12}) + (1 - \delta_d^1) (H_{t-23}^0 - H_{t-11}^0) +$$

$$(1 - \delta_c^1 - \delta_d^1)(\sum_{k=12}^{23} H_{t-k}^{j-1} - \sum_{k=12}^{23} H_{(t-1)-k}^{j-1})$$

$$= (1 - \alpha_2)(1 - \delta_d^0)(0.5\alpha_1)(H_t - H_{t-12}) + (1 - \delta_d^1) (H_{t-23}^0 - H_{t-11}^0) +$$

$$(1 - \delta_c^1 - \delta_d^1)(\sum_{k=12}^{23} H_{t-k}^{j-1} - \sum_{k=11}^{22} H_{t-k}^{j-1})$$

$$(44) \qquad = (1 - \alpha_2)(1 - \delta_d^0)(0.5\alpha_1)(H_t - H_{t-12}) + (1 - \delta_d^1) (H_{t-23}^0 - H_{t-11}^0) +$$

$$(1 - \delta_c^1 - \delta_d^1)(H_{t-23}^{j-1} - H_{t-11}^{j-1}) \qquad\qquad j = 2,....,n$$

Therefore the equation of motion for herd size is a function of the herd size and structure lagged various periods prior to t. Or in general form;

$$(4.23) \; H_t = H_t(H_{t-1}, H_{t-12}, H_{t-11}^0, H_{t-23}^0, H_{t-11}^{j-1}, H_{t-23}^{j-1})$$

Finally by assuming that the initial and terminal values of the state variables have some real number values E^0 and H^0 and, E^T and H^T respectively we can now define the optimal control problem as follows;

$$(45) \; \underset{S_t}{\text{Max}} \; NPV = \sum_{t=1}^{T} \frac{1}{(1+r)^t} \{ \pi_t^M (Y_t^q, S_t, S_{t-1}, X^e; H_{-t}) + \pi_t^L(H_t)$$

$$- C_t^S(S_t, X^e; H_t) - C_t^P \}$$

$$- \lambda_t^E (E_t^s - [E_t^{(x,y,z)} (E_{t-1}^{(x,y,z)}, S_{-1}) + E_t^e (H_t, S_t)])$$
$$- \lambda_t^H (H_t - H (H_{t-1}, H_{t-12}, H_{t-11}^0, H_{t-23}^0,$$
$$H_{t-11}^{j-1}, H_{t-23}^{j-1}))$$

subject to

$$E_0^0 = E^0 \text{ and } E_T^0 = E^T$$

and

$$H_0^0 = H^0 \text{ and } H_T^0 = H^T$$

where λ_t^E and λ_t^H are the adjoint variables for the energy and herd sub-systems, respectively.

Equation 45 demonstrates that the objective function of the dairy farmer can be specified as a function of the state variables herd size at various lags and energy available from pasture and forage sources, and the control variables, the current and previous stocking rates, and the amount of quota held in the current period.. This specification is ideal as the optimal control model can be completely defined in control and state variables.

Conclusion

In this paper we have developed a discrete optimal control model that provides the framework for a dynamic model of a dairy system. This model is based on the dynamic interactions of all the sub-systems of a dairy farm, these sub-systems being the herd dynamics, and energy demand and supply of the dairy herd. Each of these sub-systems were analysed individually and were shown to be functions of the control and state variables. The herd dynamics equation is merely a function of the herd state variable at various lags, the energy demand system is also a function of the herd state variable, and the energy supply is dependent on the energy and herd state variables and the current and previous levels of stocking rate control variable. We have also demonstrated that the milk production function summarises all the biological processes of the dairy system.

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