Dept. of
AGRICULTURAL ECONOMICS AND RURAL SOCIOLOGY

CLEMSON UNIVERSITY

# working paper



WP072782 July 1982

# MAXIMIZING PROBABILITY OF RETURN AS A METHOD TO CONSIDER RISK IN FARM PLANNING MODELS

E. H. Kaiser and B. H. Robinson*

clemson university
south carolina agricultural experiment station

department of agricultural economics
and rural sociology
clemson university
clemson, south carolina 29631

WP072782 July 1982

# MAXIMIZING PROBABILITY OF RETURN AS A METHOD TO CONSIDER RISK IN FARM PLANNING MODELS

E. H. Kaiser and B. H. Robinson*

Presented at the annual meeting of the Southern Agricultural Economics Association, Atlanta, Georgia, February 6-9, 1983.

*Assistant Professor and Professor, Department of Agricultural Economics and Rural Sociology, Clemson University.

MAXIMIZING PROBABILITY OF RETURN AS A METHOD

TO CONSIDER RISK IN FARM PLANNING MODELS

Abstract

QP and MOTAD models generate a portfolio of efficient return-risk farm plans. Often there is a great number of farm plans along the efficient frontier for the farmer to consider. If the probability distribution of return of each farm plan is used to determine an efficient return-probability frontier, the number of farm plans the farmer needs to consider may be drastically reduced. This paper presents a linear programming model that generates an efficient return-probability frontier. The farmer can then choose a farm plan based on return and probability of return, rather than on return and an absolute measure of variability.

# MAXIMIZING PROBABILITY OF RETURN AS A METHOD TO CONSIDER RISK IN FARM PLANNING MODELS

Most farm management decisions are made under conditions of risk. Quadratic programming models and minimization of total absolute deviation (MOTAD) models have often been used to consider risk in farm planning [3, 4, 5, 6, 7, 9, 10]. Both models generate a portfolio of efficient return-risk farm plans. The farmer must then choose the one farm plan on the efficient return-risk frontier that he believes will maximize his utility.

Often there is a great number of farm plans along the efficient return-risk frontier. If each farm plan specifies a production, marketing, investment and financing plan, the farmer is presented with a large amount of information. However, if the probability distribution of return is calculated for each farm plan and only those farm plans that have the highest probability of generating specific returns are presented, the number of farm plans the farmer needs to consider may be drastically reduced.

## Theoretical Models

Quadratic programming assumes the farmer orders his preferences among alternative farm plans only on the basis of expected income, E, and the associated income variance, V. Quadratic programming further assumes that the farmer is a risk averter. Given these assumptions, the rational farmer restricts his choices to those farm plans that have a minimum variance, given an expected level of income. Such farm plans are called efficient E,V farm plans and define an efficient E,V frontier over the set of all feasible farm plans (segment OM in Figure 1). The point of tangency between the efficient E,V frontier and an indifference curve defines the farm plan that will maximize the farmer's utility.

Computer codes for solving quadratic programming models have practical limits as to size, can be expensive to solve, and may not always be available. Hazell developed a linear alternative to quadratic programming which can be solved with conventional linear programming codes [5]. Hazell notes that the quadratic programming model requires _a priori_ knowledge of the expected returns for each activity and the corresponding variances and covariances. If these parameters are unknown, it is necessary to obtain estimates using time series or cross-sectional data of observed returns.

Hazell then notes that if sample data are available to estimate the variance of returns, the mean absolute income deviation may be defined as:

$$A = 1/s \sum_{h=1}^{s} \left| \sum_{j=1}^{n} (c_{hj} - g_j)\, X_j \right|$$

where h=1, 2,...,s denotes s observations in a random sample of returns, $c_{hj}$, and $g_j$ is the sample mean return of activity j measured as

$$1/s \sum_{h=1}^{s} c_{hj}.$$

A is an unbiased estimator of the population mean absolute income deviation. Hazell suggests that by using A as a measure of risk it is reasonable to consider E (expected income) and A (absolute income deviation) as the crucial parameters in the selection of a farm plan, and to define efficient farm plans as those having minimum absolute income deviations for a given expected income.

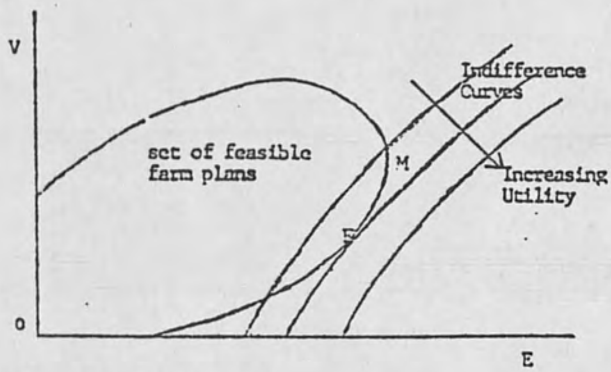


Figure 1. The optimal E,V farm plan.

Table 1. Tableau of a MOTAD[1/] model.

| | Row Type[2/] | RHS[3/] | $X_j$[4/] | Gross Returns | Cost | $TP_k$[5/] | Disposable Income | $Y_1$ | $Y_2$ | ... | $Y_s$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Resource i | L | $b_i$ | $a_{ij}$ | | | | | | | | |
| Gross return account | E | o | $-g_j$ | 1 | | | | | | | |
| Cost account | E | o | $-c_j$ | | 1 | | | | | | |
| Taxable income account | E | o | | -1 | 1 | k | | | | | |
| Tax accounting equality | E | 1 | | | | 1 | | | | | |
| Disposable income account | E | o | | | | $-DI_k$ | 1 | | | | |
| Return | G | $\lambda$ | | | | | 1 | | | | |
| $t_1$ | G | o | $d_{1j}$ | | | | | 1 | | | |
| $t_2$ | G | o | $d_{2j}$ | | | | | | 1 | | |
| . | . | . | . | | | | | | | . | |
| . | . | . | . | | | | | | | . | |
| . | . | . | . | | | | | | | . | |
| $t_s$ | G | o | $d_{sj}$ | | | | | | | | 1 |
| Objective function | N | o | | | | | | 1 | 1 | | 1 |

1/MOTAD refers to Minimization of Total Absolute Deviations.

2/L = less than or equal to; E = equal to; G = greater than or equal to; and N = nonconstrained.

3/RHS = right-hand-side; $b_i$ = level of resource i available; $\lambda$ = return required, which is parameterized from 0 to unbounded.

4/$X_{jn}$ = level of production crop activity j; $a_{ij}$ = amount of resource i used by one unit of crop production activity j; $g_j$ = gross return of one unit crop production activity j; $c_k$ = cost of one unit of crop production activity j; $d_{hj}$ = deviation of observation year h's gross return from estimated trend gross return for crop production activity j.

5/$TP_k$ = level of tax paying activity k; k = amount of taxable income used by tax paying activity k; $DI_k$ = amount of disposable income provided through tax paying activity k, $DI_k$ = k - taxes that must be paid on taxable income k.

The E,A criterion has important advantages over the E,V criterion in that it leads to a linear programming model to derive efficient farm plans. Hazell converts A to a legitimate linear programming objective function by minimizing the absolute value of the negative income deviations. The Hazell model, which has been well documented in several sources [5,7], generates a sequence of solutions of increasing expected return and mean absolute income deviation until the maximum expected return under the resource constraints has been attained. In this manner the efficient E,A frontier is generated. Since this model minimizes A, Hazell refers to it as the Minimization of Total Absolute Deviations (MOTAD) model.

Neither E,V analysis nor E,A analysis necessarily generates the farm plan which maximizes an individual farmer's utility. Rather, each of these methods of considering risk in farm management problems produces a set of minimum risk (variance or absolute deviation) farm plans for given levels of return. The particular plan among this set that will maximize an individual farmer's utility depends upon the farmer's unique utility function (i.e., risk preference). Since farmers have different utility functions, each farmer must choose the farm plan that maximizes his utility. The efficient E,V or E,A curve can be presented and the farm plans associated with several points on the curve can be described. An individual farmer must then choose the farm plan that he believes will maximize his utility.

## Empirical MOTAD Model

A MOTAD model was developed to generate an efficient E,A frontier for an owner-operator with 450 acres of cropland and a choice of eight crop plans [7]. The tableau of this basic MOTAD model is presented in

Table 1. Crop production activities use resources, add to the gross return accounting row, add to the cost accounting row, and have entries in the income deviation accounting rows. The gross return activity transfers the summation of gross returns generated by the crop production activities to the taxable income accounting row. The level of the cost activity is the summation of costs generated by the crop production activities, and this value is subtracted from the gross returns in the taxable income accounting row. The tax paying activities represent a progressive income tax function that is built into the model [11]. Use of an income tax function permits consideration of the effects of alternative decisions on after-tax or disposable income, which more accurately describes the effects of alternative decisions than does before-tax income.

Each $y_h$ activity (h=1,2,...,s) is the absolute value of the summation of the gross return deviations in observation year h, if that summation is negative. If the summation of the gross return deviations in observation year h is positive, then the level of activity $y_h$ will be zero. The absolute value of the summation of gross return deviations in observation year h is then transferred to the objective function, so that the objective function is the summation of the absolute values of the negative deviations over all observation years.

The efficient E,A curve generated by this MOTAD model is presented in Figure 2. The farm plans associated with the points on the curve are given in Table 2. The maximum disposable income is generated by the conventional linear programming solution which generated a disposable income of $16,173 (point MP16). This farm plan specified producing corn on all 450 acres. The lowest return-risk point that specified a farm

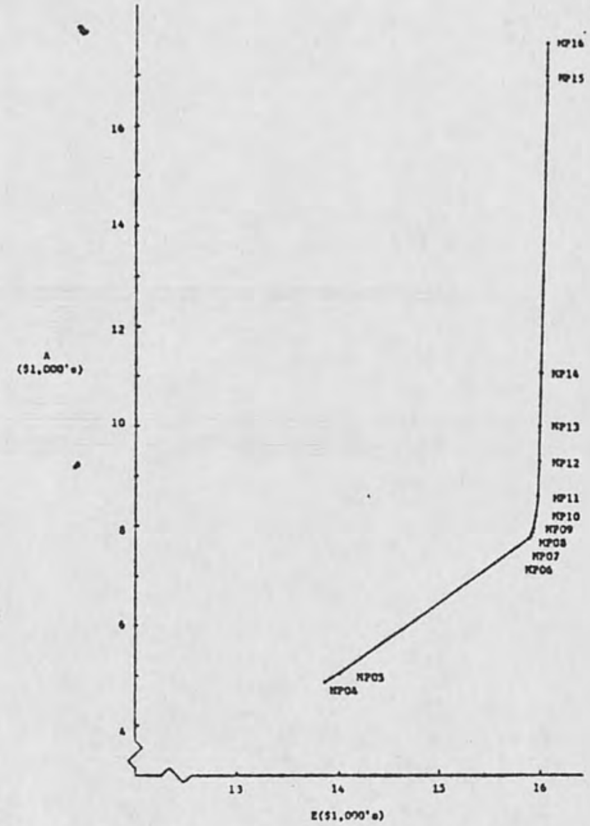


Figure 2. The Efficient E,A Frontier.

Table 2. Farm plans associated with points on efficient E,A frontier shown in Figure 2.

| (1) Farm Plan | (2) Expected Disposable Income | (3) Absolute Deviation | (4) Corn | (5) Soybeans | (6) Soybeans After Wheat | (7) Idle Land | (8) Marginal Risk (ΔA/ΔDI) | (9) ΔCorn ac/ $ of ΔDI | (10) ΔSoybeans ac/ $ of ΔDI | (11) ΔSoybeans -Wheat ac/ $ of ΔDI | (12) Estimated Standard Deviation |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | ($) | ($) | (acres) | (acres) | (acres) | (acres) | | (acres) | (acres) | (acres) | ($) |
| MP01 | 4,000 | 1,301 | | 119.8 | | 330.2 | | | | | 1,743 |
| | | | | | | | .34 | | .03 | | |
| MP02 | 8,000 | 2,661 | | 245.0 | | 205.0 | | | | | 3,566 |
| | | | | | | | .37 | | .03 | | |
| MP03 | 12,000 | 4,159 | | 383.0 | | 67.0 | | | | | 5,573 |
| | | | | | | | .39 | | .04 | | |
| MP04 | 13,873 | 4,887 | | 450.0 | | | | | | | 6,549 |
| | | | | | | | 1.43 | | -.23 | .23 | |
| MP05 | 14,000 | 5,067 | | 422.2 | 27.8 | | | | | | 6,790 |
| | | | | | | | 1.43 | | -.22 | .22 | |
| MP06 | 15,880 | 7,758 | | 11.9 | 438.1 | | | | | | 10,396 |
| | | | | | | | 1.60 | .03 | -.22 | .19 | |
| MP07 | 15,900 | 7,790 | 0.5 | 7.6 | 441.9 | | | | | | 10,439 |
| | | | | | | | 1.70 | .05 | -.22 | .17 | |
| MP08 | 15,920 | 7,825 | 1.5 | 3.3 | 445.2 | | | | | | 10,486 |
| | | | | | | | 4.40 | .44 | -.17 | -.27 | |
| MP9 | 15,940 | 7,913 | 10.2 | | 439.8 | | | | | | 10,603 |
| | | | | | | | 14.30 | 1.89 | | -1.89 | |
| MP10 | 15,960 | 8,199 | 47.9 | | 402.1 | | | | | | 10,987 |
| | | | | | | | 18.05 | 1.89 | | -1.89 | |
| MP11 | 15,980 | 8,560 | 85.7 | | 364.3 | | | | | | 11,470 |
| | | | | | | | 35.80 | 1.89 | | -1.89 | |
| MP12 | 16,000 | 9,276 | 123.4 | | 326.6 | | | | | | 12,430 |
| | | | | | | | 35.90 | 1.89 | | -1.89 | |
| MP13 | 16,020 | 9,994 | 161.2 | | 288.8 | | | | | | 13,392 |
| | | | | | | | 51.45 | 1.89 | | -1.89 | |
| MP14 | 16,040 | 11,023 | 198.9 | | 251.1 | | | | | | 14,771 |
| | | | | | | | 51.45 | 1.89 | | -1.89 | |
| MP15 | 16,160 | 17,197 | 425.3 | | 24.7 | | | | | | 23,044 |
| | | | | | | | 51.69 | 1.90 | | -1.90 | |
| MP16 | 16,173 | 17,869 | 450.0 | | | | | | | | 23,944 |

plan that used all available land was point MP04. This farm plan specified producing soybeans on all 450 acres and generated a disposable income of $13,873.

There are an infinite number of farm plans on the efficient E,A curve between point MP04 and MP16. While moving up the efficient E,A frontier results in increased disposable income, it also results in increased income variability. Column 8 of Table 2 shows the marginal risk incurred as disposable income is increased. Columns 9, 10, and 11 of Table 2 show the changes in acreages of corn, soybeans, and soybeans after wheat necessary to increase disposal income by $1 as the farmer moves up the efficient E,A curve to higher return-risk points. Given the information in Table 2, the farmer must choose the one farm plan that he believes will maximize his utility.

## Maximizing the Probability of Return

Estimating the probabilities associated with the returns generated from various farm plans is an alternative to either the quadratic programming or MOTAD models for incorporating risk into decision criteria. This approach differs from QP or MOTAD in that it seeks to maximize returns subject to specified probability level rather than minimizing income deviations or variance. Yet, both approaches are dependent on standard variability estimates.

While producers are cognizant of explanations dealing with estimated deviations in income for a specific farm plan, they frequently can better relate to estimates of the probability or likelihood of obtaining a specified income level from a specific farm plan. Thus, it seems plausible that estimates of the probability of the income levels generated from specific farm plans would provide critical information to the producer's decision process.

Two approaches are used in this paper to estimate the probabilities associated with the income generated from alternative farm plans. The first is based on estimates of the standard deviation in the incomes generated for alternative farm plans from the MOTAD analysis and calculating cumulative probability distribution functions for the farm plans on the E,A frontier. The second approach incorporates a probability function directly into a linear programming model with an objective function which maximizes returns subject to exogenously determined probability levels. Both approaches are used to generate an efficient Return-Probability (R,P) curve.

Cumulative Probability Distribution Function

The standard deviation of disposable income of each farm plan can be estimated using the absolute deviation by the following formula [5]:

$$\hat{\sigma}_{DI} = A_{DI} \cdot \left[\frac{s.\pi}{2(s-1)}\right]^{1/2}$$

where: $\hat{\sigma}_{DI}$ = estimated standard deviation of disposable income,

$A_{DI}$ = absolute deviation of disposable income,

$s$ = number of observations in time series,

$\pi = 3.14286$.

The estimated standard deviation of disposable income of each farm plan on the efficient E,A frontier is given in Table 2.

With the expected disposable income and the estimated standard deviation of disposable income of each farm plan, the cumulative distribution function of disposable income for each farm plan can be deter-

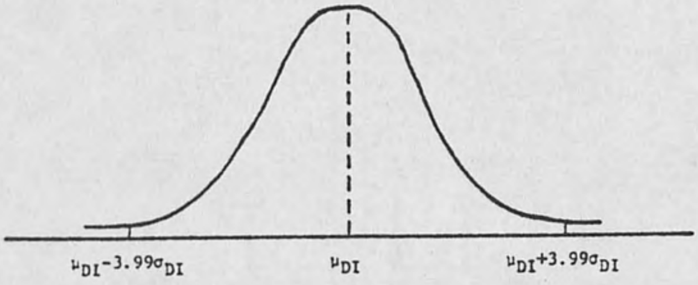


Figure 3. Probability density function of disposable income.

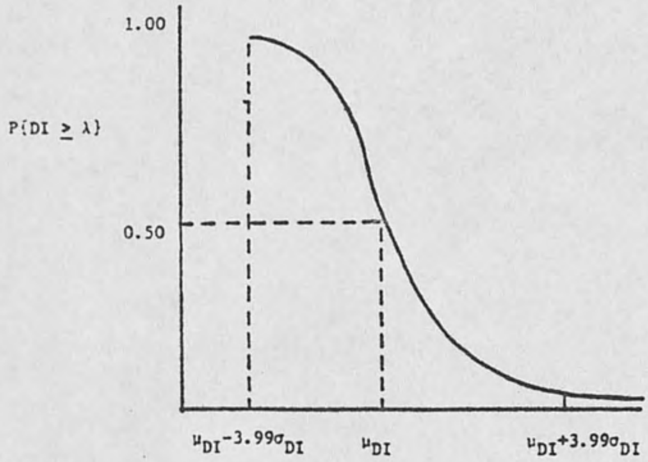


Figure 4. Cumulative distribution of disposable income.

Table 3. Various $P\{DI \geq \lambda\}$ and corresponding Z values.

| $p\{DI \geq \lambda\}$ | Values of Z |
|---|---|
| 1.00 | -3.999 |
| 0.95 | -1.645 |
| 0.90 | -1.282 |
| 0.85 | -1.037 |
| 0.80 | -0.842 |
| 0.75 | -0.675 |
| 0.70 | -0.524 |
| 0.65 | -0.385 |
| 0.60 | -0.253 |
| 0.55 | -0.126 |
| 0.50 | 0.000 |
| 0.45 | 0.126 |
| 0.40 | 0.253 |
| 0.35 | 0.385 |
| 0.30 | 0.524 |
| 0.25 | 0.675 |
| 0.20 | 0.842 |
| 0.15 | 1.037 |
| 0.10 | 1.282 |
| 0.05 | 1.645 |
| 0.00 | 3.999 |

mined. The probability density function of disposable income for a particular farm plan with expected disposable income of $\mu_{DI}$ and an estimated standard deviation of disposable income of $\sigma_{DI}$ is shown in Figure 3. The cumulative distribution function of disposable income may be defined as either $P\{DI \leq \lambda\}$ or $P\{DI \geq \lambda\}$, where $\lambda$ is some particular value of the random variable disposal income, DI. [1] In this paper cumulative distribution functions will be defined as $P\{DI \geq \lambda\}$ because it seems reasonable that farm managers are interested in knowing the probability of disposable income being above some particular value. Figure 4 shows a general cumulative distribution function of disposable income corresponding to the probability density function shown in Figure 3.

Any normal distribution of disposable income can be transformed to the standard normal distribution by the following equation:

$$Z = \frac{\lambda - \mu_{DI}}{\sigma_{DI}}$$

The standard normal distribution is normally distributed with a mean of 0 and a standard deviation of 1. This transformation allows us to compute probabilities, irrespective of the values of $\mu_{DI}$ and $\sigma_{DI}$, from a single probability table for the standard normal distribution.[2] For any particular value of the probability $P\{DI \geq \lambda\}$, $\lambda$ can be determined using the following equation:

$$\lambda = \mu_{DI} + Z\ \sigma_{DI}$$

where Z is the value of the standard normal variable corresponding to the given probability $P\{DI \geq \lambda\}$. Table 3 gives the various $P\{DI \geq \lambda\}$ values used in this paper to determine the cumulative distribution function of disposable income and the Z values corresponding to these probability levels.

In this manner the cumulative distribution function of disposable income for each farm plan specified in Table 2 was determined. These cumulative distribution functions are given in Table 4. Considering these distributions, the farmer would restrict his choice among farm plans MP04, MP08, MP09 and MP16. These farm plans define the efficient return-probability of return curve (R,P) that is graphed in Figure 5. MP04 would produce the highest possible income at probabilities of .70 through .95. MP08 would generate the highest possible income at probabilities of .60 and .65, and MP09 would generate the highest possible income at a probability of .55. Finally, MP16 would produce the highest possible income levels at probabilities of .50 and lower.

Presenting only farm plans MP04, MP08, MP09, and MP16, along with the corresponding cumulative probability distributions, would allow the farmer to choose the farm plan that would maximize his utility. Thus, the farmer would be presented with less information to consider than if the results of the MOTAD analysis (Table 2) were presented.

<u>Linear Programming Model to Determine Efficient Return-Probability Curve</u>

It is possible to determine the farm plans on the efficient return-probability curve without generating the efficient E,A frontier and determining the cumulative probability distribution for each farm plan on the efficient E,A frontier. The following linear program maximizes return subject to a probability level of receiving that return.

maximize: $\lambda$

subject to: $\lambda = \hat{\mu}_{DI} + Z\,\hat{\sigma}_{DI}$

$Z = f(P\{DI \geq \lambda\} = p) \quad (p = 0.0 \text{ to } 1.0)$

Table 4. Cumulative probability distribution functions for disposable income of farm plans on efficient E,A frontier.

| Probability that disposable income will be above given level: | MP01 | MP02 | MP03 | MP04 | MP05 | MP06 | MP07 | MP08 | MP09 | MP10 | MP11 | MP12 | MP13 | MP14 | MP15 | MP16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1.00 | -1,229 | -2,698 | -4,719 | -5,776 | -6,376 | -15,308 | -15,417 | -15,538 | -15,869 | -17,001 | -18,430 | -21,290 | -24,156 | -28,273 | -52,972 | -55,659 |
| 0.95 | 1,133 | 2,134 | 2,832 | 3,100 | 2,827 | -1,221 | -1,272 | -1,329 | -1,502 | -2,114 | -2,888 | -4,447 | -6,010 | -8,258 | -21,747 | -23,215 |
| 0.90 | 1,765 | 3,428 | 4,855 | 5,477 | 5,293 | 2,552 | 2,517 | 2,477 | 2,347 | 1,875 | 1,275 | 65 | -1,149 | -2,896 | -13,382 | -14,523 |
| 0.85 | 2,194 | 4,306 | 6,226 | 7,088 | 6,963 | 5,110 | 5,085 | 5,056 | 4,955 | 4,577 | 4,097 | 3,123 | 2,146 | 737 | -7,714 | -8,633 |
| 0.80 | 2,532 | 4,997 | 7,308 | 8,359 | 8,281 | 7,127 | 7,110 | 7,091 | 7,012 | 6,709 | 6,322 | 5,534 | 4,744 | 3,603 | -3,243 | -3,900 |
| 0.75 | 2,825 | 5,597 | 8,244 | 9,459 | 9,422 | 8,873 | 8,864 | 8,852 | 8,794 | 8,555 | 8,249 | 7,622 | 6,994 | 6,084 | 628 | 35 |
| 0.70 | 3,087 | 6,131 | 9,080 | 10,441 | 10,441 | 10,432 | 10,430 | 10,425 | 10,385 | 10,203 | 9,970 | 9,487 | 9,003 | 8,300 | 4,085 | 3,626 |
| 0.65 | 3,329 | 6,627 | 9,854 | 11,352 | 11,385 | 11,878 | 11,881 | 11,883 | 11,858 | 11,730 | 11,564 | 11,214 | 11,864 | 10,353 | 7,288 | 6,955 |
| 0.60 | 3,559 | 7,098 | 10,590 | 12,216 | 12,282 | 13,250 | 13,259 | 13,267 | 13,257 | 13,180 | 13,078 | 12,955 | 12,632 | 12,303 | 10,330 | 10,115 |
| 0.55 | 3,780 | 7,551 | 11,298 | 13,048 | 13,144 | 14,570 | 14,585 | 14,599 | 14,604 | 14,576 | 14,535 | 14,434 | 14,333 | 14,179 | 13,256 | 13,156 |
| 0.50 | 4,000 | 8,000 | 12,000 | 13,873 | 14,000 | 15,880 | 15,900 | 15,920 | 15,940 | 15,960 | 15,980 | 16,000 | 16,020 | 16,040 | 16,160 | 16,173 |
| 0.45 | 4,220 | 8,449 | 12,702 | 14,698 | 14,856 | 17,190 | 17,215 | 17,241 | 17,276 | 17,344 | 17,425 | 17,566 | 17,707 | 17,901 | 19,064 | 19,190 |
| 0.40 | 4,441 | 8,902 | 13,410 | 15,530 | 15,718 | 18,510 | 18,541 | 18,573 | 18,623 | 18,740 | 18,882 | 19,145 | 19,408 | 19,777 | 21,990 | 22,231 |
| 0.35 | 4,671 | 9,373 | 14,146 | 16,394 | 16,615 | 19,882 | 19,919 | 19,957 | 20,022 | 20,190 | 20,396 | 20,786 | 21,176 | 21,727 | 25,032 | 25,391 |
| 0.25 | 4,913 | 9,869 | 14,920 | 17,305 | 17,559 | 21,328 | 21,370 | 21,415 | 21,496 | 21,717 | 21,990 | 22,513 | 23,037 | 23,780 | 28,235 | 28,720 |
| 0.20 | 5,175 | 10,406 | 15,756 | 18,287 | 18,578 | 22,887 | 22,936 | 22,988 | 23,056 | 23,365 | 23,711 | 24,378 | 25,046 | 25,996 | 31,692 | 32,311 |
| 0.20 | 5,468 | 11,003 | 16,692 | 19,387 | 19,719 | 24,633 | 24,690 | 24,749 | 24,868 | 25,211 | 25,638 | 26,466 | 27,296 | 28,477 | 35,563 | 36,334 |
| 0.15 | 5,806 | 11,694 | 17,774 | 20,658 | 21,036 | 26,650 | 26,715 | 26,784 | 26,925 | 27,342 | 27,863 | 28,878 | 29,894 | 31,343 | 40,034 | 40,797 |
| 0.10 | 6,235 | 12,572 | 19,145 | 22,269 | 22,707 | 29,208 | 29,283 | 29,363 | 29,533 | 30,045 | 30,684 | 31,935 | 33,188 | 34,976 | 45,702 | 46,469 |
| 0.05 | 6,867 | 13,866 | 21,168 | 24,646 | 25,173 | 32,981 | 33,072 | 33,170 | 33,382 | 34,034 | 34,848 | 36,447 | 38,050 | 40,338 | 54,067 | 55,561 |
| 0.00 | 9,229 | 18,698 | 28,719 | 33,520 | 34,376 | 47,068 | 47,217 | 47,378 | 47,749 | 48,921 | 50,390 | 53,290 | 56,196 | 60,353 | 85,292 | 88,705 |

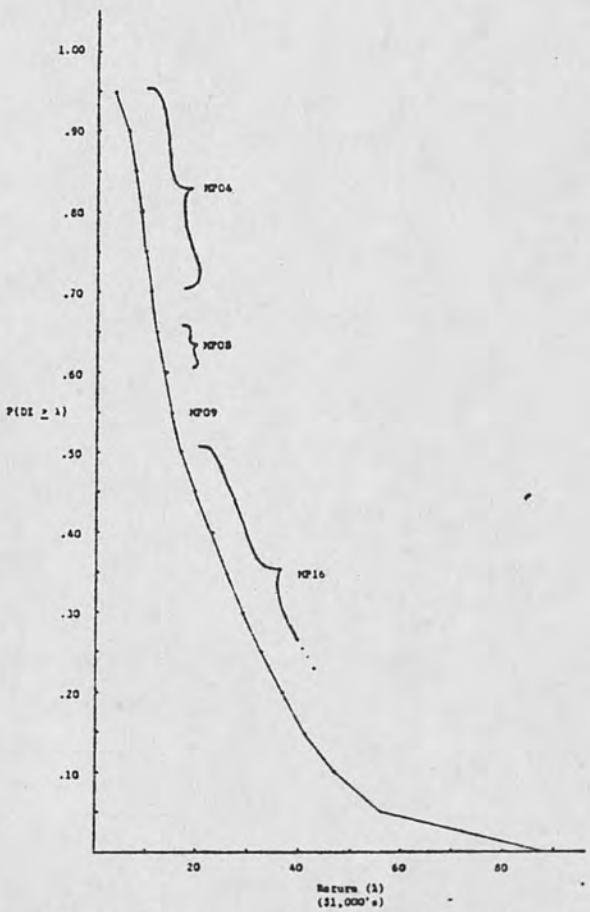


Figure 5. Efficient return-probability frontier using probability distributions of farm plans on efficient E,A frontier.

$$\hat{\mu}_{DI} = \sum_{j=1}^{n} r_j X_j$$

$$\hat{\sigma}_{DI} = A_{DI} \cdot \left[ \frac{s \cdot \pi}{2(s-1)} \right]^{1/2}$$

$$A_{DI} = 2/s \sum_{h=1}^{s} y_h^-$$

$$\sum_{j=1}^{n} (c_{hj} - g_j) X_j + y_h \geq 0 \quad \text{(for all h,h=1,...s)}$$

$$\sum_{j=1}^{n} a_{ij} X_j \leq b_i \quad \text{(for all i,i=1,...,m)}$$

$$X_j, y_h^- \geq 0$$

where: $\lambda$ = maximum return possible subject to probability p of receiving $\lambda$,

$\hat{\mu}_{DI}$ = expected disposable income,

$Z$ = standard normal variable corresponding to probability p,

$\hat{\sigma}_{DI}$ = estimated standard deviation of disposable income ,

$r_j$ = the expected return of activity j,

$X_j$ = the level of activity j,

$A_{DI}$ = absolute deviation of disposable income,

$y_h^-$ = the absolute value of the negative income deviations using observation h,

$c_{hj}$ = the return of activity j associated with observation h,

$g_j$ = the sample mean return of activity j,

$a_{ij}$ = the technical requirement of activity j for resource or constraint i,

$b_i$ = the level of resource or constraint i,

$\pi$ = 3.14286.

The objective function is the return level that can be attained with probability p. By parameterizing p from 0 to 1, a series of solutions of increasing probability and decreasing return is obtained. In this manner the efficient return-probability frontier is generated.

The tableau for the maximization of probability of return model is presented in Table 5. The first six activities and the first seven constraints shown in this tableau are identical to the MOTAD model shown in Table 1. The level of absolute deviation of return will be given in the "absolute deviation" activity. This value is then multiplied by the factor necessary to estimate the standard deviation and transferred to the standard deviation account row. The value of the estimated standard deviation is then given in the "estimated standard deviation" activity. The value of the estimated standard deviation is then multiplied by Z(p) and transferred to the $\lambda$ account row. The coefficient Z(p) is the standard normal variable corresponding to the probability level at which $\lambda$ is to be maximized. As the probability level is parameterized from 0.0 to 1.0, Z(p) is parameterized from 3.999 to -3.999 as shown in Table 3. The value of $\lambda$ is transferred from the $\lambda$ account row to the objective function. Finally, the last six activities shown in Table 5 represent the positive income or negative income that could be attained at each probability level with the farm plan that maximizes $\lambda$ subject to a probability level of p. In other words, these activities describe the cumulative probability distribution function of the farm plan that maximizes $\lambda$.

This model was used to generate an efficient return-probability (R,P) frontier for the same farm situation specified in the MOTAD model. The efficient return-probability frontier is presented in Figure 6. The

Table 5. Tableau of the maximization of probability return model.

| | Row Type | RHS | $X_j$ | Gross Returns | Cost | $TP_h$ | Disposable Income | $\bar{y}_h$ | Absolute Deviation | Estimated Standard Deviation | $\lambda$ | Positive Income p=1.00 | Negative Income p=1.00 | Positive Income p=0.95 | Negative Income p=0.95 ··· | Positive Income p=0.00 | Negative Income p=0.00 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Resource i | L | $b_i$ | $a_{ij}$ | | | | | | | | | | | | | | |
| Gross return account | E | 0 | $-g_j$ | 1 | | | | | | | | | | | | | |
| Cost account | E | 0 | $-\bar{c}_j$ | 1 | | | | | | | | | | | | | |
| Taxable income account | E | 0 | | -1 | 1 | h | | | | | | | | | | | |
| Tax accounting equality | E | 1 | | | | 1 | | | | | | | | | | | |
| Disposable income account | E | 0 | | | | $-DI_h$ | 1 | | | | | | | | | | |
| $t_h$ (h=1,2,...,s) | G | 0 | $d_{hj}$ | | | | | 1 | | | | | | | | | |
| Absolute deviation account | E | 0 | | | | | | -1 | 1 | | | | | | | | |
| Standard deviation account | E | 0 | | | | | | | $-w$ | 1 | | | | | | | |
| $\lambda$ account | L | 0 | | | | | -1 | | | $z(\bar{p})$ | 1 | | | | | | |
| Income probability = 1.00 | E | 0 | | | | | -1 | | | 3.000 | | 1 | -1 | | | | |
| Income probability = 0.95 | E | 0 | | | | | -1 | | | 1.645 | | | | 1 | -1 | | |
| ⋮ ⋮ | ⋮ | ⋮ | | | | | ⋮ | | | ⋮ | | | | | | | |
| Income probability = 0.00 | E | 0 | | | | | -1 | | | -3,000 | | | | | | 1 | -1 |
| Objective function | N | 0 | | | | | | | | | | | | | | | |

1/See footnote 2 of Table 1.
2/See footnote 3 of Table 1.
3/See footnote 4 of Table 1.
4/See footnote 5 of Table 1.
5/ $w = \left[\frac{\pi \cdot s}{2(s-1)}\right]^{1/2}$

farm plans associated with segments of the frontier and the cumulative probability distribution of each farm plan are given in Table 6. There are only three farm plans on the efficient return-probability frontier that utilize all the available land. Farm plan PP02 is the same plan as MP04 on the efficient E,A frontier and specifies producing 450 acres of soybeans. This farm plan generates the highest possible income at probability levels of .70 through .95. Farm plan PP03 has an expected disposable income of $15,936 and specifies producing 2.3 acres of corn and 447.7 acres of soybeans after wheat. This farm plan generates the highest possible income at probability levels of .55 through .65. Farm plan PP04 is identical to farm plan MP16 on the efficient E,A frontier. This farm plan has an expected disposable income of $16,173, specifies producing 450 acres of corn and generates the highest possible income at probability levels of 0.0 through .50.

As with the E,A frontier, the farmer must choose the farm plan from among PP02, PP03, and PP04 that he believes will maximize his utility. However, his choice is now among only three farm plans, rather than among a possible infinite number of farm plans on the efficient E,A frontier. Also, by presenting the information in Table 6 to a farmer and explaining the probability distributions, it seems the farmer would be better able to make the decision than if the information in Table 2 were presented. The farmer is now able to make the decision based on return and probability of return rather than on expected return and an absolute measure of variability of return.

## Summary and Conclusions

Incorporation of risk into farm decision models is not a new concept. Many models have been developed or modified to consider risk in

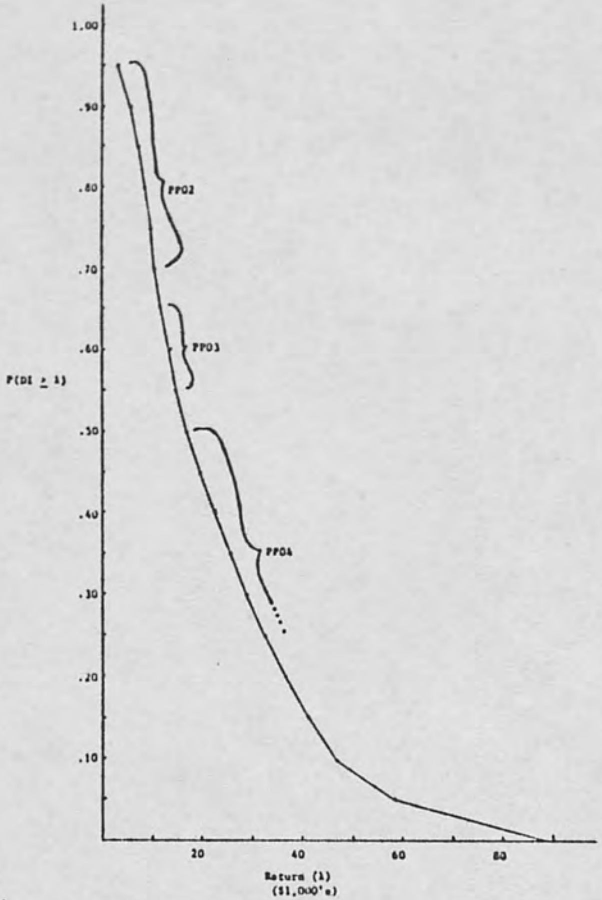


Figure 6. Efficient Return-Probability Frontier.

Table 6. Farm plans and cumulative distribution functions for disposable income of points along efficient return-probability frontier.

| Item | Farm Plan | | | |
|---|---|---|---|---|
| | PPO1 | PPO2 | PPO3 | PPO4 |
| Expected Disposable Income ($) | 0 | 13,873 | 15,936 | 16,173 |
| Absolute Deviation ($) | 0 | 4,887 | 7,853 | 17,869 |
| Estimated Standard Deviation ($) | 0 | 6,548 | 10,523 | 23,944 |
| Production Plan: | | | | |
| Corn (acres) | | | 2.3 | 450.0 |
| Soybeans (acres) | | 450.0 | | |
| Soybeans after wheat (acres) | | | 447.7 | |
| Idle Land (acres) | 450.0 | | | |
| Probability that disposable income will be above given level | | | | |
| 1.00 | 0 | $ -5,772 | $-15,634 | $-55,660 |
| 0.95 | 0 | 3,101 | -1,375 | -23,215 |
| 0.90 | 0 | 5,478 | 2,445 | -14,524 |
| 0.85 | 0 | 7,082 | 5,023 | -8,657 |
| 0.80 | 0 | 8,359 | 7,075 | -3,988 |
| 0.75 | 0 | 9,452 | 8,833 | 11 |
| 0.70 | 0 | 10,441 | 10,422 | 3,626 |
| 0.65 | 0 | 11,352 | 11,884 | 6,954 |
| 0.60 | 0 | 12,216 | 13,273 | 10,115 |
| 0.55 | 0 | 13,048 | 14,610 | 13,156 |
| 0.50 | 0 | 13,873 | 15,936 | 16,173 |
| 0.45 | 0 | 14,698 | 17,262 | 19,190 |
| 0.40 | 0 | 15,529 | 18,598 | 22,231 |
| 0.35 | 0 | 16,394 | 19,987 | 25,392 |
| 0.30 | 0 | 17,304 | 21,450 | 28,720 |
| 0.25 | 0 | 18,293 | 23,039 | 32,336 |
| 0.20 | 0 | 19,386 | 24,796 | 36,334 |
| 0.15 | 0 | 20,663 | 26,848 | 41,003 |
| 0.10 | 0 | 22,267 | 29,426 | 46,870 |
| 0.05 | 0 | 24,644 | 33,246 | 55,562 |
| 0.00 | 0 | 33,517 | 47,505 | 88,006 |

making management decisions. Quadratic programming and MOTAD models are two of the better known and more widely used procedures. Both are based on the concept that a decision maker orders preferences among alternative farm plans only on the basis of expected returns and associated income variance or deviation. Thus, the rational decision-maker restricts choices to those farm plans that have a minimum variance given an expected level of income. In each case a frontier of feasible farm plans is generated from which the farmer/decision-maker chooses according to his particular indifference function for risk. The optimum plan thus depends on a review of a potentially large number of farm plans.

The Maximization of the Probability of Return (MPR) model employs basically the same data as QP and MOTAD models and generates a frontier of feasible returns. However, the MPR model has two major advantages: (1) the number of farm plans to consider in the decision making process is considerably reduced and (2) estimates of the probability of achieving the expected income level generated by a specific farm plan appear to be more understandable by decision makers than income variability. Since the farmer decision-maker can restrict his choice among fewer alternatives, MPR results should be less confusing and permit him to evaluate the probability of the return associated with a farm plan within the dictates of his risk avoidance function. The results of the MPR model appear to be more easily incorporated into extension farm management educational programs and seem to offer a greater potential to transfer research results to ultimate user.

## References


1. Anderson, J.R., J.L. Dillon, and J.B. Hardaker, Agricultural Decision Analysis, The Iowa State University Press, Ames, Iowa, 1977.

2. Chou, Ya-lun, Statistical Analysis with Business and Economic Applications, Holt, Rinehart and Winston, Inc., New York, 1969.

3. Freund, R.J., "The Introduction of Risk into a Programming model," Econometrics, 24(2956):253-263.

4. Ganguly, Sushma S., "A Risk-Return Economic Analysis of Irrigating Field Crops in South Carolina," unpublished Ph.D. dissertation, Clemson University, Clemson, SC 1980.

5. Hazell, P.B.R., "A Linear Alternative to Quandratic and Semivariance Programming for Farm Planning Under Uncertainty." American Journal of Agricultural Economics, 53(1971):53-62.

6. McFarquhar, A.M.M., "Rational Decision Making and Risk in Farm Planning," Journal of Agricultural Economics, 14(1961):552-563.

7. Robinson, B.H., and E.H. Kaiser, "A Return-Risk Analysis of the Decision to Participate in Government Commodity Programs," South Carolina Agricultural Experiment Station Bulletin 634, September 1980.

8. Scott, John T., Jr., and Chester B. Baker, "A Practical Way to Select an Optimum Farm Plan Under Risk," American Journal of Agricultural Economics, 54(1972):657-661.

9. Stovall, John, "Income Variation and Selection of Enterprise," Journal of Farm Economics, 48(1966):1575-1579.

10. Takayama, T., and R.L. Batterham, "Portfolio Selection and Resource Allocation for Financial and Agricultural Firms with the Rand QP360 Quadratic Programming Code," AERR-117, Department of Agricultural Economics, University of Illinois, 1972.

11. Vandeputte, J.M., and C.B. Baker, "Specifying the allocation of Income Taxes, Consumption and Savings in Linear Programming Models," American Journal of Agricultural Economics, 54(1970):521-527.

**SOUTH CAROLINA AGRICULTURAL EXPERIMENT STATION,**
**CLEMSON UNIVERSITY, CLEMSON, SOUTH CAROLINA**
W. C. GODLEY, ASSOCIATE DEAN AND DIRECTOR
S. C. EXPERIMENT STATION
LUTHER P. ANDERSON, DEAN, COLLEGE OF AGRICULTURAL SCIENCES

The South Carolina Agricultural Experiment Station is a cooperative program financed from federal and state funds. It is the policy of the Experiment Station to comply fully with the regulations of Title VI, the Civil Rights Act of 1964. Complaints may be filed with the Director, S. C. Agricultural Experiment Station, Clemson University, Clemson, S. C. 29631.