Risk and Efficiency of Financial Intermediaries

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During the late 1970s and early 1980s the banking environment in the United States changed dramatically as many governmental regulations where removed from the banking sector. Banks where left with more managerial choice over decisions ranging from what to pay on deposits to interstate branching. However, the deregulation was not a pure gain for these institutions as witnessed by the savings and loan debacle. Obviously, financial intermediaries when given a free hand were also free to make mistakes.

According to economic theory two factors combine to determine the long-run viability of any economic enterprise. First, economic survival demands that the firm is efficient both technically and allocatively. Thus, the firm does not waste resources; it chooses the most efficient production process. In addition, the firm uses the right combination of variable inputs and markets the right combination of outputs. The second dimension of survival involves choices under risk. Most real world situations involve decisions based on economic variables that are not fixed or completely knowable at the time the decision is made. The randomness of these variables can have severe consequences for the firm ranging from poor performance to business failure.

Two approaches have been widely used to analyze economic efficiency at the firm level: Duality and nonparametric analysis. Duality has come to denote a statistical approach to analyzing economic efficiency. This approach typically involves estimating some average or frontier profit or cost function. Deviations from that function can then be used to examine technical and allocative efficiency. The nonparametric approach can be thought of as a "revealed technology" approach roughly akin to revealed preference approaches to demand analysis. Specifically, the revealed efficient profit or cost surface is constructed using a linear programming approach. Technical efficiency for a given firm can then be computed by the distance between the frontier formed by the linear combination of the various firms and the observed profit or cost for that firm. Under either approach, however, the research has tended to assume that both input and output prices are known with certainty.

Research into the risk efficiency of a particular portfolio is typically based on the standard Sharpe-Lintner formulation. Specifically, the Sharpe-Lintner formulation shows that given the presence of a risk-free asset the efficient portfolio of risky assets is independent of the investor's risk aversion. Hence, the asset is completely priced through its systematic risk. The risk efficiency of a given portfolio can then be tested by testing whether individual asset returns are equal to a function of the observed portfolio. These studies typically focus on

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rates of return to stock investments. Thus, they do not question whether the firm's assets that generate these returns are efficiently produced. Instead, they rely on the equity market to render decisions on technical efficiency and input allocation.

Under either approach, however, studies of efficiency typically assume away risk while analysis focusing on risk typically relegates efficiency to the same obscurity. Thus, it is entirely possible that some behavior labeled either technical or allocative inefficiency by one branch of the literature may in fact be responses to risk. On the other hand, portfolio results derived from the risk approach may neglect the effect of technical and allocative relationships.

This paper proposes three potential avenues for integrating efficiency literature and risk techniques so that lender efficiency under risk can be examined. In two of the three approaches, the model of portfolio efficiency and modified duality, we assume that input prices are known with certainty. Thus, these models focus our attention on the allocation of output. For financial intermediaries in agriculture this means looking at the choice of lending activities and services. However, the nonparametric approach will allow us to relax the assumption that input prices are known with certainty allowing for the examination of stochastic cost of funds.

Models of Portfolio Efficiency

One basic concept of efficiency from financial economics is the notion of risk efficiency. A portfolio of assets is risk efficient or mean-variance efficient if no portfolio of assets yields the same expected return for a smaller risk or variance. As an extension to financial intermediaries, a loan portfolio and set of services is risk efficient if no other combination of loans and services yields the same expected return for a smaller variance.

Roll (1979) provides an interesting starting place for a discussion of mean-variance efficiency. Specifically, Roll (1979) discusses the case of a hypothetical investor holding a portfolio of risky assets. Roll (1979) notes that a rational investor would not hold a portfolio of assets that was ex ante inefficient according to his subjective beliefs given typical results regarding expected utility. However, due to sampling error it is unlikely that a given portfolio lies exactly on the mean-variance frontier as constructed using ex post data. Thus, we are left with a statistical problem of judging whether the observed portfolio lies far enough from the ex post mean-variance frontier to conclude nonefficiency.

Roll (1979) then proposes two general approaches to determining whether a given portfolio lies on the mean-variance frontier. The first approach examines whether the given portfolio lies on the upward sloping portion of the mean-variance frontier using a brute-force technique. Specifically, for any observed portfolio w_p, Roll proposes that the researcher construct a portfolio $\tilde{w}$ on the mean-variance frontier with the same expected return based on the observed data. Given this portfolio, the test for w_p on the efficient frontier is then

$$\frac{\sqrt{T}(\tilde{w}^{1}\tilde{w} - w_p^{1}w_p)}{(w_p^{1}\tilde{w}\ \tilde{w}^{1}w_p)^{\frac{1}{2}}} \overset{a}{\simeq} N(0,1) \tag{1}$$

Unfortunately this is a large sample result.

The second line of development is the basis for most current approaches to testing for mean-variance efficiency. Specifically, Roll (1979) references Roll (1977) to conclude that if the expected return on a portfolio of assets is an exact positive linear function of the "betas" computed for the investors' portfolio, then the observed portfolio lies on the mean-variance frontier. Thus, testing for mean-variance efficiency becomes much like testing the CAPM formulation. If

$$R = r_z c + (r_p - r_z)\beta \tag{2}$$

where R is the ex ante expected return on the individual assets, r_z is the expected return on the zero beta portfolio for the observed portfolio w_p, c is an unit vector and β is the vector of estimated slop coefficients, then the observed portfolio is mean-variance efficient.

While various procedures have been proposed for testing mean-variance efficiency under the zero-beta formulation (Kandel, Shanken), other procedures based on the existence of a risk-free asset have apparently become more popular. The typical linear return formulation is

$$\begin{aligned} (\tilde{r}_{it} - r_f) &= \alpha_i + \beta_i(\tilde{r}_{pt} - r_f) + \tilde{\epsilon}_{it} \\ E[\tilde{\epsilon}_{it} &= 0] \\ E[\tilde{\epsilon}_{it}\tilde{r}_{pt}] &= 0 \\ \alpha_i &= 0 \end{aligned} \tag{3}$$

where $\tilde{r}_{it}$ is the observed rate of return on investment i in period t, r_f is the risk-free interest rate, α_i is the regression parameter used to test for mean-variance efficiency, β_i is the typical capital beta, $\tilde{r}_{pt}$ is the observed rate of return on portfolio w_p is period t and $\tilde{\epsilon}_{it}$ is the residual. MacKinlay and Richardson give the test for mean-variance efficiency of an observed asset portfolio as

$$\phi_0 = T\hat{\alpha}'\left[\left(1+\frac{\hat{\mu}_p^2}{\hat{\sigma}_p^2}\right)\hat{\Sigma}\right]^{-1}\hat{\alpha} \tag{4}$$

where $\hat{\alpha}$ is the estimated vector of $\alpha_i s$, $\hat{\mu}_p$ is the estimated average rate of return on the portfolio, $\hat{\sigma}_p^2$ is the estimated variance of the rate of return on the portfolio and $\hat{\Sigma}$ is the estimated covariance matrix for asset returns.

As an alternative to the test for mean-variance efficiency in equation (4) MacKinlay and Richardson propose a generalized method of moments estimator. The primary advantage of the generalized method of moments approach to the previous approach is the robustness of the generalized method of moments specification. First, the generalized methods of moments approach is robust with respect to nonnormality. Second, the generalized method of moments estimator can be used to test portfolio efficiency even if the error terms are serially dependent and conditionally herterscedastic.

The general method of moments attempts to select parameters so that the computed moments of the distribution are the same as some outside conditions for the moments. In this case, the conditions are derived from the expected deviation and expected deviation times the rate of return on the portfolio both equal to zero. To implement this concept, assume that there exists a sample of T time series observations, t=1,...T, on N assets, i=1,...N.

$$f_t(\delta) = \begin{bmatrix} \tilde{\epsilon}_{1t}(\alpha_1,\beta_1) \\ \tilde{\epsilon}_{1t}(\alpha_1,\beta_1)\tilde{r}_{pt} \\ \vdots \\ \tilde{\epsilon}_{it}(\alpha_i,\beta_i) \\ \tilde{\epsilon}_{it}(\alpha_i,\beta_i)\tilde{r}_{pt} \\ \vdots \\ \tilde{\epsilon}_{Nt}(\alpha_N,\beta_N) \\ \tilde{\epsilon}_{Nt}(\alpha_N,\beta_N)\tilde{r}_{pt} \end{bmatrix} \tag{5}$$

$$g_T(\delta) = \frac{1}{T}\sum_{t=1}^{T} f_t(\delta).$$

The f_t (δ) can be visualized as a sequence of 2x1 vectors for each asset. The first row of this 2x1 vector is the deviation of asset returns $\epsilon_{it}(\alpha_i, \beta_i) = r_{it} - (\alpha_i + \beta_i r_{pt})$. The second row of each 2x1 vector is the cross product of that residual with the return on the total portfolio. $g_T(\delta)$ is the computed moment of each of these measures. Hence, the generalized method of moments estimator is a procedure to equate both of these moments to zero.

However, the two original moments in equation (5) do not test for portfolio efficiency. The test for efficiency implies that α_i is equal to zero for all i. MacKinlay and Richardson then demonstrate two procedures for testing this restriction based on the generalized method of moment estimator.

The first approach involves imposing linear restriction on the estimation procedure. This test statistic is

$$\phi_1 = T\hat{\alpha}^1 [K[D_T' S_T^{-1} D_T]^{-1} K']^{-1} \hat{\alpha} \overset{a}{\sim} \chi_N^2$$

$$D_T = E\left[\frac{\partial g_T(\delta)}{\partial \delta^1}\right] \tag{6}$$

$$S_T = \sum_{l=-T}^{T} E[f_t(\delta) f_{t-1}(\delta)']$$

where $K = I_N \otimes [1 \quad 0]$ and $R\,\hat{\delta} = 2$. The second approach involves testing the over identifying restrictions.

$$\phi_2 = T g_T(\hat{\beta})' S_T^{-1} g_T(\hat{\beta}) \overset{a}{\sim} \chi_N^2 \tag{7}$$

with S_T defined as in equation (6).

Therefore the financial economics literature has developed several techniques for testing whether a particular investment portfolio is mean-variance efficient. The most natural extension of this research to rural financial intermediaries is to test the loan portfolio of a particular lender for mean-variance efficiency. However, the typical rural bank offers other product lines besides loans. For example, the bank typically offers checking accounts. The difficulty with checking accounts involves the definition of return on assets. Thus, some modifications of the standard approach outlined above may be required.

A second point that needs to be addressed involves the choice of a particular approach for testing mean-variance efficiency in local financial markets. It is obvious that the various techniques range in complexity from Roll's simple brute force technique to the generalized method of moments estimator. First, let us note that Roll expressed some skepticism about the simplest of these techniques. Thus, the choice becomes between one of the linear return models. The choice between the zero-beta and risk-free formulations hinge on the appropriateness of the risk-free rate of return for the financial intermediary in question. If the time increments are sufficiently short, the assumption of a risk-free rate of return does simplify estimation. Finally, the generalized method of moments estimator does not depend on normality or heteroscedasticity.

As a final caveat to this area of research, consider the brute force technique applied to the 1987 Federal Land Bank portfolio across districts. The actual and mean-variance efficient portfolios are given in Table 1. Both portfolios yield an expected real return of 2.48%, however, the observed portfolio yields a variance of .19% compared with the optimum variance of .04%. The difference in the portfolios yielded a test statistic of 2065.44. Hence, we would conclude that the observed portfolio is not mean-variance efficient. What went wrong? As depicted in Table 1, the observed portfolio only includes positive asset holdings while the optimum portfolio includes several short sales. A second problem involves the small sample nature of the data. These results are based on 16 observations for 12 assets. Finally, the choice of the bank's portfolio in this case is suspect. In other words, can the Federal Land Bank really choose its loan portfolios across districts? I think that these three problems, no short sales, small sample sizes and the legitimacy of the portfolio choice, will ultimately pose the greatest problem in application of mean-variance efficiency tests on financial intermediaries.

Modified Duality

A primary procedure for the study of efficiency is the Duality approach. Following Moss et al. this study shows how the typical dual approach used to quantify technical and allocative efficiency can be modified to adjust for stochastic output prices. Specifically, Moss et al. develop the dual of the certainty equivalent function assuming that input prices are known and that output prices are unknown. This certainty equivalent formulation shows how relative prices can be adjusted to provide a risk adjusted picture of allocative inefficiency. In addition, the results also indicate that the level of variable inputs will also be below the deterministic scenario in the multivariate case.

From a rural credit market perspective these results will be useful in understanding the allocation of resources to produce loans and services. For example, if the deterministic duality results indicate that commercial banks are allocatively inefficient because they don't issue as many agricultural loans as mortgage notes, adjusting for relative riskiness of each class may explain the deviation. Similarly, a smaller than profit-maximizing loan portfolio may simply indicate risk aversion on behalf of the bank.

Moss et al. proposed a model for a firm that produces three non-negative outputs $y=(y_1,y_2,y_3)$ using n non-negative inputs $x=(x_1,\ldots x_n)$. Assuming a nonempty, closed, and convex production technology with free disposal, T, and that T satisfies the stated regularity conditions, the technically efficient input-output combinations can be represented by an asymmetric transformation function $y_3=G(y_1,y_2,x)$ where G is assumed to be concave, continuous from above, increasing in x, and decreasing in y_1 and y_2 (Diewert). Given the above technology, we assume that the producer chooses levels of x and y such that all input prices $w=(w_1,\ldots w_n)$ and the price of the third output p_3 are known with certainty. Further we assume that output prices p_1 and p_2 are distributed normal with means μ_1 and μ_2, zero covariance and variances σ_1 and σ_2 respectively.[1] The profits of the firm are then given by

$$\pi(y_1,y_2,x) = p_1y_1+p_2y_2+p_3G(y_1,y_2,x)-w\cdot x. \qquad \textbf{(8)}$$

Thus, profits are distributed normally with a mean

$$\bar{\pi}(y_1,y_2,x) = \mu_1y_1+\mu_2y_2+p_3G(y_1,y_2,x)-w\cdot x \qquad \textbf{(9)}$$

and variance

$$\sigma_{\pi} = y_1^2\sigma_1^2+y_2^2\sigma_2^2. \tag{10}$$

Assuming that the producer is risk averse and that his preferences can be expressed as a negative exponential utility function, the certainty equivalent of the risk gamble can be expressed as

$$R = \bar{\pi}(y_1,y_2,x) - \frac{\rho}{2}\sigma_{\pi}^2 \tag{11}$$

where R is the certainty equivalent and ρ is the risk aversion coefficient (Featherstone and Moss). The producer is assumed to maximize the certainty equivalent of production which is a monotonic transformation of the expected utility.

The first order conditions for maximization of the certainty equivalent are given by

$$\begin{aligned}
\frac{\partial R}{\partial y_1} &= \mu_1 + p_3\frac{\partial G}{\partial y_1} - \rho y_1\sigma_1^2 = 0 \\
\frac{\partial R}{\partial y_2} &= \mu_2 + p_3\frac{\partial G}{\partial y_2} - \rho y_2\sigma_2^2 = 0 \\
\frac{\partial R}{\partial x_i} &= p_3\frac{\partial G}{\partial x_i} - w_i=0
\end{aligned} \tag{12}$$

where the absolute value of $\frac{\partial G}{\partial y_j}$ is the marginal rate of product transformation (RPT) between y_3 and y_j ($j=1,2$). These conditions do not coincide with the usual first order conditions under certainty. Specifically, translating the first order conditions in equation (12) indicate that the optimum rate of product transformation depends on the risk premium

$$\begin{aligned}
RPT_{13} &= \frac{\partial G}{\partial y_1} = -\frac{\mu_1}{p_3} + \rho\frac{y_1\sigma_1^2}{p_3} > \frac{\mu_1}{p_3} \\
RPT_{23} &= \frac{\partial G}{\partial y_2} = -\frac{\mu_2}{p_3} + \rho\frac{y_2\sigma_2^2}{p_3} > \frac{\mu_2}{p_3}
\end{aligned} \tag{13}$$

These results suggest that under risk, the decision maker will produce fewer units of y_1 and y_2 and more units of y_3.

The result in equation (13) means that the measure of allocative output efficiency must be adjusted to account for changes in relative risk. One reformulation of the relative risk problem is to develop two coefficients α_1 and α_2 such that
where α_1 and α_2 represent the relative price risk for the two outputs. Hence, the allocative efficiency between y_1 and y_3, and y_2 and y_3 is dependent upon the relative risk of each activity and the decision maker's risk aversion coefficient. The allocative efficiency between

$$RPT_{13} = \frac{\partial G}{\partial y_1} = -\frac{p_1/\alpha_1}{p_3}$$

(14)

$$RPT_{23} = \frac{\partial G}{\partial y_2} = -\frac{p_2/\alpha_2}{p_3}$$

y_1 and y_2 becomes

$$RPT_{12} = \frac{\frac{\partial G}{\partial y_1}}{\frac{\partial G}{\partial y_2}} = -\frac{\frac{\mu_1}{\alpha_1}}{\frac{\mu_2}{\alpha_2}}. \qquad \textbf{(15)}$$

However, the additional insight into the efficiency of financial intermediaries results from the input choice decisions. Specifically, the optimal choice of inputs now becomes

$$\frac{\mu_1}{\alpha_1}\frac{\partial y_1}{\partial x_i} = w_i \quad (i=1,..n)$$

$$\frac{\mu_2}{\alpha_2}\frac{\partial y_2}{\partial x_i} = w_i \quad (i=1,..n) \qquad \textbf{(16)}$$

$$p_3\frac{\partial y_3}{\partial x_i} = w_i \quad (i=1,...n).$$

Hence, the amount of each input used will be less than under price certainty.

Given the adjusted first order conditions, the next question that arises is how to apply or utilize this construction. One approach would be to substitute the optimal levels of outputs and inputs into the certainty equivalent function to yield a theoretical optimal certainty equivalent for a given vector of prices, risk aversion coefficients, and variances. Observed levels of certainty equivalence could then be used to measure the allocative and technical inefficiency of a particular firm. However, certainty equivalence is an economic unobservable largely due to our inability to quantify the risk aversion coefficient. Thus, a direct test for allocative and technical inefficiency under this framework may rest on the ability to estimate the risk aversion coefficient simultaneously with the technology.

Another approach to quantifying economic inefficiency within this framework is to adjust the normal profit maximization conditions for risk aversion. Specifically, output price risk does not lead to a solution that is off the profit frontier. The producer will still strive to be technically efficient for a different point on the profit surface. In addition, the firm will still attempt to produce that point using the lowest cost combination of inputs. Thus, the critical difference would appear to be quantifying the change in optimal output allocation appropriate due to relative riskiness (i.e., the ratio of the variances or investment beta).

Figure 1 demonstrates the change in allocation along the production possibilities frontier. The point of tangency for ray r_1 is the optimum allocation between outputs y_1 and y_3 taking the expected price of output 1 as deterministic and substituting its mean μ_1. The tangency at ray r_2 is then determined by maximizing the certainty equivalent as depicted in equation (15). The distance between the tangency at ray r_1 and the parallel line drawn through the tangency at ray r_2 is then taken as allocative inefficiency assuming profit maximization. However, as we have demonstrated above the tangency at point r_2 is actually efficient due to relative risk.

Adjustments to Nonparametric Efficiency

Finally, the oldest approach to analyzing efficiency involves nonparametric analysis of input and output combinations. In this approach observed input-output relationships are used to create an efficient frontier. Differences between observed profits or costs and the optimum profit or cost are then used to measure inefficiency.

Following the riskless construction proposed by Färe et al. assume that the kth firm produces a vector of outputs $\mathbf{u}^k$ using a vector of inputs $\mathbf{x}^k$. A nonparametric profit function can then be derived

$$
\begin{aligned}
&Max \quad \sum_{m=1}^{M} r_m u_m - \sum_{i=1}^{l} P_i x_i \\
&\sum_{k=1}^{K} z^k u_m^k \geq u_m \quad m=1,\ldots M \\
&\sum_{k=1}^{K} z^k x_i^k \leq x_i \quad i=1,\ldots l \\
&\sum_{k=1}^{K} z^k x_i^k \leq x_i \quad i=l+1,\ldots N \\
&\sum_{k=1}^{K} z^k = 1
\end{aligned}
\tag{17}
$$

such that the input vector has been partitioned so that x_i $i=1,..l$ are variable inputs and x_i $i=l+1,\ldots N$ are fixed inputs. This maximization problem then depicts the optimal linear combination of firm outputs, or that combination of firm outputs that will maximize profit for a given level of outputs. Färe et al. then introduce an expenditure constraint following Lee and Chambers to analyze the effect of credit constraints on agricultural efficiency.

Extension of the basic approach to Färe et al. to include risk involves replacing the typical profit or cost objective function with an objective that considers risk. For example, it is not difficult to imagine three different objective functions for three stylized agricultural lenders. One type of lender could be a sole proprietorship or closely held corporation such as a commercial bank. This form of organization may be modeled with the certainty equivalent of the negative exponential discussed in the preceding section. Another general class of financial intermediary may be larger, publicly held commercial banks whose objective is to maximize their stock value. Finally, the objective function for a cooperative or government agency may involve minimizing expected cost given service demands.

Assuming negative exponential preferences within a small range around the observed allocations, the certainty equivalent for a given set of outputs and inputs can be expressed as

$$R = [r_1 r_2 \dots r_m p_1 p_2 \dots p_l] \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_m \\ -x_1 \\ -x_2 \\ \vdots \\ -x_l \end{bmatrix} - \frac{\rho}{2} \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_m \\ -x_1 \\ -x_2 \\ \vdots \\ -x_l \end{bmatrix} \Omega \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_m \\ -x_1 \\ -x_2 \\ \vdots \\ -x_l \end{bmatrix} \qquad \textbf{(18)}$$

such that Ω is the variance matrix for input and output prices. Thus, imposing the objective function in equation (18) onto the technology constraints from equation (17) yields a model for nonparametric risk analysis under uncertainty. Specifically,
Like the duality approach, this model could be quantified exactly given knowledge of the decision maker's risk preferences.

The parametrization of equation (19) requires observed input and output levels for agricultural and nonagricultural banks. For example, information on the outputs produced by each bank (i.e., farm loans, car loans, checking accounts) and the inputs used (i.e., sources of capital, buildings, employees) combine to specify the technical tradeoffs. To quantify the objective function requires some idea of the expected input and output prices along with the variance matrix for those prices. In addition, the formulation in equation (18) will require a risk aversion coefficient.

Once the data have been collected, equation (19) can be solved for each firm in the sample. Specifically, the firm's fixed inputs will be used for the right hand sides of equation (19) and the objective function maximized. This maximization will yield the maximum certainty equivalent available to the bank given its fixed inputs. The difference between the two certainty equivalents yields a measure of risk inefficiency for the bank.

$$Max \quad R = [r_1 r_2 ... r_m p_1 p_2 ... p_l] \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_m \\ -x_1 \\ -x_2 \\ \vdots \\ -x_l \end{bmatrix} - \frac{\rho}{2} \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_m \\ -x_1 \\ -x_2 \\ \vdots \\ -x_l \end{bmatrix} \Omega \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_m \\ -x_1 \\ -x_2 \\ \vdots \\ -x_l \end{bmatrix}$$

$$\sum_{k=1}^{K} z^k u_m^k \geq u_m \quad m=1,...M \tag{19}$$

$$\sum_{k=1}^{K} z^k x_i^k \leq x_i \quad i=1,...l$$

$$\sum_{k=1}^{K} z^k x_i^k \leq x_i \quad i=l+1,...N$$

$$\sum_{k=1}^{K} z^k = 1.$$

One alternative would be to utilize the observed input and output allocation to estimate a risk aversion coefficient. For each bank in the sample we will have an observed combination of inputs and outputs. If we assume that the decision maker knows his risk aversion coefficient, all the misallocation becomes technical inefficiency. The process of determining inefficiency would then become a two step process. The first step would be to determine the risk aversion coefficient by minimizing the difference between the actual certainty equivalent and the computed certainty equivalent. Afterwards, the approximated risk aversion coefficient could be used to compute economic inefficiency.

Another possibility is that the firm is publicly traded. Under this scenario we assume that management operates in a way that maximizes the stock value of the firm. Hence, we are particularly interested in the linkage between stock prices and firm behavior. One way to model this linkage is by looking at a firm's stock as an option on future income and valuing that option using some variant of the Black-Scholes pricing model. The difficulty in this scenario is in determining the stochastic process depicting the yield on the lending portfolio over time. One suggestion would be to model this using some generalized autoregressive conditional heterscedasticity model of loan returns.

A second alternative is to develop some form of a capital asset pricing relation based on the choice of lending portfolio and services offered by the bank. The argument for this approach would be that covariance between the return of a particular bank and the market portfolio is determined in part by its choice of business activities. Hence, as those business activities change the covariance should change. This basic approach is reminiscent of the approach used by Chen et al. to explain variations in stock returns based on interactions with the business cycle.

Finally, the cooperative farm credit system and federal lending agencies probably allocate funds neither to maximize expected utility or stock values. In one case, the farmer owned cooperative can be hypothesized to minimize the cost of credit delivery subject to a safety constraint such as minimizing the probability of requiring federal assistance. Federal lending agencies (i.e., Farmer's Home Administration), on the other hand, probably have the most difficult to research objective function because they involve policy.

Conclusions

The problem of modeling economic efficiency under risk is an important, but unsettled problem in economics today. Typically, studies that focus on production efficiency ignore the effect of risk potentially biasing their results towards inefficiency. Financial economic models of risk efficiency, however, typically fail to take basic production relationships into account relegating technical and scale efficiency to a similar obscurity. This paper examined three approaches to integrating risk into models of efficiency. First, we discussed how a typical mean-variance efficiency model from portfolio theory could be applied to financial intermediation. However, in a naive application of one such model to an agricultural banking scenario certain shortcomings of this formulation became evident.

The second framework discussed by this paper was an adjusted duality approach as explained in Moss et al. This approach demonstrates how output allocation and input decisions may be altered by risk in output prices. To correct for this change, Moss et al. suggest that the output price line be rotated to account for relative risk. In addition, these results indicate that the input levels under risk will be lower than under certainty.

Finally, we examined how risk could be integrated into a nonparametric efficiency study. One important observation for this type analysis is that the industry may be comprised of several types of firms with different objective functions. Hence, the objective function for a small closely held corporate bank may be to maximize expected utility while a larger bank with publicly traded equities may wish to maximize the wealth of their shareholders.

Endnotes

1. In actuality the zero covariance assumption simplifies the calculus, but is probably not necessary. In addition, it is probably an inappropriate assumption for financial intermediaries who lend money and provide services in markets that are probably highly interrelated. The previous section of the paper is based, to a large extent, on correlation between investment opportunities. A promising compromise between the zero covariance scenario and a full covariance matrix may involve correlation of individual assets to some common market portfolio as in the single index model proposed by Sharpe.

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Table 1: Observed and Efficient Farm Credit System.

Observed	Mean-Variance Efficient
0.025300	-2.237795
0.058300	-5.841611
0.109400	10.163746
0.076900	-12.540223
0.041000	-5.108758
0.089900	4.573212
0.135300	6.050411
0.103200	-3.468169
0.096600	13.836239
0.074800	-3.640210
0.114800	0.273872
0.074500	8.085711