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Geographical Diversification: An Application of Copula Based CVaR

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Introduction

The foundations of quantitative risk management can be traced back to the Markowitz derived the “optimal portfolio”. For many years this was the basis for financial risk management. In the recent years, quantitative risk management has relaxed some of the restrictions and assumptions imposed by Markowitz’s theory. One example is the assumption of normal or elliptically distributed returns. Recent research has shown that it cannot be assumed that returns are normally distributed(Hatherley and Alcock, 2007). With this being the case, it is not appropriate to solely utilize the risk management tools that rely on normal distributions. The field of financial risk management has adapted these new techniques and furthered the quantitative risk management field. One example of these new techniques is the use of copulas.

The increasing role of copulas in risk management can be seen in the burst of research that has taken place concerning the topic. The quantitative finance research has used it extensively in the past(Clemen and Reilly, 1999, Embrechts, et al., 2002). Just recently has the use of copulas extended into the agricultural economics literature focusing on price-yield distributions and insurance applications (Vedenov, 2008, Zhang, et al., 2007, Zhu, et al., 2008). With this increase in popularity and usage comes the question of how do we as applied economists implement this tool into our modeling? Or even more appropriate would be, “how could producers benefit from or use this statistical technique?” One of the natural extensions of implementing copulas into a modeling framework is portfolio optimization.

The objective of this paper is to examine how the use of copulas could be used in the decision making process. Specifically, this study will implement copulas into a Value-at-Risk framework. The paper will be organized as follows. The role of risk management in agriculture and its evolution will be addressed in the first section. Next, a look at traditional dependency measures used in risk management will be discussed. Following this, a brief overview of the theory of copulas and VaR (Value at Risk) will be presented. This will include a discussion of the common copulas used, and a look at the Gaussian copula used in this paper. Next is a description of the data used for this paper and the model description. The final section will consist of both the results and conclusions.

Evolution of Risk Diversification in Agriculture

The role of risk management in agriculture takes places in many different ways. Technology, contracts, financial instruments, off-farm income, and insurance are all tools that have shown to be widely used and effective in managing risk (Mishra and Lence, 2005). Another tool that has found some usefulness in risk management is that of diversification. The role of diversifying risk was pioneered by Harry Markowitz in 1952. His seminal piece, "Portfolio Selection", laid the foundation for future work in risk management through diversifying a portfolio (Markowitz, 1952). Markowitz's study focused on maximizing expected returns by minimizing variance. He showed that by minimizing variance, a diversified portfolio is preferred to a single asset that is purely maximizing expected returns. His application of the efficiency locus and efficiency frontier laid the foundation for future portfolio diversification problems.

In that same year, an agricultural economist, Earl O. Heady published “Diversification in Resource Allocation and Minimization of Income Variability” in the *Journal of Farm Economics* (Heady, 1952). Heady argued that there were two different aspects of diversification. The first involved a farmer planning under certainty and following the simple economic rule of maximizing profits. The second aspect was that of minimizing variance. This involved working with the unknown variables of price and yield. Although both are separate issues, both must be considered when discussing a diversification strategy, argued Heady. According to Heady, diversification could be accomplished in two ways. The first way would be for a farmer to increase his/her resources. The second way would be for the amount of resources to remain fixed but now the use of the resources would be changed or used for other enterprises.

In 1956, Rudolf J. Freund introduced the concept of risk programming (Freund, 1956). He used the tools of linear programming to illustrate the differences in outcomes when a risk factor was included and not included. Freund applied this to a farm diversification problem in North Carolina. The optimum crop mix changed significantly between the two separate procedures.

An early application of portfolio theory was used when analyzing the levels of grain that should be stored at an elevator (Heifner, 1966). Heifner compared the possible seasonal revenues to returns on stocks. He used the same approach as Markowitz by using the E-V method. Using quadratic programming, the author calculated the efficient frontier of inventory levels. But as noted by the author, the limitation of this approach is now choosing from those efficient levels.

Building on both the work of Freund and Heady, S. R. Johnson published, “*A Re-Examination of the Farm Diversification Problem*”, in 1967 (Johnson, 1967). Johnson analyzed how portfolio theory could be applied to three different cases of the agricultural portfolio problem relying on the separation theorem presented by Tobin (Tobin, 1958). The first case involved allowing for different lease rates. Johnson argued that there must be some risk-free alternative, thus the farmer should be able to lease his/her land. The second case involved looking at long-run diversification problem. Or in other words, the amount of land ownership can vary. The third case involved additional restrictions on the feasible set (such as government payments).

In 1979, further research into the application of portfolio theory to agriculture was examined (Lindon and Brake, 1979). These authors found that many of the assumptions of portfolio were not applicable to agriculture. For example as said by the authors, “Most physical production processes, however, are not linear; asset choices are not always completely divisible; output is rarely certain.” Lindon and Brake tried to correct for these assumptions by extending the theory of asset fixity and asset liquidity to accommodate applications to agriculture.

Early work in this area focused on enterprise diversification at the farm level, but just as the agriculture industry has changed and is changing (Blank, 2002), so are the methods of analyzing risk. Many of these methods have mirrored those that have been applied to the corporate firm (Katchova, 2005). Katchova examined the returns to diversification at the farm level. Relying on ARMS data, she echoed the same statement as Peterson et al. al., that agriculture firms are becoming more diversified. Her research question was whether or not there was a decrease in value based on the level of

diversification. Analyzing a time period from 1999 to 2001, Katchova found that the discount for commodity diversified agricultural producers was 9.4% and for commodity and livestock diversified firms the discount was 5.4% when compared to specialized firms.

The implementation of Value-at-Risk as a risk management tool has been well documented (Basak and Shapiro, 2001, Stambaugh, 1996, Szegö, 2005). VaR has seen a limited number of applications in the agricultural risk management area. As agriculture becomes more industrialized, the effectiveness of measures such as VaR will become more relevant (Boehlje and Lins, 1998). Manfredo and Leuthold applied the VaR framework to a cattle feeding context (Manfredo and Leuthold, 2001). The authors compared alternative estimation methods of the VaR measure. They found that when minimizing the risk of the portfolio, the correlation (dependence) measures were more important to model performance than the volatilities.

The above examples focus on traditional enterprise diversification and looking at the discount of diversification. Another type of diversification is geographical diversification. Geographical diversification is defined as the degree to which a firm's operations in a particular industry is concentrated in one region or spread over many regions. Geographical diversification is not a new method of risk management. The banking industry and real estate investors have used this tool extensively to manage portfolio risk. Liang and Rhoades (1988) studied the impact of geographical diversification in the banking industry (Liang and Rhoades, 1988). The study was motivated by the changes in banking regulations that were taking place in the late 80's. Many banks had begun to expand beyond state borders because regulations had changed

and now banks were able to expand into different regions. The authors argue that geographical diversification will reduce insolvency risk but in turn may increase operating risk through increased management costs and the issues surrounding the acquisition of a new firm. To test this hypothesis, the authors looked at 5,500 banks over the time period of 1976 to 1985. They tested the impact of geographic diversification on overall diversification. They found that indeed geographic diversification reduces insolvency risk but caution must be taken because of the potential increase in operating risk which could offset any potential gains from geographic diversification.

In another study, the impact of geographic diversification was specifically applied to small banks that were acquired by larger banks (geographically diversified)(Rose and Wolken, 1990). The results of the mergers provided no long run advantage for the small banks. But, in the short run, the merger provided some opportunities for entry into new markets.

Industry in general has researched this problem as well. One study examined the differences between sector diversification and geographic diversification (Ehling and Ramos, 2006). The authors examined industries that were within the Eurozone. The authors argued that with the implementation of the Euro, the gains of geographic diversification would be diminished. Using a mean-variance efficiency test (Basak, et al., 2002), the authors tested whether companies were better off by diversifying in the two different manners. The authors found that the results depended on the constraints imposed on the models. If short-selling constraints were imposed, then geographic diversification outperforms the sector diversification. If the problem is unconstrained, then the two strategies are statistically equivalent.

In another article, Kim and Mathur examined the impact of geographical diversification on firm performance (Kim and Mathur, 2007). The author's results suggested that geographical diversification increased operating costs but also increased ROE and ROA when compared to industrially diversified firms. These results suggest that there are some possible gains from geographic diversification. Other research has also looked at the impacts of geographic diversification (Carrieri, et al., 2004, Qian, 2002).

On the agricultural side, the results of geographical diversification have provided conflicting answers. On the one hand, it was shown that by diversifying geographically, a grape grower could increase profits by producing in the US and in Chile (Kreuger, et al., 1999). Another study looked at diversification within Central Illinois (Nartea and Barry, 1994). The authors analyze the costs and returns of geographical diversification in Central Illinois. The question being answered here is whether or not this is a legitimate risk management strategy for individual grain growers in Illinois. The costs that the authors analyzed were the increased transportation costs, monitoring costs, and the loss due to poor machinery coordination. They then compare this to the increased returns received. The authors concluded that there was no realizable gain from diversifying geographically in Central Illinois.

Dependence Measures

Linear correlation is the main measure of dependence in many of the more prolific finance theories. For example, the Capital Asset Pricing Model (CAPM) and the Arbitrage Pricing Theory (APT) rely on correlation (Embrechts, et al., 2002). In addition, Markowitz's portfolio theory relies on correlation as the measure of dependence. In these

cases, correlation is an appropriate measure because of the assumption of multivariate normally distributed returns.

Pearson correlation or linear correlation is defined by:

$$\rho_{ij} = \frac{\text{cov}(X_i, X_j)}{\sqrt{\text{var}(X_i)\text{var}(X_j)}}$$

where $\text{cov}(X_i, X_j) = E(X_i X_j) - E(X_i)E(X_j)$

When the underlying distributions are normal or elliptical, this correlation measure is appropriate. If returns are not normally distributed, estimates of the efficient portfolio using the mean-variance approach could be erroneous (Hatherley and Alcock, 2007).

Some of the issues with using linear correlation as a dependence measure are:

1. If two random variables are independent, the correlation coefficient is zero.

The opposite of this cannot be true, if the two random variables are uncorrelated, that does not imply independence.

2. Correlation is invariant under strictly linear transformations. This is not true under nonlinear strictly increasing transformations.
3. Linear correlation is only defined for finite variances.

The task at hand is to incorporate these alternative structures of dependence¹ into the portfolio model. The impact of using alternative dependence measures has recently begun to be researched heavily in the finance and insurance fields (Patton, 2002). This has opened the door to re-examine portfolio problems using these alternative dependence

¹ Embrechts et al formally defined dependence as: “summarizes the dependence structure between two random variables in one number.”

structures. One method that has been recommended is the use of multivariate copulas (Hennessy and Lapan, 2002).

Copula Theory

Copulas are used to model multivariate distributions. An extensive treatment of copulas can be found in numerous books and research articles (Patton, 2002). For the purpose of this paper, a basic treatment of copulas will be sufficient to lay the foundation for future applications. A copula function is formally defined as (Patton, 2002):

a **copula** is a multivariate cumulative distribution function defined on the n -dimensional unit cube $[0, 1]^n$ with the following properties:

1. The range of $C(u_1, u_2, \dots, u_n)$ is the unit interval $[0, 1]$;
2. $C(u_1, u_2, \dots, u_n) = 0$ if any $u_i = 0$, for $i = 1, 2, \dots, n$.
3. $C(1, \dots, 1, u_i, 1, \dots, 1) = u_i$, for all $u_i \in [0, 1]$

By definition there are an infinite number of copulas that may be generated. In the field of risk management, one family of copulas that has been used extensively is that of the Archimedean copula (Hennessy and Lapan, 2002). The Archimedean has become used extensively for these applications because of the relative ease of calculating the copula.

Sklar's Theorem

Although the application of copulas to statistical problems is relatively recent, the theory behind copulas was developed in 1959 (Sklar 1959). Sklar's Theorem states (Nelsen, 2006):

Let H be a joint distribution function with margins F and G . Then there exists a copula C such that for all x, y in \bar{R} ,

$$H(x,y) = C(F(x),G(y))$$

If F and G are continuous, then the copula function C is unique. If F and G are not continuous, then C is uniquely determined on RanF x RanG. In addition, if C is a copula and F and G are distribution functions, then the function H is a joint distribution function with margins F and G.

In theory, there exist an infinite number of copula functions. Previous research has focused on some families of copula functions (Patton, 2002). The purpose of this research is to not elaborate exclusively on these functions². The Gaussian Copula will be used for this research.

Gaussian Copula

An extension of the multivariate normal distribution is that of a Gaussian Copula. The convenience of the Gaussian copula is that it can be used to model multivariate data that may exhibit non-normal dependencies and fat tails. The Gaussian Copula is formally defined as:

$$C(u_1, \dots, u_n; \Sigma) = \Phi^K(\Phi^{-1}(u_1), \dots, \Phi^{-1}(u_n); \Sigma) \quad (1)$$

The copula function $C(u_n)$ is defined by the standard multivariate normal distribution (Φ^K) and the linear correlation matrix (Σ). When $n = 2$, equation 1 can be rewritten in the following manner.

$$C(u_1, u_2) = \int_{-\infty}^{\Phi^{-1}(u_1)} \int_{-\infty}^{\Phi^{-1}(u_2)} \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left\{-\frac{(r^2 - 2\rho rs + s^2)}{2(1-\rho^2)}\right\} dr ds \quad (2)$$

² For a complete review of copula theory refer to Joe, H. *Multivariate Models and Dependence Concepts*. Monographs on Statistics and Probability. Edited by D. R. Cox. London: Chapman & Hall, 1997, Nelsen, R. B. *An Introduction to Copulas*. 2 ed. Springer Series in Statistics. New York: Springer, 2006.

ρ is the linear correlation between the two variables.

The copula density function is now derived in the following manner.

$$c(\Phi(x_1), \dots, \Phi(x_n)) = \frac{f^{Gaussian}(x_1, \dots, x_n)}{\prod_{i=1}^n f^{Gaussian}(x_i)} \quad (3)$$

Equation (3) can then be rewritten using the definitions of the Gaussian functions.

$$\begin{aligned} & \frac{1}{(2\pi)^{n/2} |\Sigma|^{1/2}} \exp\left(-\frac{1}{2} x' \Sigma^{-1} x\right) \\ = & \frac{1}{\prod_{i=1}^n \sqrt{2\pi}} \exp\left(-\frac{1}{2} x_i^2\right) \end{aligned} \quad (4)$$

Simplifying equation (4) results in the final equation for the copula density function.

$$= \frac{1}{|\Sigma|^{1/2}} \exp\left(-\frac{1}{2} \zeta' (\Sigma^{-1} - I) \zeta\right) \quad (5)$$

VaR Theory

The foundations of VaR can be traced back to 1952 (Roy, 1952). Roy defined in his seminal article the safety first theory. The safety theory included the use of a shortfall constraint that specified a given probability level of disaster. Some forty years later, Philippe Jorion built on that theory and called it VaR (Jorion, 1996). Value at risk can formally be defined as (McNeil, 2005):

Given some confidence level $\alpha \in (0, 1)$. The VaR of a given portfolio is given at the confidence level α is given by the smallest number l such that the probability that the loss L exceeds l is no larger than $(1 - \alpha)$.

$$VaR_\alpha = \inf \{l \in \mathfrak{R} : P(L > l) \leq 1 - \alpha\} = \inf \{l \in \mathfrak{R} : F_L(l) \geq \alpha\}$$

One of the major drawbacks of using VaR is that it is not coherentⁱ when the data is not normally distributed (Stoica, 2006).

An alternative to the VaR that is shown to be coherent is the CVaR (conditional value at risk)(Alexander, et al., 2006, Rockafellar and Uryasev, 2000). The CVaR is defined as:

$$CVaR = \inf_{\alpha} (\alpha + (1 - \beta)^{-1} \mathbb{E}([F_L(l) - \alpha]^+))$$

When the CVaR is continuous, it can be written as the conditional expectation of the loss:

$$CVaR = (1 - \beta)^{-1} \int_{F(l) \geq \alpha} F_L(l) p(S) dS$$

Where $p(S)$ represents the probability density for S .

Data

Three geographically distinct areas were chosen for this application. The locations were chosen based on harvesting windows and distance criteria. Pampa, Texas, Akron, Colorado, and Big Sandy, Montana were chosen for the analysis. All three areas grow Dryland wheat. For this analysis, yields for both spring and fall plantings were used. County level yields and prices were collected from 1973 until 2006 (USDA, NASS). These were used to calculate a gross annual return for each location.

The feasibility of the harvesting windows and the costs associated with each individual enterprise are based off of previously estimated work (Wolfley, 2008). Wolfley estimated farm-level operating costs for each area. These costs estimates were then used to calculate net-annual returns for each farm. These net returns form the basis of the VaR measure. In this case, the VaR measure can be thought of as the probability of a dollar loss per acre.

Results

Using both the traditional VaR and the copula CVaR, a portfolio optimization was performed minimizing the respective risk measures. The results represent the VaR measures if the portfolio is equally weighted between the three production regions (Table 2), the results of the copula CVaR are more conservative than both the traditional VaR and the copula based VaR. This represents the methodology of the CVaR in emphasizing expected shortfall or the lower tail. The copula VaR varies more at the 99% level than the traditional VaR. This is as expected as the Gaussian Copula will capture the tails more than the traditional mean-variance approach. The VaR measures can be interpreted as there is a 5% probability that the per/acre returns for the farm portfolio will be \$4.00 under both the copula-VaR and the traditional VaR. Using the copula-CVaR, there is a 5% probability that losses will exceed \$8.84 per/acre.

Optimizing the portfolio based on minimizing CVaR, the optimal portfolio did not include any production within Texas. The optimal portfolio consisted of 47% in Colorado and 53% in Montana (Table 3). Given these results, there is a 1% chance that there will be \$15.35 loss per acre. At 5% level, there is no expected loss. This can be compared to the VaR measures for all three areas with no diversification (Table 4). At the 5% level for all individual areas, there is an expected loss. For Texas, there is an expected loss of \$34.42 per/acre, Colorado an expected loss of \$8.94, and for Montana an expected loss of \$4.95.

Conclusions

The role of VaR in corporate finance theory and research has been well established. The role of VaR in agricultural finance and agribusiness applications has

seen only a few applications. This paper provides an application for implementing VaR into an agribusiness decision making. As agriculture becomes more industrialized, the role of risk measures such as VaR will become more utilized. In this case it was applied to geographical diversification and also modifying the traditional VaR estimation by incorporating a copula dependence parameter into the VaR estimation. In addition, an alternative risk measure was also calculated, CVaR. The CVaR, unlike VaR, is a coherent risk measure. Thus it does not suffer from many of the shortcomings of the VaR. In this report, geographical diversification relates to producing crops in three different regions. The land portfolio consisted of Dryland wheat production acres in Texas, Colorado, and Montana. Based on the VaR and the CVaR, the portfolio was optimized based on minimizing the expected loss based on historical gross revenues and estimated production costs. The results showed that diversification could be reduced by producing in all three areas. The CVaR optimization consisted of only producing in Montana and Colorado.

The results of this report do not take into consideration the costs that could be involved with geographical diversification. Transportation and management issues are some of the main costs involved with geographical diversification. This could be a topic for further research. In addition, future estimation could incorporate alternative copulas into the model could provide better risk estimates.

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Table 1. Summary Statistics

Statistic	Big Sandy, Montana	Pampa, Texas	Akron, Colorado
Mean	127.32	21.53	101.91
Std. Dev.	87.80	54.80	78.66
Min	-21.72	-37.35	-15.28
Max	354.54	230.36	320.45
CV	68.96	254.47	77.19

Table 2. Equally-Weighted Portfolio Results

%Level	Copula-VaR	Copula-CVaR	Traditional VaR
99%	-14.65	-19.19	-3.42
95%	3.97	-8.84	4.06
90%	18.44	18.59	22.32

Table 3. CVaR Optimal Portfolio Results

%Level	Copula-VaR	CVaR	Traditional VaR
99%	-13.39	-15.35	4.69
95%	9.13	4.50	16.5683
90%	28.50	7.56	46.98

Table 4. VaR for Individual Regions

VaR	Texas	Colorado	Montana
90%	-30.38	1.83	23.55
95%	-34.42	-8.94	-4.95
99%	-36.99	-14.61	-19.95
CVaR			
90%	-34.21	-8.10	21.01
95%	-36.02	-12.65	14.64
99%	-37.18	-14.99	3.24

Figure 1. Annual Wheat Yield

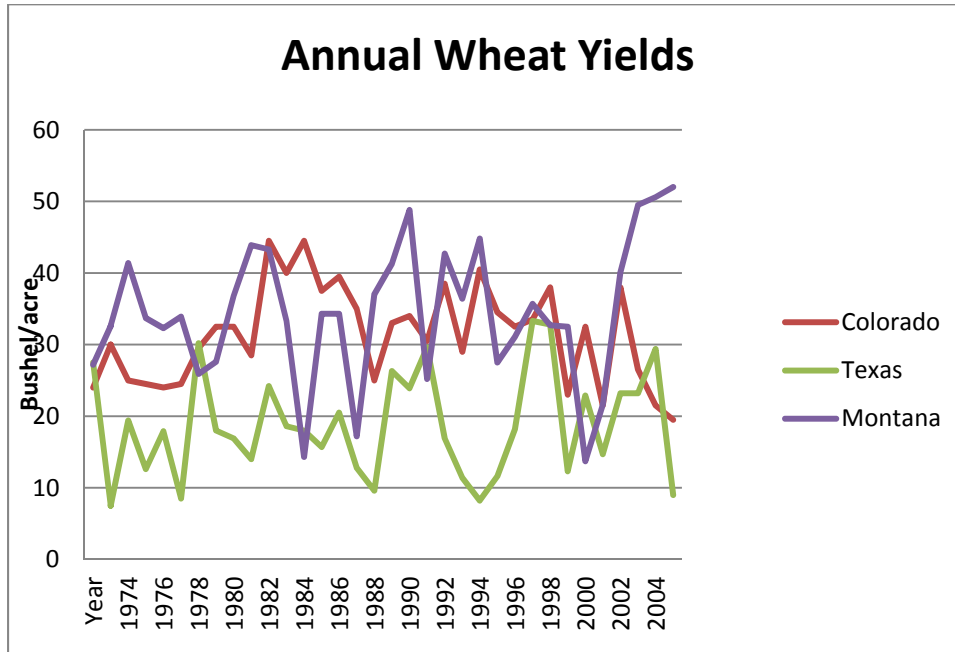


Figure 2. Annual Wheat Returns

