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This paper presents a dynamic crop rotation model that shows how crop yield and price volatility could impact crop mix and acreage response under crop rotation considerations. Specifically, a discrete Markov decision model is utilized to optimize producers' crop rotation decision within a finite horizon. By maximizing net present value of expected current and future profits, a modified Bellman equation helps develop optimum planting decisions. This model is capable of simulating crop rotations with different lengths and structures. Specifically, the corn-soybeans rotations were simulated using the crop rotation model.

Key words: Crop rotation, Acreage response, Bellman equation.

Introduction

In the United States, crop rotation has been a very popular agricultural practice for many decades. Crop rotation is a practice of planting different crops on the same farm land for sequential seasons. Agronomically speaking, crop rotation could reduce the risk of disease and pest damage while maintaining soil quality for crop growth. In other words, crop rotation is a substitute to some external inputs such as fertilizer or pesticide. For economic considerations, crop rotation helps reduce input costs and improve soil productivity, therefore increasing expected profit which dominates acreage response. Crop rotation benefit is believed to be induced by the agronomic interrelationship between different field crops. A prevalent example is the corn-soybeans rotation, where soybeans provide a key nutrient for corn growth. Furthermore, crop rotation also helps reduce greenhouse gas emission, since it is a substitute for nitrogen fertilizer. Overall, crop rotation can maintains or improve crop yield by controlling for disease and pests and promoting soil nutrients.

Agricultural producers' acreage response is an important determinant of agricultural supply. Acreage response is largely constrained by crop rotation considerations. Switching from a crop rotation scheme to continuous cropping to take advantage of high crop prices could make farmers worse off in the long run since yield loss due to continuous cropping could decrease profit. For example, in recent years many producers have allocated more acreage to corn planting in response to the corn price boom due to ethanol demand. Even though an immediate short run profit could be gained in some cases, the gain in corn price might not be able to offset the yield loss from continuous cropping in the long run. Therefore, crop choice and acreage response are complex decisions with both agronomic and economic considerations. Without considering the effects of crop rotations on long-term crop yields and profit, producers' planting decision models may be misspecified and misinformed. An interesting research question is: Considering crop rotation effects, how will a profit-maximizing producer's acreage response be altered by crop price volatility? This research question is expected to be of current interest because of increased crop price volatility in recent years.

Literature Review

Crop rotation has been of great interests to both agricultural producers and policy makers for many decades. This topic has also been intensively investigated by researchers. Crop rotation studies generally focus on two major categories: agronomic and economic modeling.

Agronomists concentrate on estimating yield response to crop rotation, and sometimes, the tradeoff between yield response and external inputs. Agronomists conduct these studies by controlling external factors such as soil type, fertilizer level and some other agronomic factors. The agronomic literature generally indicates that crop rotation practices could enhance crop yield while reducing input demand. Therefore, crops grown during last season could alter this season's crop yield and input demand depending on if producers decide to stay with a rotation scheme or skip it. Johnson et al. (1998) estimated that cotton and peanut yields from the cotton-peanut rotation were 26% and 10% greater, respectively, than those from monoculture over a 7-year study in Georgia. In an agronomic study based in Michigan, Roberts and Swinton (1995) demonstrated that crop rotation could increase corn yields by 16 percent comparing to continuous cropping. Vyn (2006) reported that in Indiana, corn-soybeans rotation enhanced corn yields by about 6%. Overall, yield response results vary across almost all agronomic studies.

Disagreement of agronomic results indicates that crop rotation is largely affected by various external factors such as soil type and fertilizer input, therefore increasing the difficulty of developing an economic crop rotation model.

Acreage response is largely constrained by crop rotation considerations. Expected profitability will be altered by crop rotation effects of reducing input demand and improving productivity. However, crop rotation effects were surprisingly omitted by most previous acreage response studies.

Even for acreage response models considering crop rotation, crop rotation was usually used as an additional variable to help estimate acreage response. For example, many researchers incorporate a lagged acreage variable in the econometric acreage response model trying to represent the effects of crop rotation (Bewley, Young, and Colman 1987; Weersink, Cabas, and Olale 2010). This lagged acreage variable only captures rotational constraints, while the mechanism of the crop rotation effects to acreage response behavior was not represented, such as how producers' acreage responses were dynamically altered by price and yield volatility under crop rotation considerations. The reason for this inactive incorporation of crop rotation into acreage response studies is believed to be the lack of a mature economic structural model of crop rotation. Without a usable and correct crop rotation model, it is hard for researchers to incorporate these effects into an acreage response study correctly.

Some researchers have incorporated dynamic considerations into crop rotation and acreage response models. Orazem and Miranowski (1994) estimated a dynamic model to study price effects on acreage response. The effects of current crop choice on future soil productivity were also considered. However, this research focused on how future prices affected current acreage allocations. Dynamically speaking, there is indeed a connection between future prices and current acreage allocations. However, following most economic models of crop rotation in the literature, we argue that previous acreage allocations and current prices should dominate current acreage allocations.

In general, previous studies have not adequately incorporated crop rotation into acreage response models. The reason for this gap in the literature, we believe, is that previous studies have lacked a structural model of crop rotation based on economic theory. Without a usable and theoretically-correct crop rotation model, it is hard for researchers to effectively incorporate crop rotation into acreage response models.

Economic studies of crop rotation are relatively limited compared to agronomic studies. Economic studies may be more limited because of the complexity of crop rotation effects which include interconnections between various factors. Furthermore, many effects of rotations are not completely understood by agronomists. Many economic techniques have been applied for crop rotation modeling. Among various economic modeling approached for crop rotation, linear programming has been one of the most prevalent approaches.

An early study of crop rotation using linear programming was conducted by El-Naze and McCarl (1986). The major contribution of their research is allowing the model to determine freely the optimal long run rotation while most other researchers modeled predetermined rotations. Multiple year crop rotations were modeled using an annual equilibrium linear programming. It assumes sequential crops planting on the same land for continuous seasons. However, most producers actually plant all crops in crop rotation simultaneously in the same season with the purpose of reducing production risk and balancing labor load.

Detlefsen (2004) modeled crop rotation with network modeling. Detlefsen’s model provides a visual representation of the crop rotation problem. While it shows an alternative to previous linear programming approaches with certain advantages, it is still limited in only optimizing a one year return.

Hennessy (2006) developed a crop rotation economic model to analyze and separate the interconnected crop rotation effects of yield-enhancement and input-saving carry-over effects. The model was developed by considering both one-year rotation effects and multi-year rotation effects. However, this model does not consider how producers' sequential decision making will be altered by crop rotation effects. Also, Hennessy's model focuses on choosing among rotations instead of allocating acreage to crops within one rotation. Switching between rotations has higher input costs; therefore it is unrealistic for most small producers.

Livingston, Roberts and Rust (2009) examined crop choice as a dynamic optimization problem over an infinite time horizon. Their work is believed to be the first in the literature to consider crop rotations in a dynamic optimization framework. A simple crop rotation model was developed to analyze farmers' response to expected revenue given crop rotation considerations. However, only the simple corn-soybeans rotation was modeled. The whole model was based on one field grown either in corn or soybeans for sequential seasons. In a real farm, producers would prefer to grow all crops in rotation simultaneously which helps to reduce production risk. The situation with both crops planted is more complicated. Another limitation of their model is that it is calibrated by specific agronomic data from Northeast Iowa. The model they develop is most salient to that region of the country and nearby regions with similar soils and climate. It is not apparent that their model could be easily applied to other regions with different external environments. The final results were the optimal choice of crops given previous crops grown and current fertilizer use. This result provides useful decision rules for corn-belt farmers trying to decide between planting corn or soybeans in any given year, however, a multi-period decision analysis was not delivered.

Methodology

Economic analysis of crop rotation schemes plays a dominant role in acreage response studies. Various approaches developed in recent decades have broadly expanded people's knowledge about economic modeling of crop rotation. However, due to the complexity of crop rotation systems, economic models generally have various limitations, and therefore it is difficult to utilize these models in actual case studies. This study attempts to contribute to the literature by providing a dynamic optimization crop rotation model with a general structure. This model was designed to have minimum agronomic restrictions, such as soil type, yield response, and previous crops grown so that future research could easily adjust the model for use on any crop rotation system with various external environments. It also considers multi-year rotation carry-over effects which were barely addressed in previous studies.

To our knowledge, no literature exists pertaining to crop rotation structural modeling incorporating a Bellman equation to maximize net present value of returns. Therefore, in this study, we focus on the overall research question: What is the optimal cropping plan over multiple periods considering the economics of crop rotation in a dynamic framework?

In the remainder of this paper, we will first develop a dynamic theoretical model with one-year carry-over effects. This model will then be extended to include two-year carry-over effects, followed by a case study with application to the corn-soybeans rotation.

In our model, we study three types of rotation systems. A-B denotes the rotation with crop A and crop B repeatedly planted after each other on the same farm land for sequential seasons. A-A-B denotes the rotation with repeated schemes of crop A planted for two seasons and crop B planted for one season on the same farm land. A-B-C denotes the rotation with repeated schemes of crop A planted during the first season, crop B planted during the second season, crop C planted during the third season on the same farm land. Agricultural producers are assumed to be price-takers and profit-maximizers. Considering crop rotation effects on yield response, producers intend to maximize net present value of returns for an infinite horizon by allocating crop acreage for each season.

In this study, the discrete time and discrete state Markov decision model is modified to simulate the crop rotation optimization process. The original Markov decision model has the following structure: in every period t, an agent observes an economic state s_t, takes an action x_t, and earns a reward $f(s_t, x_t)$ which depends on both the state of the system and the action (Miranda and Fackler 2002). This process could be converted into the crop rotation process as follows. In the beginning of a planting season, a producer observes the crops planted on the land during last season and decides which crops to plant on the same land for the current season. Producers are making discrete decisions assuming that each field could only plant one type of crop. The expected crop yield depends on both the previous planting state and current planting decision. For example, if corn and soybeans were each planted on two equally sized farm land tracts during last season, and a producer decides to follow the corn-soybeans rotation by flipping the crop planted on the two tracts, then expected corn yields could be maintained at the original level. However, if the producer decides to plant corn on both tracts due to increased corn price, one of the expected corn yields will be reduced due to continuous cropping (see figure 1).

Expected input and output prices are assumed to be exogenous. Even expected yields are assumed to be exogenous; only the yield response level (the yield under continuous cropping compared to the yield under rotation) is assumed endogenous to current and previous crop choices. Since crop rotation practices are adopted by most producers, we assume that exogenous expected yields are the yield under rotation. The yield under continuous cropping will depend on the yield response level.

The discrete Markov decision model used in this study is analyzed using the dynamic programming methods developed by Richard Bellman (Bellman 1957). The Bellman equation helps to optimize sequential decisions to balance an immediate reward against expected future rewards. With a finite horizon, the Bellman equation is written as:

$$(1) \qquad \boldsymbol{V_t(s) = \max_{x \in X(s)} \{ f(s,x) + \delta \textstyle\sum_{s' \in S} P(s'|s,x) V_{t+1}(s') \}, \; s \in S, t = 1, 2, \ldots, T}$$

where $V_t(s)$ is the maximum attainable sum of current and expected future rewards, given that the system is in state s in period t, x is the control variable. $f(s,x)$ is the immediate reward:

$$(2) \qquad \boldsymbol{\delta \textstyle\sum_{s' \in S} P(s'|s,x) V_{t+1}(s')}$$

is the expected future reward.

The objective function for the producer is maximizing the sum of current and expected future farm returns considering the crop rotation for *T* years. It is also assumed that the current season's crop yield will be known with certainty once both the last and current season's planting decisions are known. Therefore, the crop rotation is considered to be a finite horizon, deterministic problem in this study.

The producer makes planting decisions by looking at the crops planted during last season; therefore, we take crop yield at time *t-1* as the state variable at time *t*. This state variable includes both the crop choice and crop yield. We assume that the yield response level during last season has no impact on this season's yield response level, only the crop choice matters. To be specific, the actual state variable in this model is the profit where exogenous input and output prices are included. To simplify the notation, we say that the combined crop choice and the yield response level is our state variable:

(3) $$\boldsymbol{y_{t-1}} \in \{\boldsymbol{y}, \boldsymbol{ym}\}$$

where y denotes the yield of crop y under crop rotation, and ym denotes the yield of crop y reduced yield under continuous cropping.

We assume that the producer plants alternative crops simultaneously during the same season and switch crops for the next season. Therefore, the size of state space varies according to rotation length. For a rotation with two crops such as A-B, the number of elements in the state space is nine which includes all possible combinations of yield and reduced yield for crop A and crop B. AM-BM is not considered as an element of the state space for rotation A-B. AM-BM indicates that both A and B are harvested with reduced yield due to continuous cropping, so the crops planted for the last season must be A and B. While both crop A and crop B were planted for two sequential seasons, a rational producer will switch the lands for A and B and obtain crop rotation yield A-B, but not continuous cropping yield AM-BM. Therefore, AM-BM is not a possible yield scenario, thus:

(4) $$\boldsymbol{y_{t-1}} \in (\mathbf{A-B}, \mathbf{A-BM}, \mathbf{AM-B}, \mathbf{A-A}, \mathbf{A-AM}, \mathbf{AM-AM}, \mathbf{B-B}, \mathbf{B-BM}, \mathbf{BM-BM})$$

For crop rotations with longer length or more crops, the number of elements in the state space will be more. A-A-B has 16 elements and A-B-C has 100 elements in their state spaces. The control variable is:

(5) $$\boldsymbol{x} \in \{\boldsymbol{A}, \boldsymbol{B}, \dots, \boldsymbol{N}\}$$

where $A, B, \dots, N$ denotes alternative crops in a crop rotation scheme.

Based on the state variable and the control variable denoted above, the modified Bellman equation for crop rotation could be written as:

$$(6) \qquad \boldsymbol{V_t(y_{t-1}) = \max_{x \in X(\pi(y_{t-1}))}\{\pi(y_{t-1}, x) + \delta V_{t+1}(g(y_{t-1}, x))\}, y_{t-1} \in Y, t = 1, 2, \dots,}$$

where $V_t(y_{t-1})$ is the maximum attainable sum of current and expected futures farm returns, given that system is in state y_{t-1}in period t, x is the crop choice for the current season, $\pi(y_{t-1}, x)$ is the current season farm return, and $\delta V_{t+1}(g(y_{t-1}, x))$ is the expected future farm returns.

The state transition function $g(y_{t-1}, x)$ denotes how the current state y_{t-1} transits in the state space based on the current season crop choice x. $g(y_{t-1}, x)$ in this model could be better understood by visually inspecting figure 2. Again, the simplest crop rotation A-B was chosen to demonstrate the state transition process for this model.

Each of the two crops is planted on two tracts denoted by cells. The left column denotes the current state, which is the crop choice and yield response level during the last season. The right column denotes the next state depending on the current planting decision. Lines connecting the two columns denote planting decisions. Solid lines denote crop rotations, long dash lines denote growing crop A on both tracts, long dash dot-dot lines represent growing crop B on both tracts. This figure illustrates how the state variables (crop choice and yield response level during last season) transit with the control variables (planting decisions).

Figure 2 visually demonstrates the state transition function for the simplest crop rotation A-B. The previous crop choice and yield response level transits to the specific current yield response level, depending on the current planting decision. At the beginning of the current season, a producer considers crops planted during the last season, making the choice between three alternative planting decisions: planting A on both tracts, planting B on both tracts or planting both A and B on both tracts.

In order to maximize the net present value of the return, the producer optimizes his or her planting decision based on the crop planted during last season and this season's expected yield. For example, row 2 of the left column means both A and B were planted during the last season, while A was harvested with rotational yield, and B was harvested with continuous yield. If the producer decides to plant A on both tracts during the current season, the expected current yield level will transit to row 5 on the right column where A was harvested with rotational yield on one tract and continuous yield on the other tract.

The state transition function, the reward function and the Bellman equation for the A-B rotation are listed in the Appendix. The above illustration of state transition could also extend to other crop rotation types such as A-A-B and A-B-C. As alternative crops in the crop rotation increases, the number of elements in the state space also largely increases. Compared to nine elements for the A-B rotation, the A-A-B rotation has 16 elements and the A-B-C rotation has 40 elements in their state spaces. Their structure figure, the state transition function, the reward function and the Bellman equations are listed in the Appendix as well.

It should be noted that the above dynamic optimization models derived for A-B, A-A-B and A-B-C rotations have one strong assumption: the crop yield response level at time *t* only depends on the crop planted at time *t-1* and the planting decision at time *t*. However, this assumption is unlikely to be valid for some crops, for which the crop yield response level depends on crops planted at both time *t-1* and time *t-2* and the planting decision at time *t*.

Therefore, we extend the previous model by considering the last two crops grown instead of just the last crop grown.

The control variable for the new model is still the current crop choice. The state variable now changes from last season's crop choice and yield response level to the same two variables for the last two seasons. As mentioned earlier, the yield response level is still not completely understood by agronomists. Although agronomic yield response level results are available in many previous studies, their values vary by area, by crop and some other unknown agronomic factors. In the model only considering last season's impact, the crop planted in the same land for two sequential seasons could be categorized into either the same crop or a different crop for the A-B rotation.

For the model considering the last two seasons' impact, the crop on the same land for three sequential seasons could be categorized into four scenarios: A-B-A, B-A-A, A-A-A, B-B-A (assuming crop A will be planted for the current season). The yield response level for A is believed to be different for all these four scenarios. However, we are not able to value these four yield response levels due to the lack of agronomic evidence. We will assign different appropriate values to these yield response levels for the model simulation.

Since the state variable is now more complicated, the number of elements in the state space also greatly increased. Take the A-B rotation as an example. There were nine elements in the state space for the old model only, while there will be 27 elements in the state space for the new model. Specifically, there are nine different states by crops, and each crop has three yield response level scenarios, combining into a total of 27 states. For example, the crop state A-B|A-B could come from three possible previous states: A-B|A-B, A-A|A-B, and B-B|A-B. Therefore, given the fact that A and B are planted during this season, there are three possible yield response level scenarios depending on previous states. The state transition figure for the A-B rotation with two-season effects is illustrated in figure 3.

Furthermore, the same approach can be applied to other crops rotation types such as A-A-B and A-B-C to extend the model. The number of elements in the state space also greatly increases. It could be summarized that for each rotation type, the number of elements in the state space for the last season is the square of their possible crop combinations, for the last two seasons it is the cubic of their possible crop combinations. A-B has three crop combinations: A-B, A-A and B-B. A-A-B has four combinations: A-A-B, B-B-A, A-A-A and B-B-B. A-B-C has ten combinations: A-B-C, A-A-B, B-B-A, A-A-C, C-C-A, B-B-C, C-C-B, A-A-A, B-B-B and C-C-C. Compared to 27 elements for the A-B rotation, the A-A-B rotation has 64 elements and the A-B-C rotation has 1,000 elements in their state spaces (see Table 1). For simplicity, their transition functions, reward functions, and modified Bellman equations will not be demonstrated.

We extend the crop rotation model with one-season effects to two-season effects (if desired, we also could extend the model to three-season effects or even longer). However, we argue that the crops planted at three seasons earlier have insignificant effects on current crop yields. Thus, we only derive the model considering two-season effects in this study.

Assumptions

As an initial assessment to apply the Bellman equation on acreage response considering crop rotation, several major assumptions have been made in the economic models derived in this study.

This model does not presume any soil types or natural factors that could affect yield response levels. It is designed to be able to apply to various external conditions. Expected yield and expected input and output prices are all exogenous, while yield response levels are endogenous. We assume producers to be price takers. Therefore, their response in acreage will not cause dynamic price responses.

It is assumed that each phase of a rotation system is grown every year. For example, the A-B rotation means producers grows both crop A and crop B in the same season. If the rotation continues the following season, a producer will flip the farm land tracts planted for crop A and crop B. Also, A-A-B means the producer plants crop A on two tracts and crop B on one tract. A-B-C means the producer simultaneously plants crop A, B and C on three different tracts in the same season. This assumption simulates the real situation on most farms. Also, this assumption helps separate the effects of the rotation system on yields from that of variable weather factors.

Two types of dynamic crop rotation models were developed with each assuming the number of previous seasons that could affect this season's yield response level. For the first model, we assume that only last season's crop could impact this season's yield response. As mentioned earlier, that could be false for some crops. Therefore, we develop the second model where we assume that last two seasons' crop could impact this season's yield response.

We assume that the producer uses constant external inputs such as fertilizer and pesticides for different seasons. In reality, farmers could do crop rotations while adjusting external inputs simultaneously in order to maximize returns. Continuous cropping yields could be made similar to yields of rotational crops if producers upgrade inputs such as fertilizer. Crop rotations could thus either improve yields with fixed input, or save inputs with fixed yields. However, the interconnections between fertilizer inputs and yields with crop rotation are relatively complex. As a first attempt to incorporate crop rotation process into the Bellman equation, external inputs were fixed for simplicity. That is, the producer will not change inputs after switching from rotating crops to continuous cropping.

It is assumed that certain tracts of farm land are only used to plant certain crop rotations or continuous crops for that particular rotation. Other crops will not be planted on these tracts. We also assume there is no land use change and producers will not introduce new crop varieties into the system. It is not necessary to use percentage share to represents a producer's acreage response, since his or her response occurs plot by plot.

Simulation

MATLAB was used to simulate the dynamic crop rotation model developed in this study. MATLAB utilizes the CompEcon toolbox to solve for discrete time/discrete variable dynamic programming problem (Fackler 2010). Given the terminal value of $V_{t+1}(g(y_{t-1}, x))$, the decision is solved recursively by repeated application of the Bellman equation. MATLAB compares the value of $V_{t+1}(g(y_{t-1}, x))$ for each time *t*, and provides the optimal decision for each period.

The value for each $V_{t+1}(g(y_{t-1}, x))$ includes current and discounted future rewards. The current reward for each period is a producers' immediate profit:

$$\boldsymbol{\pi}_{\boldsymbol{it(d)}} = \sum_{\boldsymbol{i=1}}^{\boldsymbol{N}} (\boldsymbol{P}_{\boldsymbol{it}} \boldsymbol{Y}_{\boldsymbol{it(d)}} - \boldsymbol{C}_{\boldsymbol{it}}) \tag{7}$$

The above profit function is the profit summation for crops planted under planting decision *d.* Take corn-soybeans rotation as an example. We assume previous crops planted on two farm land tracts were corn and soybeans. There will be three possible decisions *d* for the current season: keeping a rotation system, planting all corn or planting all soybeans. If keeping a rotation system is decided, the current expected profit will be corn profit and soybeans profit, both with rotation yields. If growing all corn is decided, the current expected profit will be corn profit with rotation yields and corn profit with continuous yields. If planting all soybeans is decided, the current expected profit will be soybean profit with rotation yields and soybeans profit with continuous yields. MATLAB will then compare three profit bundles and pick the one with the highest value as the optimal decision for period *t.* However, this will be true only for the last period T where there is no future reward. For any other period *t,* MATLAB compares three profit bundles with each adding their future rewards given by the Bellman equation value at period *t.*

The models developed above were simulated on corn-soybeans for the A-B structure. Specifically, both the one-season effects and two-season effects models were simulated. Yield response levels are summarized from previous empirical studies. The corn-soybeans rotation yield response level is retrieved from a compilation of all known published data comparing corn after corn to a corn-soybeans rotation in the U.S. by Erickson (2008). We simply take the average of all data compiled by Erickson (2008) which is 7.8%, meaning that the continuous corn yield is on average 7.8% lower than the corn rotation yield. The continuous soybeans yield response is 14.5 % lower than the soybeans rotation yield. Since most producers use crop rotation systems, we assume that expected yields are the equal to the rotation yields. Continuous yields are discounted based on this assumption.

The expected input and output prices and expected yields are all retrieved from USDA ten-year agricultural projections. We simulated the individual producers' planting decisions under USDA projections of prices and yields. The producers are assumed to be profit-maximizers and price-takers. It is assumed that the producer owns two equally sized farm land tracts. At the beginning of each period, the producer decides which crop to plant on each cropland based on price expectations, the crop planted last season and related yield expectations.

Based on current USDA projections for the next five years, the producers will plant corn for all tracts. As long as the USDA corn price projections are higher than 98% of the current level, producers produce all corn. The upper and lower bound of corn price percentage changes for all crop rotations are -12% and -16%. If corn prices decreased by over 30%, then producers will not rotate crops and instead grow all soybeans.

Now we run another A-B model simulation with two previous crops considered. Compared to the A-B model only considering the last crop, the yield response level is more complicated. We need to decide the yield response of crop A after A-B for last two periods, or after B-A, A-A, B-B. There are four yield response levels for crop A given different crops combinations for the last two periods which is the same for crop B. To the best of our knowledge, agronomic results for these complicated yield response levels are not available. We therefore make several assumptions. We assume crop A after B-B has the full yield, crop A after A-B has a 5% reduction in yield, crop A after B-A has a 10% reduction in yield, crop A after A-A has a 15% reduction in yield. The same assumption was made for crop B. We use USDA yield projections for the next five years again. As long as USDA corn price projections are higher than 108% of current level, producers produce all corn. If corn prices decreased by over 30%, then producers will not rotate crops and instead grow all soybeans. No level of corn price change can be found for pure crop rotation practices.

Conclusion

In this study, a dynamic crop rotation model was developed to connect expected profit to acreage response. Specifically, a modified Bellman equation was used for dynamic optimization, and the crop rotation model is actually a part of its transition function.

The crop rotation model was developed for both the one-season effects and two-season effects. The simulation results indicate that by considering the one-season effects, continuous corn cropping is the optimized choice. For the two-season effects, corn-soybeans rotation is the optimized choice. These results indicate that two-season effects are more stable and producers should prefer to choose a mixed cropping scheme.

The complexity of interactions is inherent in a crop rotation system. This crop rotation ignored the interactions between crop yield and fertilizer usage by using empirical yield responses. Future research could improve this model by including fertilizer usage. Furthermore, while it is commonly agreed that rotational effects varied by region, the effect of differences in soil types and other natural factors were not considered. Again, an improvement of this crop rotation model should allow the input of soil types and other natural factors.

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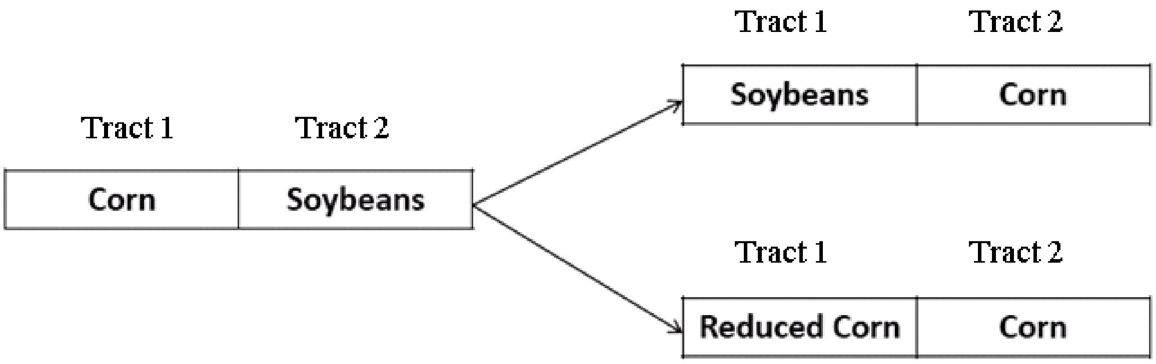

Figure 1. Two current planting scenarios based on previous crops planted

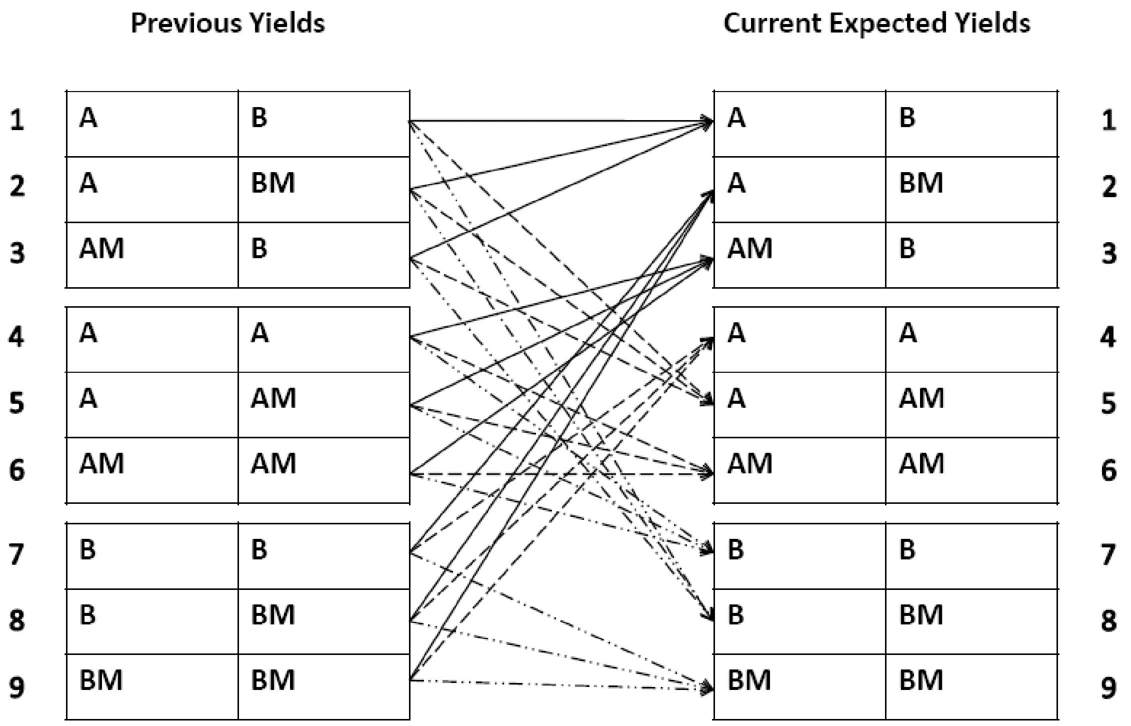

Figure 2. Illustration of the transition function of A-B rotation

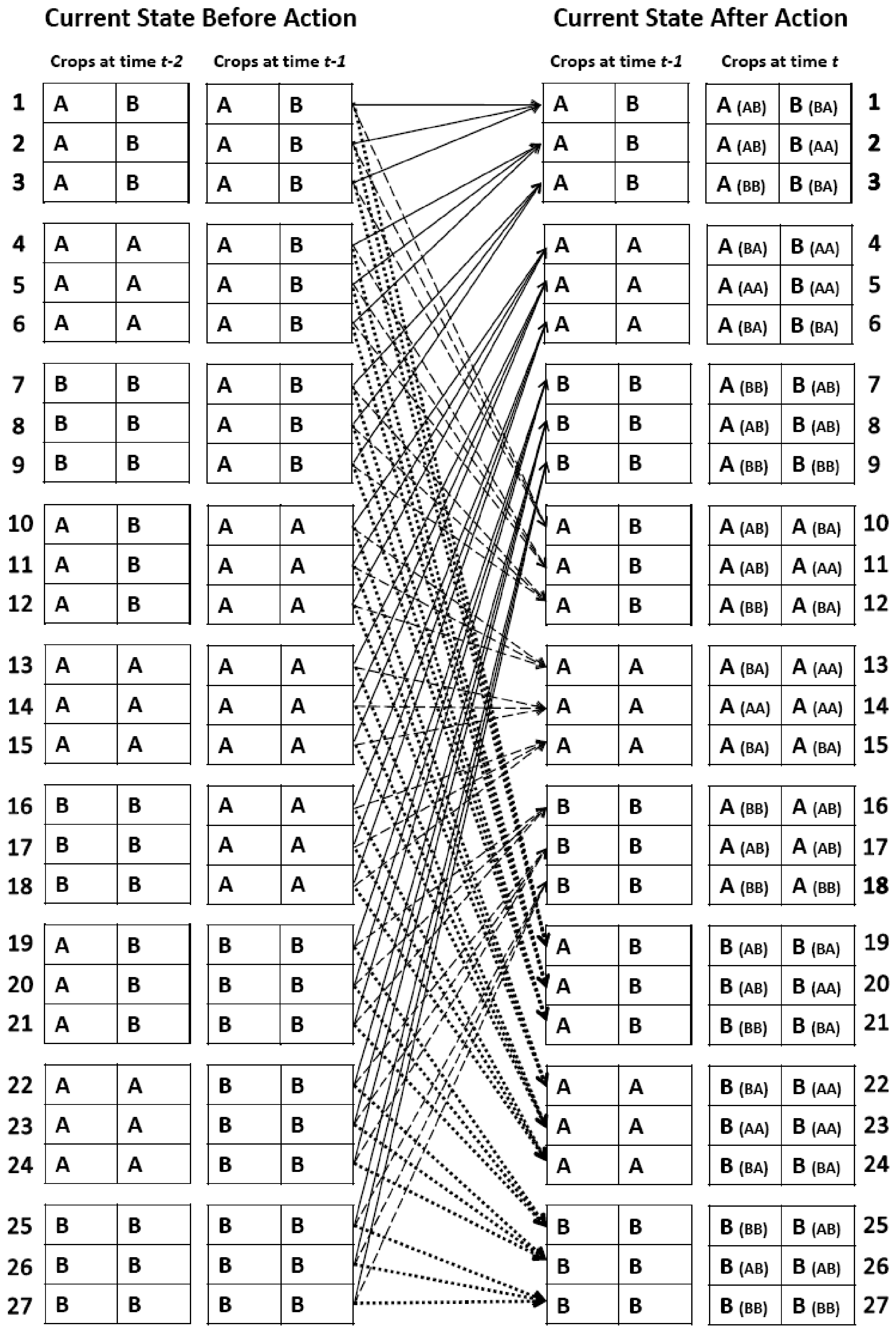

Figure 3. A-B rotation with two-season effects

Table 1. The Number of Elements in the State Spaces for Different Rotations

	Last Season			Last two seasons		
Rotation Type	A-B	A-A-B	A-B-C	A-B	A-A-B	A-B-C
No. of elements in the state space	9	16	100	27	64	1000

Table 2. Break-Even Corn Price Range for different crop mixes

	Corn-Corn	Corn-Soybeans	Soybeans-Soybeans
One-season effect	c > 98%	88%>c>84%	c < 70%
Two-season effect	c > 108%	N/A	c < 70%

Note: c is the percentage of USDA corn price projections

Appendix

$\boldsymbol{The\ Reward\ Functions}$:

$The\ state\ variable\quad Y \in \{C|S, C|SM, CM|S, C|C, C|CM, CM|CM, S|S, S|SM, SM|SM\},$

$t \in \{1,2,3,\dots,n\}$

$The\ action\ variable\quad x \in \{crop\ rotation, all\ corn, all\ soybeans\}$

$$f(Y_{t-1}, x)$$

$$= \begin{cases} (P_c - C_c)Y_c + (P_s - C_s)Y_s\,, & if\ Y_{t-1} = (C|S, or\ C|SM, or\ CM|S)\ and\ x = rotation; \\ (P_c - C_c)Y_c + (P_c - C_c)Y_{cm}\,, & if\ Y_{t-1} = (C|S, or\ C|SM, or\ CM|S)\ and\ x = corn; \\ (P_s - C_s)Y_s + (P_s - C_s)Y_{sm}\,, & if\ Y_{t-1} = (C|S, or\ C|SM, or\ CM|S)\ and\ x = soybeans; \end{cases}$$

$$f(Y_{t-1}, x)$$

$$= \begin{cases} (P_c - C_c)Y_{cm} + (P_s - C_s)Y_s\,, & if\ Y_{t-1} = (C|C, or\ C|CM, or\ CM|CM)\ and\ x = rotation; \\ (P_c - C_c)Y_{cm} + (P_c - C_c)Y_{cm}\,, & if\ Y_{t-1} = (C|C, or\ C|CM, or\ CM|CM) and\ x = corn\ ; \\ (P_s - C_s)Y_s + (P_s - C_s)Y_s\,, & if\ Y_{t-1} = (C|C, or\ C|CM, or\ CM|CM) and\ x = soybeans; \end{cases}$$

$$f(Y_{t-1}, x)$$

$$= \begin{cases} (P_c - C_c)Y_c + (P_s - C_s)Y_{sm}\,, & if\ Y_{t-1} = (S|S, or\ S|SM, or\ SM|SM) and\ x = rotation; \\ (P_c - C_c)Y_c + (P_c - C_c)Y_c\,, & if\ Y_{t-1} = (S|S, or\ S|SM, or\ SM|SM) and\ x = corn; \\ (P_s - C_s)Y_{sm} + (P_s - C_s)Y_{sm}\,, & if\ Y_{t-1} = (S|S, or\ S|SM, or\ SM|SM) and\ x = soybeans; \end{cases}$$

$\boldsymbol{The\ Transition\ Functions}$:

$$g(Y_t, x) = \begin{cases} 9t + 1, & if\ Y_t = (C|S, or\ C|SM, or\ CM|S)\ and\ x = rotation; \\ 9t + 5, & if\ Y_t = (C|S, or\ C|SM, or\ CM|S)\ and\ x = corn; \\ 9t + 8, & if\ Y_t = (C|S, or\ C|SM, or\ CM|S)\ and\ x = soybeans; \end{cases}$$

$$g(Y_t, x) = \begin{cases} 9t + 3, & if\ Y_t = (C|C, or\ C|CM, or\ CM|CM)\ and\ x = rotation; \\ 9t + 6, & if\ Y_t = (C|C, or\ C|CM, or\ CM|CM)\ and\ x = corn; \\ 9t + 7, & if\ Y_t = (C|C, or\ C|CM, or\ CM|CM)\ and\ x = soybeans; \end{cases}$$

$$g(Y_t, x) = \begin{cases} 9t + 3, & if\ Y_t = (S|S, or\ S|SM, or\ SM|SM)\ and\ x = rotation; \\ 9t + 4, & if\ Y_t = (S|S, or\ S|SM, or\ SM|SM)\ and\ x = corn; \\ 9t + 9, & if\ Y_t = (S|S, or\ S|SM, or\ SM|SM)\ and\ x = soybeans; \end{cases}$$

The Bellman Equation:

$V(Y_{t-1}) = \max\{(P_c - C_c)Y_c + (P_s - C_s)Y_s + \delta V(9t + 1),\ (P_c - C_c)Y_c + (P_c - C_c)Y_{cm} +$

$\delta V(9t + 5)$,

$(P_s - C_s)Y_s + (P_s - C_s)Y_{sm} + \delta V(9t + 8)\}, \qquad if\ Y_{t-1} = (C|S, or\ C|SM, or\ CM|S)$

$V(Y_{t-1}) = \max\{(P_c - C_c)Y_{cm} + (P_s - C_s)Y_s + \delta V(9t + 1),\ (P_c - C_c)Y_{cm} + (P_c - C_c)Y_{cm} +$

$\delta V(9t + 5)$,

$(P_s - C_s)Y_s + (P_s - C_s)Y_s + \delta V(9t + 8)\}, \qquad if\ Y_{t-1} = (C|C, or\ C|CM, or\ CM|CM)$

$V(Y_{t-1}) = \max\{(P_c - C_c)Y_c + (P_s - C_s)Y_{sm} + \delta V(9t + 1),\ (P_c - C_c)Y_c + (P_c - C_c)Y_c +$

$\delta V(9t + 5)$,

$(P_s - C_s)Y_{sm} + (P_s - C_s)Y_{sm} + \delta V(9t + 8)\}, \qquad if\ Y_{t-1} = (S|S, or\ S|SM, or\ SM|SM)$

Note: C-Corn S-Soybeans CM-Corn with reduced yield SM-Soybeans with reduced yield

$P_c - Corn\ price \quad P_s - Soybeans\ price \quad Y_c - Corn\ yield \quad Y_s - Soybeans\ yield$

$C_c - Production\ cost\ of\ Corn \quad C_s - Production\ cost\ of\ soyneans$