YIELD FUTURES, PRICE FUTURES AND AGRICULTURAL LENDING RISKS

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by Calum G. Turvey* and Govindaray N. Nayak

Abstract

This study analyzes the joint hedging decision of a Canadian firm in U. S. based price and yield futures on farm business, financial and total risk. The key results of this study are that jointly hedging price, yield, and foreign exchange can reduce more revenue risk than hedging only with price futures. The results imply that a hedge constructed to provide revenue assurance to U.S. producers or Canadian producers hedging on the U.S. exchange can reduce risk. Assuming that producers are willing to compensate reduced business risk with increased financial risk, the introduction of yield index futures may encourage some farmers to increase financial risk. However, for these producers already highly leveraged the use of yield index futures can lead to lower total risk.

Introduction:

The introduction of yield index futures on the CBOT provides a diverse opportunity for agricultural producers, crop insurers, and marketers to hedge against crop yield and revenue losses. In the original documentation regarding yield futures it was also suggested that lenders would benefit in that the probability of loan losses will be lower as downside agricultural risks are mitigated. However, in the general context of yield and price futures the mechanism by which business risk is reduced is not so straight forward, especially if the futures hedger is an off-shore client, such as a grower in Ontario Canada hedging on the U.S. market.

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The purpose of this paper is to explore the relationship between hedging with future contracts and farm capital structure. From a Canadian perspective the simultaneous hedging of prices and yields is desirable, but is somewhat complicated by foreign exchange risk. Thus we present a simultaneous hedging model of price, yield and foreign exchange. This model can be generalized to the specific hedging decision of a U.S. producer as well. The intent of presenting the simultaneous hedging model is to show how these instruments can be used to effectively reduce business risk. The reduction of business risk is important since the extent of financial risk depends critically on the relationship between business risk and capital structure. Hence, before proceeding to the generalized hedging model we present a simpler model which illustrates how capital structure affects hedging decisions, and in turn how hedging interacts with capital structure to reduce financial risk.

In the next section the use of yield and price futures is discussed in general terms. Next, a simple model of hedging and capital structure is presented. This is followed by a hedging model which includes yield, price, and foreign exchange, and compares incremental risk reduction associated with using the hedge instruments, collectively or individually. Finally the paper presents some concluding comments on the effects of hedging on lending risk.

Background

Traditional hedging theory is often associated with the writings of Keynes (1930) and Hicks (1946) who characterized hedging as the act of transferring risk from risk-averse hedgers to more risk-tolerant speculators. A second theory of hedging considers the profit motives of the hedger. Working (1953) was the first to offer this alternative concept of hedging. The profit maximization concept integrated with traditional theory of risk reduction, led to considering hedging in the context of portfolio management (Leuthold *et al.*, 1989). Johnson (1960) and Stein (1961) were the first to argue within this theoretical framework that hedgers enter the futures market for the same reason an investor enters any market - to attain the highest return for a given risk level. Johnson's formulation of the theory of hedging suggested that hedging and speculative activities are often combined in the actions of a decision maker. Stein (1961) outlined a theory explaining the allocation between hedged and unhedged holding of stocks. Given a utility map relating expected returns and risk, the optimal combination of hedged to unhedged stocks can be found. The optimal hedging proportion provides the maximum attainable level of utility for the hedger.

McKinnon (1967) developed a theory of futures utilization by primary producers as a hedge against production and price risks. His derivation of the optimal hedge follows from the assumption that producers wish to minimize the variance of income, concluding that yield risk management is as important as price risk management in maximizing expected utility. In a recent work, Heifner and Coble (1996) and Vukina, Li, and Holthausen (1996) extended the hedging theory to include both price and yield futures to manage price and yield risks.

The most common rationale for government involvement in agriculture is the incompleteness of contingent markets and the failure of the private sector to provide contingent instruments, such as crop insurance, to farmers. The economic consequence of incomplete contingent markets is a misallocation of resources from their most useful or profitable ends, as a means to reduce risk. However, the introduction of price futures and the introduction of options on price futures in 1984 (Heifner *et al.*, 1993) provided partial contingent markets. Price futures and options provide risk protection, but, as discussed above McKinnon (1967) argues that risk protection is incomplete because of yield risk. To many farmers, crop insurance is available through government organizations at less than actuarial cost. However, as of June 1995, the Chicago Board of Trade (CBOT) has offered yield futures and options contracts. Collectively the private sector now offers a potentially complete contingency market. Providing that yield contracts maintain sufficient liquidity to be effectively priced, price plus yield futures and options can be continued into a variety of revenue assurance derivative instruments.

The first set of crop yield contracts to come on board were Iowa corn yield futures and options (on 2 June, 1995). Shortly after the introduction of the Iowa contract, the CBOT followed up with corn yield contracts for Ohio, Illinois, Indiana, Nebraska, and the U.S. average and has plans to introduce contracts on wheat and soybeans at a later date (McNew, 1996). With crop yield futures, users can lock in a crop yield for a growing season. This is a temporary substitute for a later yield-based commitments (CBOT, 1995), or they can hedge the revenue of a given acreage by combining yield contracts with futures price contracts (Vukina *et al.*, 1996).

The potential for offshore (e.g., Canada) end users hedging yield risk as direct crop /revenue insurance or as crop reinsurance holds promise. However, even though Canadian crop yield risk would be systematically correlated with those of the mid western United States an effective hedge cannot ignore foreign exchange which contribute significantly to basis risk on dollar denominated contracts. For the Canadian hedger (insurer or reinsurer), the problem really entails the simultaneous hedging of commodity prices, crop yields, and currency (foreign exchange).

The Effect of Capital Structure on the Hedging Decision

In the general context of risk measurement, total risk can be defined as the sum of business risk plus financial risk. As posited by Collins, total risk can be measured independently of capital structure by the variability of the return on assets (ROA). The relationship between total risk and business risk is defined by the incremented increase in the variability of equity return due to financial leverage, $\delta = D/A$. The rate of return on assets is determined by cash prices, yields, variable costs, the gains from hedging, debt and the growth rate in asset values.[1] Mathematically this is stated as:

$$\text{(1)} \qquad \tilde{R}_A = \frac{[(Y\tilde{p} - c(Y) + (F_1 - \tilde{f}_2)h + rD]}{A} + g$$

Where p = the stochastic cash price,

Y = the total production,

c(Y) = the cost function increasing in Y such that $c'(Y) > 0$, and $c''(Y) \leq 0$,

F_1 = the initial futures price,

f_2 = the terminal futures price,

h = the bushels of crop hedged,

r = the rate of return on bonds,

D = the dollar value of debt,

g_r = the growth rate in asset values, and

A = the dollar value of assets.

Assuming that the only stochastic variables are the commodity price and the terminal futures price, the variability in the return on assets is given by:

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[1]This model is developed from previous published ideas by Turvey and Baker (1989, 1990). Gaps in the proof can be found in Turvey (1989) and Baker (1989, 1990) and a *caveat* on the approach can be found in the comment by Gaspar *et. al.* (1992) and the reply by Turvey and Baker (1992).

(2) $$\sigma_A^2 = \frac{\sigma_A^2 Y^2 + \sigma_f^2 h^2 + 2Yhp\sigma_p\sigma_f}{A^2}$$

where

σ_p^2 = the variance of profit from the cash position is described fully by the cash price variance,

σ_f^2 = the variance of profit from the hedge position is described fully by the variance of f_2, and

P = the correlation coefficient between cash and futures prices.

Defining leverage, δ as the ratio of debt to assets, the expected utility on utility on equity returns is given by equation **(3)** which assumes that utility is defined by a negative exponential utility function with constant risk aversion measured by λ, and σ_A^2 is normally distributed.

(3) $$E[U] = [\tilde{R}_A - i\delta]\frac{1}{[1-\delta]} - \frac{\lambda}{2}\sigma_A^2[\frac{1}{1-\delta}]^2$$

Substituting equation **(1)** and **(2)** into equation **(3)**, taking the derivative of **(3)** with respect to h and Y, and using Cramer's rule to simultaneously solve for optimal h^* and Y^*, gives:

(4) $$h^* = \frac{A[1-\delta][\sigma_p^2[F_1 - \tilde{f}_2] - (\tilde{p} - c'(Y))p\sigma_p\sigma_f]}{\lambda[\sigma_p^2\sigma_f^2 - p^2\sigma_p^2\sigma_f^2]}$$

(5) $$Y^* = \frac{A[1-\delta][(\tilde{p} - c'(Y))\sigma_f^2 - [F_1 - \tilde{f}_2]p\sigma_p\sigma_f}{\lambda[\sigma_p^2\sigma_f^2 - p^2\sigma_p^2\sigma_f^2]}$$

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(5) $$Y^* = \frac{A[1-\delta][(\tilde{p} - c'(Y))\sigma_f^2 - [F_1 - \tilde{f}_2]p\sigma_p\sigma_f}{\lambda[\sigma_p^2\sigma_f^2 - p^2\sigma_p^2\sigma_f^2]}$$

Equations **(4)** and **(5)** provide the conventional result that optimal hedging h^* will increase with price volatility. In addition, **(4)** and **(6)** reveal the h^* will increase with financial leverage as gains from the hedged position mitigate financial risk, while leverage generally reduces output as increased business risk increases financial risk.

The effect of debt on the optimal hedging position has thus far been viewed in terms of given capital structure described by the debt-to-asset ratio δ. An important consideration, however, is how the optimal debt-to-asset ratio, δ^*, adjusts to hedging. Following the calculus applied to **(4)** it is anticipated that the decrease in business risk due to farmers' use of futures would permit an increase in the amount of debt relative to assets. To see this, we differentiate **(3)** with respect to δ to get Collin's (1985) result for the optimal leverage ratio:

$$\textbf{(6)} \qquad \delta^* = 1 - \frac{\lambda \sigma_A^2}{[E[R_A] - i]}$$

Differentiating **(6)** with respect to λ gives the tautological result that, for R_A greater than i, the amount of debt relative to assets decreases as risk aversion increases. Differentiating **(6)** with respect to h yields:

$$\text{(7)} \qquad \frac{\partial \delta^*}{\partial h} = \frac{\lambda}{[E[R_A] - i]} \frac{\partial \sigma_A^2}{\partial h} + \frac{\lambda \sigma_A^2}{[E[R_A] - i]^2} \frac{\partial E[R_A]}{\partial h}$$

For a hedge that reduces business risk (i.e. $\partial \sigma_A^2 / \partial h < 0$) and increases the expected returns to assets ($\partial E[R_A] / \partial h > 0$) the optimal leverage ratio will increase as hedging increases. This result holds even if $\partial E[R_A] / \partial h$ equals zero and, depending on the amount of risk reduction, even if $\partial E[R_A] / \partial h$ is less than zero. Specifically, it can be stated that leverage will increase with the amount hedged as long as:

$$\frac{\sigma_A^2}{[E[R_A] - i]} \geq \frac{\partial \sigma_A^2}{\partial E[R_A]}$$

Because $\partial\sigma_A^2$ is non-positive, leverage will increase or remain unchanged if $\frac{\sigma_A^2}{[E[R_A]-i]}$ is positive. Hence, if the return on assets is expected to increase with hedging, then more debt can be obtained. However, in many cases the return on assets will decrease. The maximum decrease in the return on assets is given by the strict equality:

$$\partial E[R_A] = \partial\sigma_A^2 \frac{[E[R_A]-i]}{\sigma_A^2}$$

If the return on assets decreases below this value, then the amount of debt relative to assets will also decrease. Thus there is a threshold decrease in the return on assets, which outweighs the benefits of risk reduction; financial risk increases relative to business risk and the optimal amount of debt relative to assets decreases. Under fairly plausible conditions, hedging strategies can cause an increase in the farmer's debt-to-asset ratio. The results corroborate, in part, the risk balancing hypothesis of Gabriel and Baker (1980) and Collins (1985). As business risk is decreased through farmers' use of futures, there may be an induced leverage effect that increases financial risk. But there is the additional possibility that decreases in the return on assets outweigh the benefits of risk reduction. In this situation, financial risk increases as a result of decreased returns, which includes a decrease in financial leverage.

The Simultaneous Hedging of Yield, Price and Foreign Exchange

As discussed earlier the introduction of yield index futures and options provides a new mechanism for hedgers to reduce risk. For the financially leveraged farm reduction in business risk becomes more important with debt, whereas business risk reduction may provide an adverse incentive for producers to increase debt. If a simultaneous price plus yield hedge reduces business risk more than each hedge used individually, or in the case of a simultaneous price, yield and foreign exchange hedge reduces risk more than each individual or paired hedge then one may expect that the use of yield contract will have a high demand among highly leveraged farms, while an increased demand for debt may be observed for high equity farms. Consequently the purpose of this section is to derive the simultaneous hedge ratios and measure the marginal and combined impacts on risk reduction.

The decision problem faced by a Canadian hedger who is an offshore hedger in U.S. futures is different from the U.S. hedger who is a domestic hedger. The offshore hedger must also consider fluctuating exchange rates (Thompson and Bond, 1985) and may have to consider currency futures in addition to price and yield futures.

To reduce the complexity of the model, we assume that an individual farmer has a fixed production opportunity so that his planting decision is made exogenously and is not affected by future prices[2]. A single period decision process is considered. The hedging decision is made in the beginning of the period and the future positions are closed at the end of the period. All the lower case letters indicate variables which are random and upper case letters indicate variables which are known (unless stated otherwise). The notation is defined as follows[3]:

- K is the total acres of a particular crop planted,
- p is the local cash price at the end of the period,
- z is the individual farm yield per acre,

[2]Farmers would normally make their planting decision for a particular crop dependent on expected futures prices (McKinnon, 1967). However, here we wish to isolate the problem of hedging decision in price, yield and currency futures markets given a planting decision.

[3]The ideas and mathematical derivation in this section are drawn from Nayak and Turvey (1997)

- h is the price futures market position,
- F_1 is the futures prices at the beginning of the period,
- f_2 is the futures price at the end of the period,
- g is the yield futures market position,
- Q_1 is the futures yield at the beginning of the period,
- q_2 is the futures yield at the end of the period,
- M is the yield futures contract multiplier[4],
- c is the currency futures market position,
- E_1 is the futures exchange rate at the beginning of the period,
- e_2 is the futures exchange rate at the end of the period, and
- e_r is the spot exchange rate at the end of the period.

The revenue (income) stream of the offshore farmer at the end of the period based on his decision to hedge revenue risk by trading in price, yield and currency futures (expressed in local currency) is given by[5]:

$$\text{(8)} \quad HR = R + h\,(F_1 - f_2)\,e_r + g\,M\,(Q_1 - q_2)\,e_r + c\,(E_1 - e_2)\,.$$

where HR is the hedged revenue, and R = K*p*z is the spot revenue at the end of the period. Both R and HR are stochastic at the beginning of the period, when the decision to hedge is taken. Now by defining $f = F_1 - f_2$, $q = Q_1 - q_2$, $e = E_1 - e_2$, equation 1 can be rewritten as:

$$\text{(9)} \quad HR = R + hfe_r + gMqe_r + ce\,.$$

The variance of the hedged revenue can be written as:

$$\text{(10)} \quad \sigma_{HR}^2 = \sigma_R^2 + h^2\sigma_{fe_r}{}^2 + g^2M^2\sigma_{qe_r}{}^2 + c^2\sigma_e^2 + 2h\sigma_{R,fe_r} + 2gM\sigma_{R,qe_r} + 2c\sigma_{R,e} + 2hgM\sigma_{fe_r,qe_r} + 2hc\sigma_{fe_r,e} + 2gMc\sigma_{qe_r,e}\,,$$

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[4] For example, in case of corn yield contracts, M equals $100 for each bushel per acre settlement. Because of the space limit, the details of the yield futures contract is not given in this paper. For details the readers may refer to CBOT, 1995 or Vukina *et al.*, 1996).

[5] Positions in the futures market (short or long) come from the signs of h, g, and c. If h and g are positive then the hedger is short in price and yield futures and if c is negative then he/she is long in Canadian dollar currency futures.

where,

$$\sigma_R^2 = Var(R); \quad \sigma_{fe_r}{}^2 = Var(fe_r); \quad \sigma_{qe_r}{}^2 = Var(qe_r); \quad \sigma_e^2 = Var(e);$$
$$\sigma_{R,fe_r} = Cov(R,\ fe_r); \quad \sigma_{R,qe_r} = Cov(R,\ qe_r); \quad \sigma_{R,e} = Cov(R,\ e);$$
$$\sigma_{fe_r,qe_r} = Cov(fe_r,\ qe_r); \quad \sigma_{fe_r,e} = Cov(fe_r,\ e); \quad \sigma_{qe_r,e} = Cov(qe_r,\ e).$$

The variances and covariances of these composite terms can be calculated from the means, variances and covariances of the original variables using the expressions provided by Bohrnstedt and Goldberger (1969), Anderson (1958), and Goodman (1960) for the variances and covariances of products of normally distributed variables.

The hedging decision is obtained by minimizing the variance of the hedged revenue in equation **(10)** with respect to h, g and c, the price futures, yield futures and currency futures market positions, respectively[6].

$$\textbf{(11)} \quad h_{pze}^{RM} = \frac{1}{1 - \rho_{fe_r,qe_r}^2 - \rho_{fe_r,e}^2}\left(- \frac{\sigma_{R,fe_r}}{\sigma_{fe_r}^2} + \frac{\sigma_{R,qe_r}\sigma_{fe_r,qe_r}}{\sigma_{fe_r}^2\sigma_{qe_r}^2} + \frac{\sigma_{R,e}\sigma_{fe_r,e}}{\sigma_{fe_r}^2\sigma_e^2} \right)$$

$$\textbf{(12)} \quad g_{pze}^{RM} = \frac{1}{M(1-\rho_{fe_r,qe_r}^2-\rho_{fe_r,e}^2)}\left(-\frac{\sigma_{R,qe_r}}{\sigma_{qe_r}^2} + \frac{\sigma_{R,fe_r}\sigma_{fe_r,qe_r}}{\sigma_{fe_r}^2\sigma_{qe_r}^2} - \frac{\sigma_{R,e}\sigma_{fe_r,qe_r}\sigma_{fe_r,e}}{\sigma_{fe_r}^2\sigma_{qe_r}^2\sigma_e^2} + \frac{\sigma_{R,qe_r}\sigma_{fe_r,e}^2}{\sigma_{fe_r}^2\sigma_{qe_r}^2\sigma_e^2} \right)$$

$$\textbf{(13)} \quad c_{pze}^{RM} = \frac{1}{(1-\rho_{fe_r,qe_r}^2-\rho_{fe_r,e}^2)}\left(-\frac{\sigma_{R,e}}{\sigma_e^2} + \frac{\sigma_{R,fe_r}\sigma_{fe_r,e}}{\sigma_{fe_r}^2\sigma_e^2} - \frac{\sigma_{R,qe_r}\sigma_{fe_r,qe_r}\sigma_{fe_r,e}}{\sigma_{fe_r}^2\sigma_{qe_r}^2\sigma_e^2} + \frac{\sigma_{R,e}\sigma_{fe_r,qe_r}^2}{\sigma_{fe_r}^2\sigma_{qe_r}^2\sigma_e^2} \right)$$

[6] Traditionally, it is assumed that agents are risk averse and hence there is no speculative component in hedge and hedging decisions are arrived at by minimizing the risk as measured by the variance. Benninga, Eldor and Zilcha (1984) have shown that minimum variance hedge is consistent with expected utility maximization under some relatively plausible conditions.

where $\rho^2_{fe_r qe_r} = \dfrac{\sigma^2_{fe_r qe_r}}{\sigma^2_{fe_r}\sigma^2_{qe_r}}$, and $\rho^2_{fe_r e} = \dfrac{\sigma^2_{fe_r e}}{\sigma^2_{fe_r}\sigma^2_{e}}$ are the square of the correlation coefficient between the futures price and the contract underlying yield, adjusted for local currency, and the square of the correlation coefficient between futures price (in local currency) and currency futures, respectively.

The first term in the bracket of the price hedge **(11)** reflects the position in the price futures contracts required to minimize the variability of revenue associated with the fluctuation of local price. The second term results from the presence of yield risk and use of yield futures to hedge the revenue risk. It considers the covariance between price and yield and thus any natural hedge that arises because of the negative correlation of price and yield is explicitly considered. The third term arises from the presence of currency risk in the use of both price and yield futures. It considers the covariance between local cash price and exchange rate and also futures price and exchange rate. The role of these interactive terms will be made clear in the next section by considering a U.S. based firm's hedging decision and comparing it with a Canadian (off-shore) firm's decision. Yield and currency hedges can also be explained in a similar way.

The risk minimizing hedge decision of a Canadian firm, in the presence of exchange rate uncertainty, indicates that the perceived variances and covariances of spot revenue (thereby spot price and yield) and futures prices in the domestic currency are not the same as the variances and covariances of those in U.S. dollar terms. For example, the perceived covariance of revenue and futures price in domestic currency (σ_{R,fe_r}) is the sum of covariance between revenue and exchange rate and revenue and futures price (in U.S. dollar) weighted by the expected futures price and expected exchange rate, respectively. In the case of a U.S. firm, it would be the covariance between revenue and futures price ($\sigma_{R,f}$) only. Consequently, a Canadian firm must view the risk arising from participation in futures market differently than a U.S. firm, and the risk minimizing hedges are derived with the perception that covariances between price and exchange rate and price futures and exchange rate futures are non zero.

Value of Hedging and Risk Reduction

Typically, hedging is valued indirectly by means of hedging effectiveness measures (Ederington, 1979; Cicchetti *et al.*, 1981; Dale, 1981; Hill *et al.*, 1983; Hill and Schneeweis, 1982; Wilson, 1983; Junkus and Lee, 1985). In the case of a minimum-variance hedge, hedging effectiveness is measured by the reduction in the variance of the revenue.

Since an emphasis in this paper is on the role of price, yield and currency futures markets in revenue risk management within a particular crop year, reduction in the variance of the revenue is used to compare different combinations of hedging instruments. The purpose of this section is to evaluate hedging effectiveness by considering several combinations of risks and futures contracts. In what follows, four such cases are presented in order to analyze the possible use of price, yield and currency futures for revenue risk reduction.

Case 1: Use of only price futures

When only price futures is used to manage revenue risk, the risk minimizing hedge is given by equation **(14)**, and the risk minimizing hedged revenue variance is:

$$\textbf{(14)} \qquad (\sigma_{HR_p}^{RM})^2 = \sigma_R^2 + (h_p^{RM})^2 \sigma_{fe_r^2} + 2h_p^{RM} \sigma_{Rfer}$$

Risk reduction (RR_p) from hedging is the difference between the unhedged revenue variance and hedged revenue variance[7]:

$$(15) \qquad RR_p = \sigma_R^2 - (\sigma_{HR_p^{RM}})^2 = \frac{\sigma_{R,fe_r}^2}{\sigma_{fe_r}^2} \geq 0$$

Since RR_p is non-negative, equation **(15)** implies that even with the joint presence of price, yield and currency risk, it is possible to reduce the revenue risk just by trading in price futures.

Case 2: Use of price and yield futures

The risk minimizing hedge ratios from price and yield futures, without trading in currency futures, can be obtained from equations **(11)** and **(12)**. Without hedging in currency futures market, $\sigma_{fe_r,e} = 0$, equations 11 and 12 reduce to:

$$(16) \qquad h_{pz}^{RM} = \frac{1}{1 - \rho_{fe_r,qe_r}^2} \left(- \frac{\sigma_{R,fe_r}}{\sigma_{fe_r}^2} + \frac{\sigma_{R,qe_r}\sigma_{fe_r,qe_r}}{\sigma_{fe_r}^2\sigma_{qe_r}^2} \right)$$

$$(17) \qquad g_{pz}^{RM} = \frac{1}{M\,(1-\rho_{fe_r,qe_r}^2)} \left(- \frac{\sigma_{R,qe_r}}{\sigma_{qe_r}^2} + \frac{\sigma_{R,fe_r}\sigma_{fe_r,qe_r}}{\sigma_{fe_r}^2\sigma_{qe_r}^2} \right) .$$

The risk minimizing hedged revenue variance is:

[7] Contact authors for the derivation..

$$\text{(18)} \quad \left(\sigma_{HR_{pz}}^{RM}\right)^2 = \sigma_R^2 + \left(h_{pz}^{RM}\right)^2\sigma_{fe_r}^2 + \left(g_{pz}^{RM}\right)^2 M^2\sigma_{qe_r}^2 + 2h_{pz}^{RM}\sigma_{R,fe_r} + 2g_{pz}^{RM}M\sigma_{R,qe_r} + 2h_{pz}^{RM}g_{pz}^{RM}M\sigma_{fe_r,qe_r}$$

Risk reduction (RR_z) that can be attributed to yield futures is the difference between the price futures hedged revenue variance, and both price and yield futures hedged revenue variance[8].

$$\text{(19)} \quad RR_z = \left(\sigma_{HR_p}^{RM}\right)^2 - \left(\sigma_{HR_{pz}}^{RM}\right)^2 = \frac{1}{\left(1 - \rho_{fe_r,qe_r}^2\right)\sigma_{qe_r}^2}\left(\frac{\sigma_{R,fe_r}\sigma_{fe_r,qe_r}}{\sigma_{fe_r}^2} - \sigma_{R,qe_r}\right)^2 \geq 0$$

RR_z is positive because ρ_{fe_r,qe_r}^2 is positive and less than 1. This indicates that by trading in yield futures, in addition to trading in price futures, it is possible to reduce more risk than by trading only in price futures. This analytical result is supported by the empirical study of Tirupattur, Hauser and Chaherli (1995). They have shown, by simulating revenue functions under different crop marketing scenarios, that yield futures in conjunction with price futures reduces more risk than using either of the two contracts alone. Vukina, Li and Holthausen (1997) have also come up with a similar analytical proof for a U.S. based firm.

Case 3: Use of price, yield and currency futures

This case is analyzed as an intermediary step in analyzing the revenue risk reduction when currency futures are also used. The risk minimizing hedges in price, yield and currency futures are presented in equations **(11)**, **(12)** and **(13)**, respectively. The risk minimizing hedged variance is:

[8] Contact authors for the derivation.

$$(20)\quad (\sigma_{HR_{pze}}^{RM})^2 = \sigma_R^2 + (h_{pze}^{RM})^2\sigma_{fe_r}^{\ 2} + (g_{pze}^{RM})^2M^2\sigma_{qe_r}^{\ 2} + (c_{pze}^{RM})^2\sigma_e^2 + 2h_{pze}^{RM}\sigma_{R,fe_r} + 2g_{pze}^{RM}M\sigma_{R,qe_r} + 2c_{pze}^{RM}\sigma_{R,e} + 2h_{pze}^{RM}g_{pze}^{RM}M\sigma_{fe_r,qe_r} + 2h_{pze}^{RM}c_{pze}^{RM}\sigma_{fe_r,e}\ .$$

Risk reduction (RR_{ze}) due to use of yield futures and currency futures when all three futures markets are used is the difference between price futures hedged revenue variance (case 1) and the variance of hedging revenue using all three futures contracts[9].

$$(21)\quad RR_{ze} = \left(\sigma_{HR_p}^{RM}\right)^2 - \left(\sigma_{HR_{pze}}^{RM}\right)^2 = \frac{1}{\theta\sigma_{qe_r}^2}\left(\frac{\sigma_{R,fe_r}\sigma_{fe_r,qe_r}}{\sigma_{fe_r}^2} - \sigma_{R,qe_r}\right)^2 + \frac{1}{\theta\sigma_e^2}\left(\frac{\sigma_{R,fe_r}\sigma_{fe_r,e}}{\sigma_{fe_r}^2} - \sigma_{R,e}\right)^2 - \frac{1}{\theta\sigma_{fe_r}^2\sigma_{qe_r}^2\sigma_e^2}\left(\sigma_{R,qe_r}\sigma_{fe_r,e} - \sigma_{R,e}\sigma_{fe_r,qe_r}\right)^2$$

where, $\theta = 1 - \rho_{fe_r,qe_r}^2 - \rho_{fe_r,e}^2$. The derivation in the above equation cannot generally be signed because the sign of θ is ambiguous and the magnitude of the third component relative to the first two components cannot readily be determined. The final outcome depends upon the magnitude of these terms and leaves open the possibility that hedging currency futures could possibly be risk enhancing.

The risk reduction (RR_e) due to currency futures can be isolated by using the risk reduction measures in **(19)** and **(21)**.

$$(22)\quad RR_e = RR_{ze} - RR_z = \left(\frac{1}{\theta} - \frac{1}{\theta_1}\right)\frac{1}{\sigma_{qe_r}^2}\left(\frac{\sigma_{R,fe_r}\sigma_{fe_r,qe_r}}{\sigma_{fe_r}^2} - \sigma_{R,qe_r}\right)^2 + \frac{1}{\theta\sigma_e^2}\left(\frac{\sigma_{R,fe_r}\sigma_{fe_r,e}}{\sigma_{fe_r}^2} - \sigma_{R,e}\right)^2 - \frac{1}{\theta\sigma_{fe_r}^2\sigma_{qe_r}^2\sigma_e^2}\left(\sigma_{R,qe_r}\sigma_{fe_r,e} - \sigma_{R,e}\sigma_{fe_r,qe_r}\right)^2$$

[9] Contact authors for the derivation.

5. Yield and currency futures ($\sigma_{R,fe_r} = 0, \sigma_{fe_r,qe_r} = 0$)

$$RR_{ze} = \frac{\sigma^2_{R,qe_r}}{\sigma^2_{qe_r}} + \frac{\sigma^2_{R,e}}{\sigma^2_{e}} \geq 0$$

5.1 RR attributable to currency futures when both yield and currency futures are used

$$RR_{e} = \frac{\sigma^2_{R,e}}{\sigma^2_{e}} \geq 0$$

* The terms in the parenthesis are the covariances set to zero in equation (23).

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